Modelling hurricane track memory

Problem presented by

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Executive Summary

It has been observed that hurricanes that are close in time often follow similar paths. If this can be shown to be statistically significant, it could have implications for how insurance premiums are calculated in areas of the US prone to hurricanes.

We developed two independent path distance metrics and while one suggested that sequential storms within a given hurricane season are more likely to follow each other than any other pair of storms within that season, this conclusion was not supported by the other metric.

Some considerations of how local and large scale air pressure gradients might affect hurricane paths were considered. A point vortex model in the presence of a steering flow field was developed and used to simulate the path of two time displaced vortices. In order for the vortices to follow each other they had to be relatively weak compared to the steering flow field. At realistic vortex strength, the trajectories became chaotic.

In summary, our metrics provided conflicting evidence towards the notion of hurricane track memory. A large-scale steering flow field did not appear to provide sufficient explanation for hurricanes following each other, though this does not preclude hurricane track memory being due to localised physical changes following a large storm.

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1 Introduction

1.1 Background

(1.1.1) The hurricane seasons of 2004 and 2005 led to extreme losses for the (re)insurance industry. It appeared as though certain large scale atmospheric structures (such as the 'Bermuda High') were steering storms into the US. If, under some conditions, hurricane tracks are conditional on either previous hurricanes, or other climate variables this could be very significant for the insurance industry. It would suggest that storms may cluster which would lead to a larger variance in financial results than is typically modelled. If the conditions for clustering are expected to either be more or less prevalent under a climate changing world again this is very significant to the insurance industry over the coming decades.

(1.1.2) US landfalling hurricanes result in huge losses to the insurance industry. For instance, Katrina caused USD 45bn worth of damage in 2005 and in the same year, Rita caused USD 6bn and Wilma USD 11bn worth. In 1992, a single hurricane (Andrew) caused USD 22bn and in 2004 a collection of hurricanes caused USD 25bn worth of damage.

(1.1.3) The insurance industry typically models hurricane arrival rates as a Poisson process and this feeds into further modules that compute the 1 in 200 Value at Risk index, i.e. the value that is at risk in a single year with probability less than 1 in 200.

(1.1.4) Recent storm seasons in the US appear to result in at least some hurricanes following a similar path. One could think of this as storms having a property which makes it more likely that further storms will follow in their wake. If this is indeed the case, then the underlying statistical process assumed to be generating the storms is not well aligned with what is actually occurring.

1.2 Problem Statement

(1.2.1) Within a hurricane season is there a tendency, under some conditions, for groups of hurricane tracks to follow a large scale steering pattern? Can the steering pattern be identified in some sense?

(1.2.2) What is the unconditional probability a steering pattern will exist in a given year? Can this probability be made conditional on large scale climate variables with any skill?

Category	Wind speed
Tropical depression	0-38
Tropical storm	39-73
Cat 1	74-95
Cat 2	96-110
Cat 3	111-130
Cat 4	131-155
Cat 5	156+

Table 1: Saffir-Simpson hurricane scale. Note that wind speed refers to the one-minute sustained wind speed in knots and that a Tropical Depression is defined as such only if the wind circulation is closed.

2 Literature and orientation towards the problem

2.1 Some facts about hurricanes

(2.1.1) Hurriances are classified according to the Saffir-Simpson scale, as shown in Table 1.

(2.1.2) As can be seen, a hurricane is defined as a storm with a sustained wind speed of 74 knots or higher. Hurricane formation requires a number of factors: sufficiently warm seas (at least 26.5°C, to at least a depth of 50m); high humidity in the low to middle troposphere; Coriolis Force powerful enough ($>$ 300 miles from equator); low vertical shear ($<$ 22 knots).

(2.1.3) Maximum hurricane windspeed is reasonably well correlated with a low central pressure, as shown in Figure 1.

(2.1.4) Hurricanes have a very definite *season* as shown in Figure 2.

2.2 Supporting literature

(2.2.1) There are operational hurricane forecast models using both statistical and physics-based approaches, see [4].

(2.2.2) The forecasting of where a storm makes landfall is greatly improved by modelling the hurricane core as an elliptical vortex in a strain field instead of a point vortex.

(2.2.3) The number of hurricanes that make landfall on US is almost the same over a long term average (decadal). The North Atlantic Oscillation (NAO, or Bermuda high) is caused by pressure difference between Iceland and Azores and is parameterised as the NAO index (the difference in pressure between Iceland and the Azores). With statistical significance, hurricanes make landfall South of Florida when the index is less than 0, more often

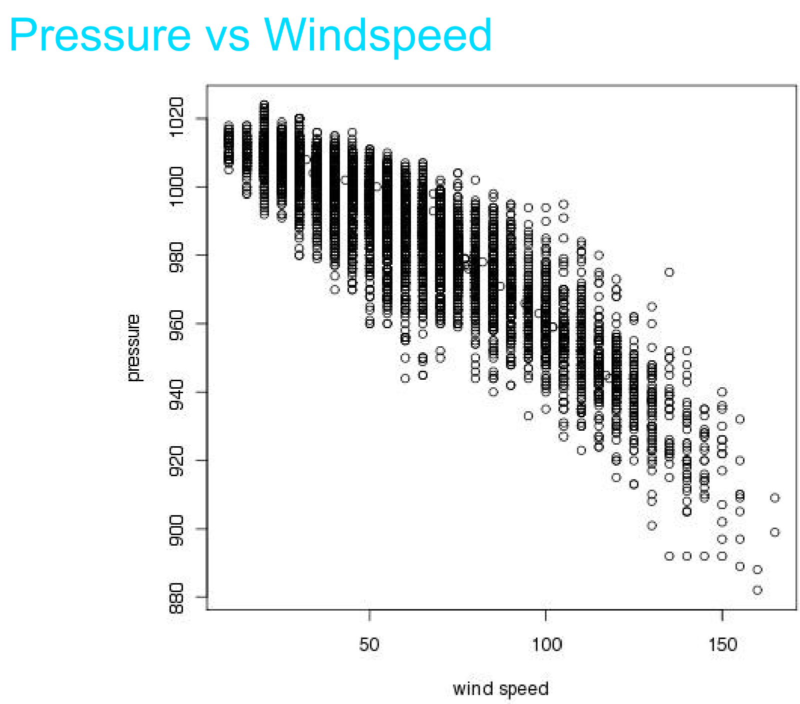

Figure 1: Pressure (mBar) vs wind speed (knots) for all hurricanes in the HURDAT data set (1851-2009). Note the strong correlation, but further note that the deepest recession ever recorded did not give rise the the highest wind speed ever recorded.

North of Florida otherwise [1]. Another candidate for long-term atmospheric variations could be the Quasi-biennial oscillation or the El Nino Southern Oscillation (ENSO).

(2.2.4) Sea surface temperatures, which one would also expect to have a long coherence time, are an important driver of hurricane intensity. They might also have an effect on the trajectory, either directly or by influencing the air pressure and humidity over the sea.

(2.2.5) A review of different atmospheric/hurricane models as they have been developed between 1998 to 2005 is provided by [2]. Some models concentrate on weather conditions (which is not so interesting because this is a very localised, short term effect for which we have no reliable historic record). Others concentrate on pressure inside the hurricane, while others discuss the transition from the uncoupled atmospheric model to the coupled atmospheric model. The discussion of some of the models is rather specialised.

(2.2.6) The likely path the hurricane will follow is considered by [3] (in the con-

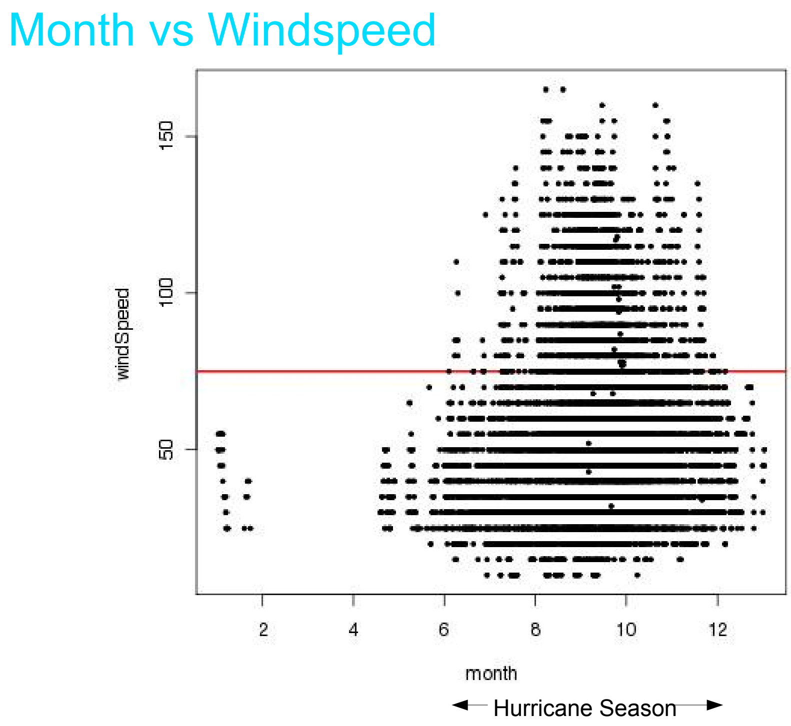

Figure 2: Maximum sustained storm windspeed recorded by month (i.e. Jan=1 to Dec=12) for all of the hurricane data in the HURDAT data set (1851-2009). The solid line shows the threshold point of 74 knots above which a storm is considered to be a hurricane.

text of Pacific cyclones). The authors propose that there are internal and external factors affecting the path. Internal factors include the speed of the storm, storm intensity, storm size and the Coriolis parameter. External factors include ambient pressure field and a parameter called *surface friction*. Of most interest is the ambient pressure field, as we have access to reliable data. The proposition is that a hurricane going Westward in an anti-cyclonic pressure gradient field climbs the high pressure gradient initially. There is a turning point as the storm crosses a line of equal latitude with the centre of the pressure gradient, thereafter it starts to descend the pressure gradient as it continues to track North. However, the paper details many counter examples where such simple rules are not followed.

2.3 Possible approaches towards the problem

(2.3.1) This could be approached as a data-driven problem, or by looking at physical mechanisms that might persist over a hurricane season (or both).

(2.3.2) Weather forecasters can generate a huge range of possible tracks from best ensemble forecasts, even for a storm that is currently in existence with fairly well known atmospheric conditions. The best that forecasters can currently do is to propose a *cone of uncertainty*, but it is not to be supposed that the hurricane is most likely to follow the central path of that cone. Some of these background hurricane models used by Met Office are described at [6].

(2.3.3) Vast amounts of data for statistical analysis are readily available from NOAA's National Geophysical Data Center [5] and [7]. A useful starting data set is the HURDAT data set which contains around 40000 samples of hurricane data since 1851 [8].

(2.3.4) Addressing ourselves to the problem statement (1.2) the first task is to undertake analysis of the available data (e.g. [8]) to establish whether or not there is consistent steering patterns within a single season, for at least some seasons. If such a pattern can be identified, the subsequent task is to relate seasons with the steering pattern and seasons without this pattern to explanatory physical atmospheric phenomena.

2.4 Work plan

(2.4.1) Our approach then is to look at historical data in the first instance to see how much evidence exists for hurricanes creating 'tracks' for other hurricanes to follow. The first task is to create a definition of 'track distance' that is reasonably consistent, so that we can then focus attention on specific seasons where clumping of hurricane tracks seems to have occurred.

(2.4.2) Using these distance metrics, can we identify storms that are close and contrast these with storms that are distant from each other? Presuming that we can identify such tracks, this leads to a number of interesting follow-on questions.

(2.4.3) Are larger hurricanes more or less likely to follow paths? This is of great practical importance. If larger hurricanes carve their own path regardless, then any track following effect will not have a great impact on insurance losses as smaller storms cause massively less damage than larger ones. On the other hand if larger storms carve a path that smaller ones follow then this may compound the disaster of a large hurricane.

(2.4.4) Consideration should also be given to broad isobar landscape. The Met Service starts with data points over a very non-uniform grid (i.e. collected by volunteer ships and planes historically, but more recently collected by satellite). Atmospheric modelling is then used to interpolate this across a uniform grid. Unfortunately these data are too coarsely sampled to see hurricanes (the grid spans the entire globe).

(2.4.5) Of course, isobars vary in three dimensions (although they are usually only drawn on a 2-d map). Thus isobars define a manifold surface. In practice, one considers 2-d maps of isobars at different heights. From 2-d pressure data - can we infer the *typical path* a hurricane will follow? Can a geometric analogy be sought, considering areas of high pressure as hills and low pressure as valleys and thinking of a hurricane following a path by rolling down the resulting hill, perhaps with a random likelihood of jumping out of a particular valley?

(2.4.6) Using the closeness measure and the topology suggested by the 2-d pressure data we can check if actual hurricanes in HURDAT ([8]) follow the 'typical path' suggested. We can then answer the following related questions. Which pressure level (if any) tends to drive the storms (e.g. 500 HPa or 750 HPa)? Call this the *driving pressure level*. Does the driving pressure level change depending on storm size? Do large storms get driven, or do they make their own mind up? This point is developed further in Section 4.1.

(2.4.7) Assume a pair of time sequential storms A and B, where storm B is generated within m miles of storm A's track and within w weeks of the time A was at that point. Does storm B tend to follow the path of storm A? If so, then is this just the typical path, caused by the driving pressure, or does hurricane A add steering over and above the ambient atmosphere? What range of values of m are valid? What range of values of w are significant?

(2.4.8) The pressure data is in netCDF format. This needs to be converted to a format suitable for direct use in Matlab, R, etc.

3 Data analysis: finding steering patterns

3.1 Track closeness measures: definition

(3.1.1) The group considered two different ways of computing track difference between two storms. The first track distance metric, the *area ratio metric*, was based on defining an area around each hurricane path and computing the ratio of the intersecting area to the total combined area swept out by each storm. The second track distance metric, the *absolute area metric*, was based on computing the area (or a proxy for the area) between the two curves.

(3.1.2) Both metrics rely on some higher order decision making, such as: over what length do we measure the path length; how do we normalise the metrics so that long storm paths and short storm paths can be treated in an equivalent way? If paths diverge dramatically at the end of the path period (or converge from distant points to the same path) then this is interesting behaviour, but our metrics may not distinguish this behaviour from two storms that seem to behave independently of each other over their entire length. Additionally, the first metric requires a parameter definition to convert the 1-d hurricane path into a swept 2-d area.

(3.1.3) The closeness of pairs of storms was measured only between latitudes 20° and 40° North.

(3.1.4) Not all storms transitted the 20° and 40° lines of latitude. To prevent errors or distortions, the area ratio metric excluded such storms from consideration. In HURDAT, covering the years 1851-2009, there are 76 years in which there is at least one pair of storms measurable by the area ratio metric. There is a combined total of 259 storms in those years, leading to 374 possible pairings of storms.

(3.1.5) The absolute area metric considered storms that only partially transited these latitude boundaries by introducing a normalisation factor, which is fully described later in bullet point (3.4.3). The absolute area metric could then consider 156 years in which there is at least one pair of qualifying storms, leading to 4761 possible pairings over all storms in the HURDAT data set.

(3.1.6) An improvement on this rather arbitrary latitude bracketing would be to consider the Caribbean as the start point. East of this area the steering pattern of the Azores anti-cyclone is the dominant pattern, which will distort the metric.

(3.1.7) A further consideration for either metric is to consider the pair of hurricanes A and B. Suppose hurricane A is long (and occurs at time t), and hurricane B is short and occurs at time $s > t$). B's track follows a portion of A's exactly, but they then diverge. We would like to ensure that either metric considers A and B to be close. This could perhaps be achieved if we normalise by the length of the shortest path. Currently, neither metric addresses this problem.

3.2 Area ratio metric

(3.2.1) The area ratio metric defines an epsilon-width buffer region around the path of each hurricane. For any two hurricanes the area of the buffer region intersection is calculated. This area is then divided by the combined area of both buffer regions to give a number between 0 and 1.

(3.2.2) More formally, let two hurricane paths be given as $p_1(x, y)$ and $p_2(x, y)$. Let $[p(x, y) \pm \epsilon]$ represent the 2-dimensional buffer region defined by two curves lying either side of $p(x, y)$, a constant distance ϵ away. Then define two area measures based on the intersection and union of the two buffer regions generated by each pair of hurricane paths,

$$\begin{aligned} A_N &= [p_1(x, y) \pm \epsilon] \cap [p_2(x, y) \pm \epsilon] \\ A_U &= [p_1(x, y) \pm \epsilon] \cup [p_2(x, y) \pm \epsilon] \end{aligned} \tag{1}$$

The area ratio metric, m_A, is computed as

$$m_A = A_N / A_U \tag{2}$$

By definition, we see that $0 \leq m_A \leq 1$.

(3.2.3) The main implementation simplification was that the buffer regions were discretized and interpolated onto a grid, which led to significant coding problems. In particular, calculating area was done by checking if a point was in a buffer region, which was defined by two boundary curves. However due to the interpolation, sometimes these curves were ill-defined, making this difficult. The solution used was to discretize the curves onto a 10000×10000 grid, while using a 100×100 grid for the area calculation. The effect of these simplifications is to introduce some rounding (or quantization) errors; however it is believed that such errors should be small enough so as not to effect the results materially.

3.3 Area ratio metric in action

(3.3.1) Application of the metric in the case of hurricanes Rita and Katrina is shown in Figure 3. Figure 4 shows the actual paths followed by these two storms.

3.4 Absolute area metric

(3.4.1) A simple area measure can be computed as follows. Define Δx to be the longitude difference of two curves at a given latitude, y. An incremental area ΔA can then be defined between y and $y + \Delta y$ as,

$$\Delta A = |\Delta x \Delta y| \tag{3}$$

so giving the total area between the curves as our metric,

$$m_B = \sum \Delta A \tag{4}$$

There are several practical implementation problems and solutions discussed below.

(3.4.2) The data in HURDAT ([8]) are not uniformly sampled, so it is not possible to compute the metric directly. The solution used was to interpolate each curves longitude at fixed intervals of latitude $y_i, \{0 < i < n\}$. Interpolation of the longitude for equal intervals of latitude was preferred

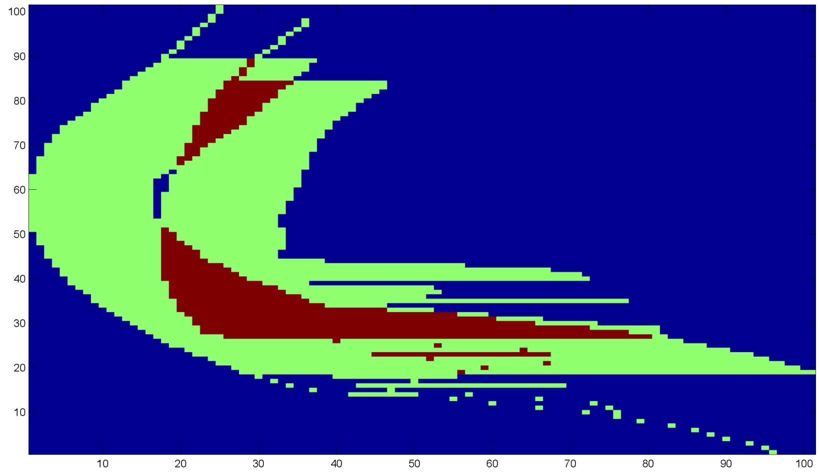

Figure 3: Showing the regions of intersection (A_N) in red of the paths of hurricanes Rita and Katrina, with the green regions displaying the non-intersecting region $A_U \setminus A_N$. The value of the metric in this case was $m_A = 0.3289$ with $\epsilon = 2$. The actual paths followed by Rita and Katrina are shown in Figure 4.

because storms are more likely to be monotic in their procession through lines of latitude (they frequently double back on lines of equal longitude) and non-monotonicity introduces errors into the interpolation function. All such errors (due to non-monotonicity in latitude) are ignored by the metric, but are not thought to be of great importance to a first order. Clearly this needs to be fixed at some point, if further use is to be made of the code.

(3.4.3) As previously described in bullet point (3.1.3), both metrics consider storms between latitudes 20°N and 40°N. The area metric also considers storms that do not fully transit through these latitudes by dividing by a normalisation factor, which we now describe. Define $x_{1,2}$ and $y_{1,2}$ to be (respectively) the pairwise longitude and latitude series of the pair of storms under consideration, lying between 20°N and 40°N. The range of longitude (k_x) and latitude (k_y) for the storms under consideration is then given as,

$$\begin{aligned} k_x &= \min[\max(x_1), \max(x_2)] - \max[\min(x_1), \min(x_2)] \\ k_y &= \min[\max(y_1), \max(y_2)] - \max[\min(y_1), \min(y_2)] \end{aligned} \quad (5)$$

The normalisation factor, z, is then computed as,

$$z = \sqrt{k_x^2 + k_y^2} \quad (6)$$

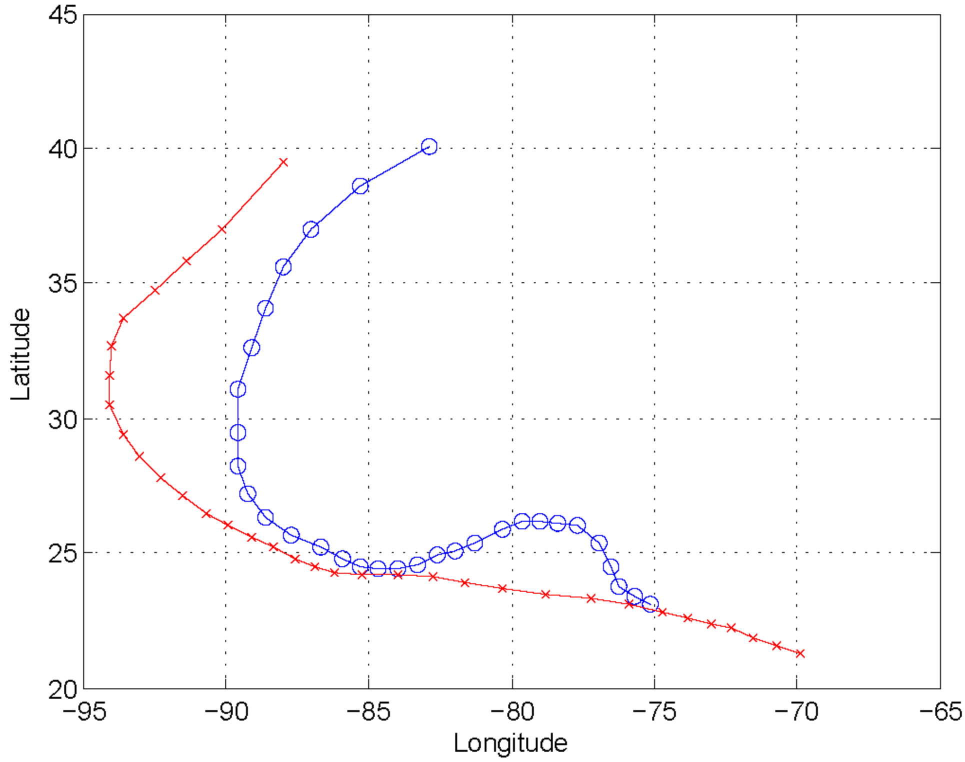

Figure 4: The actual paths followed by Rita (crosses) and Katrina (circles) that are used as inputs to the area ratio metric calculation of Figure 3.

Effectively, we approximate the valid storm path by a straight line in the latitude-longitude plane, and divide the area metric by the length of this line.

3.5 Absolute area metric in action

(3.5.1) Using the absolute area metric (m_B) we consider each possible pairing of storms within a given season. Figure 5 shows the minimum distance between a pair of storms in each year over the HURDAT data set.

3.6 Metric comparison and sequential storm hypothesis testing

(3.6.1) We compared the storms shown in Table 2. Corresponding plots for two of these contrasting storms are are shown in Figure 6 (Jeanne and Frances, $m_A = 0.0901$, $m_B = 0.76$) and Figure 7 (Gustav and Hanna, $m_A = 0.0$, $m_B = 5.32$).

(3.6.2) Scatterplots of the storm distance are shown for the area ratio metric

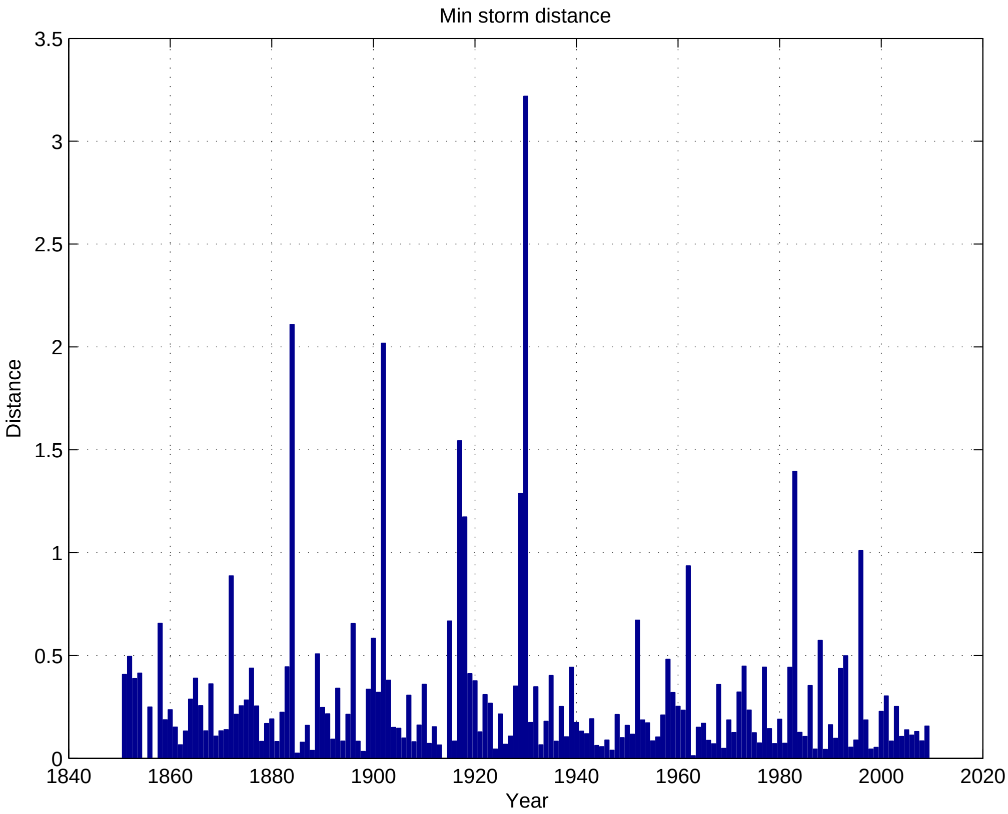

Figure 5: Using the absolute area metric m_B we consider each possible pairing of storms within a given season. What is plotted is m_B for the pair of storms within the season that gave the minimum distance in that year.

Year	Storm Pair	m_A	m_B
2004	Charlie and Ivan	0.1476	0.96
2004	Jeanne and Frances	0.0901	0.76
2004	Jeanne and Ivan	0.1145	1.61
2005	Katrina and Rita	0.1027	0.67
2008	Gustav and Hanna	0.0	5.32

Table 2: Comparison of some recent storms using the m_A and m_B metric. Recall that the m_A metric varies between 0 (no match) and 1 (perfect match). The m_B is a distance metric such that $m_B \geq 0$. A low value of m_B is a good match, higher values indicate more distance curves.

(m_A) in Figure 8 and for the absolute area metric (m_B) in Figure 9. For a storm pair to be considered, storms must be within the same season and the start time of both storms have to be separated by no more than 35 days. (Thirty-five days was chosen somewhat arbitrarily, but seemed like a reasonable time span within which path memory effects should be

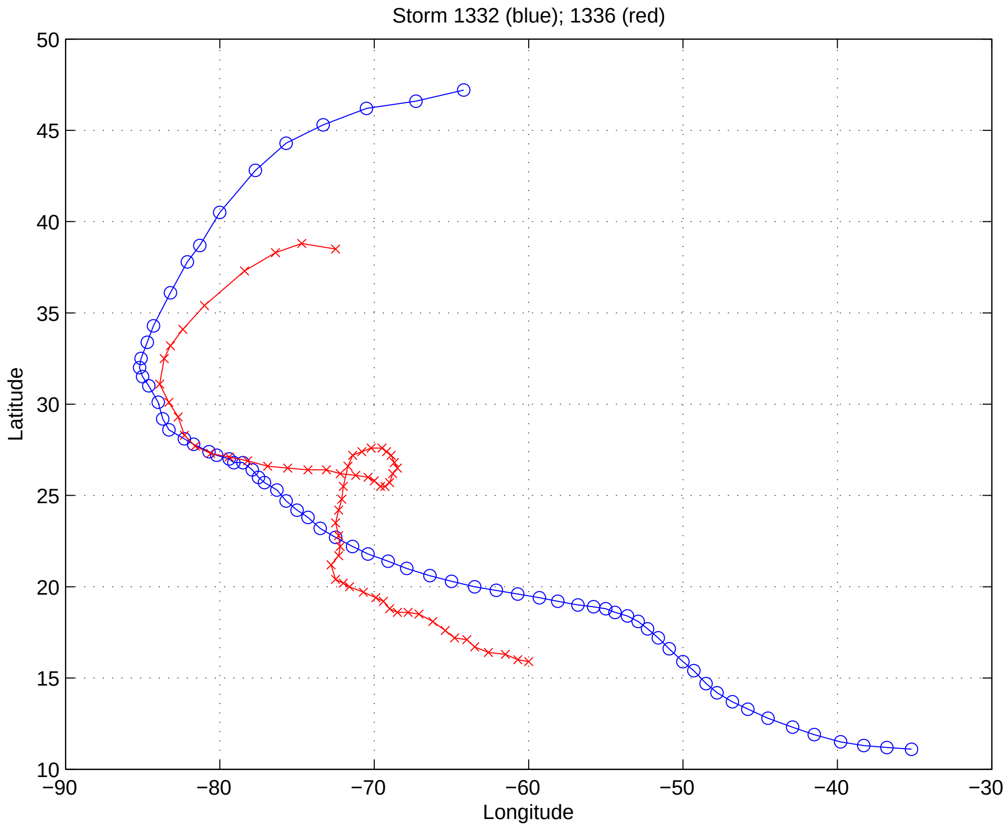

Figure 6: Storms Jeanne (1332) and Frances (1336), following a a similar path. In this case, $m_A = 0.0901$ and $m_B = 0.76$. For further comparisons, see Table 2.

apparent.) As can be seen, these scatterplots seem to give little support to the theory of track memory. However, it must be borne in mind that there may be a hidden subgroup within this seemingly random pattern, a subgroup that could be identified by some other measure (e.g. starting location, or time of year, or wind speed, or some combination of these properties).

(3.6.3) We tested the hypothesis: H_0: Hurricanes are equally likely to be similar to ANY other hurricane within the season; H_1: A hurricane is MORE likely to be similar to its nearest neighbour (i.e. the hurricane which immediately precedes or follows it) than it is to other hurricanes within the season.

(3.6.4) Using the absolute area metric, we reject the null hypothesis H_0 at the 99% level, supporting the theory of hurricane path memory. However, using the area ratio metric, we accept the null hypothesis with most choices of parameters.

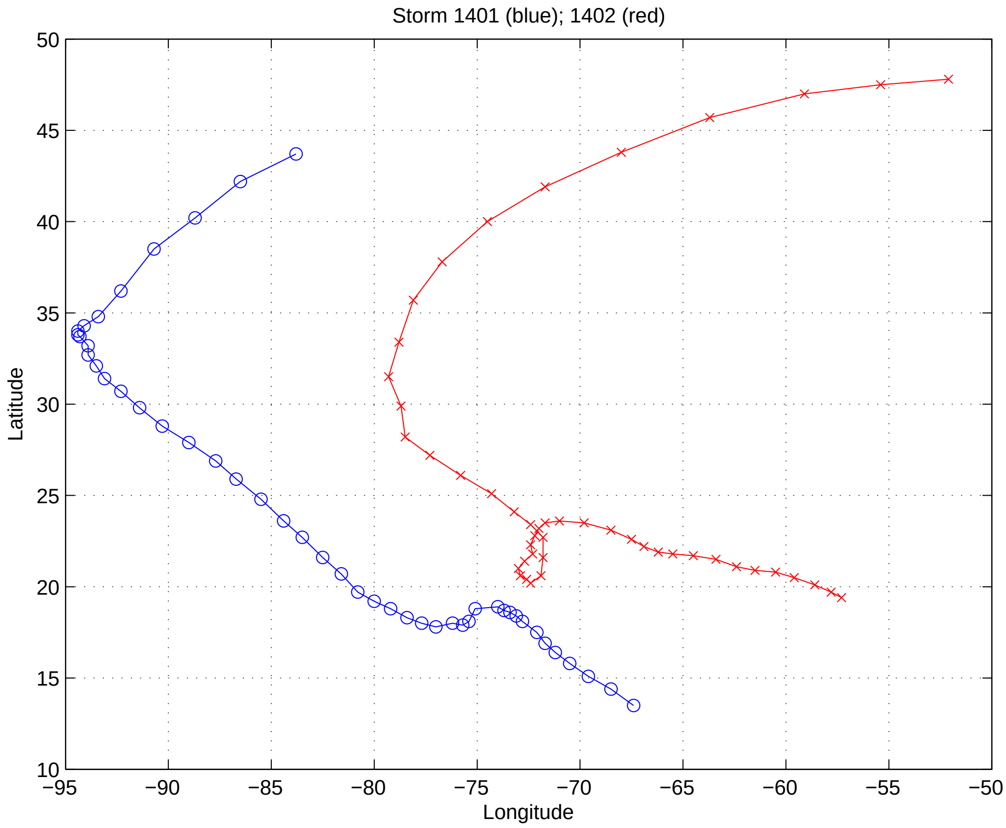

Figure 7: Storms Gustav (1401) and Hanna (1402), which are relatively far apart. In this case, $m_A = 0.0$ and $m_B = 5.32$. For further comparisons, see Table 2.

(3.6.5) In summary, the absolute area metric seems to suggest there is evidence that sequential hurricanes follow each other within-season over and above what might be expected by chance, but this is not confirmed by the area ratio metric.

3.7 Hypothesis testing: details

(3.7.1) The details of how this hypothesis testing was carried out for the m_B metric are as follows. Identify the most similar storms in each season over the entire data set in HURDAT, so resulting in 76 pairings[1]. Compute the total number of possible *sequential* storm sequences in the data ($k_s = \sum n_i - 1$, where n is the number of storms in year i). Further compute the total possible number of intra-season pairings that can occur ($k_u = \sum n_i(n_i - 1)/2!$). Compute the probability that a sequential pairing will arise for the most similar storms in a given season, assuming independence,

[1]Due to simplifying heuristics in the computation of the m_B metric, not all potential pairings can be considered.

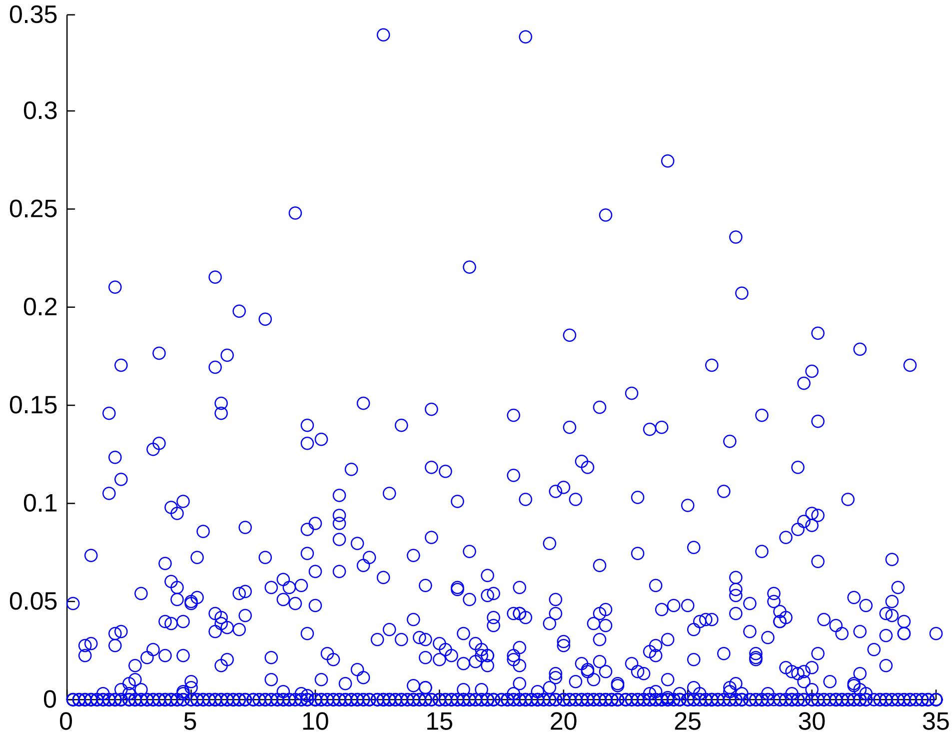

Figure 8: Scatter plot using the area ratio metric, m_A (no storm pair had a value of greater than 0.35). The x-axis shows the number of days between the storm pair under consideration.

i.e. $p_i = k_s/k_u$. Compute the number of times a sequential pairing *actually* occurs and divide this by the number of trials (i.e. number of years, 76 in this case) to get p_a.

(3.7.2) For the years 1851-2009 the probability of the most similar storms also being sequential is $p_i = 0.2168$, assuming independence between the storms. However, using the absolute area metric (m_B) to identify the closest storm pairing in any season, we find that the actual probability of the closest storms being sequential is $p_a = 0.2564$.

4 Track modelling: explanations and hypotheses

4.1 Evidence from air pressure data

(4.1.1) In the long list of parameters that are assumed to determine the track of a hurricane it is commonly assumed that the air pressure plays a major role. We want to use the available data on the daily air pressure in our area of interest to investigate this hypothesis and, if valid, detect driving patterns. Our area of interest is the North Atlantic, between latitudes

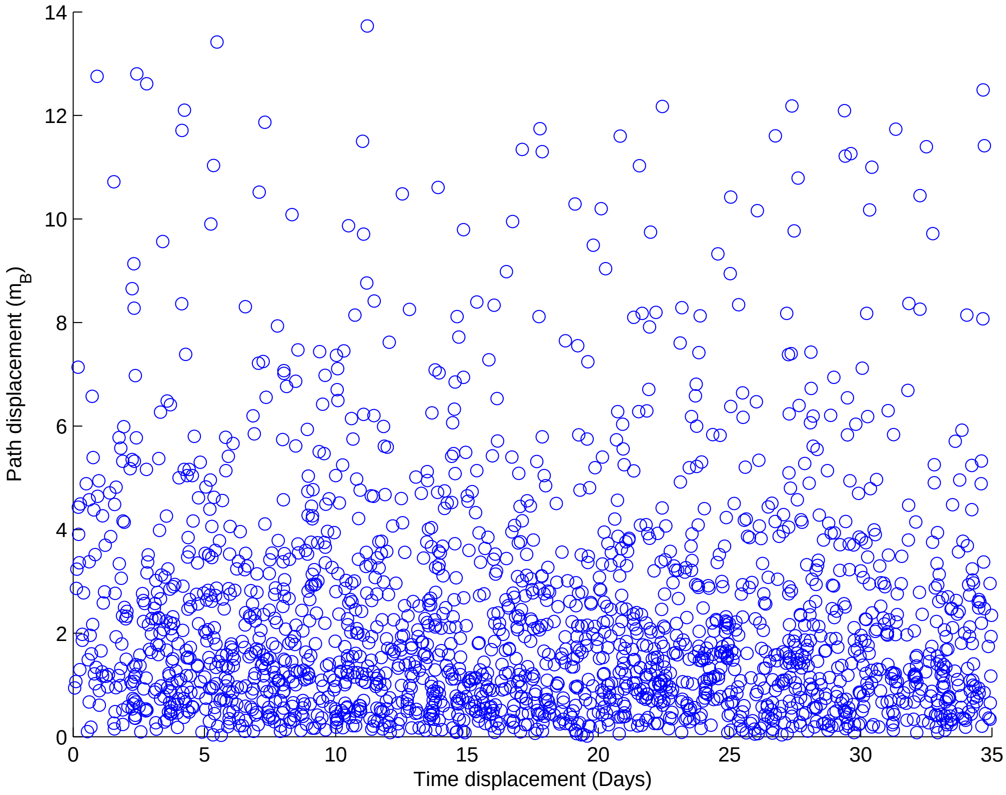

Figure 9: Scatter plot using the absolute area metric, m_B. The x-axis shows the number of days between the storm pair under consideration.

20° and 40°, as described in bullet point (3.1.3). A number of research questions come to mind.

(4.1.2) Can we determine and visualize the pressure that a hurricane experiences during its journey? In particular: Does the *experienced pressure* decline over time?

(4.1.3) Can we determine and visualize the pressure landscape locally around the hurricane track? In particular: Does the hurricane tend to move locally in the direction of the lowest pressure?

(4.1.4) Unfortunately there are several issues that complicate the investigation of the two research questions above. First, hurricanes are 3d-objects and thus experience several air pressures at the same time. As noted in bullet point (2.4.6), amongst all of these pressures, we can ask if there is a driving level (possibly depending on the size of the hurricane). If not, can we then detect more subtle properties of the pressure field and assess their influence on the storm track? These other properties might include such things as the position of the largest pressure gradient with respect to height in the vicinity of the storm.

(4.1.5) For the second research question, we also have to take into account that a moving hurricane has some momentum. So even if a hurricane *prefers* areas with a lower pressure, we may not necessarily see a strict decline of the hurricane's *experienced pressure* over time. This phenomena might be quite apparent if the direction of the steepest decent of the pressure landscape does not coincide with the direction of the momentum of the hurricane.

(4.1.6) We can ask similar questions when regarding two hurricanes following a similar path. Can we compare the *experienced pressures* of close storms? Can we compare the pressure landscape of two close storms? Are these *experienced pressures* and pressure landscapes similar to each other? If so, this would strengthen the hypothesis that two hurricanes tend to follow each other's path. Interestingly enough, this would allow the following competing explanations (which could be resolved if we are able to compare the pressure landscapes of similar storms).

(4.1.7) Perhaps two hurricanes are likely follow each other's path, because the first hurricane influences the pressure landscape around its track in a way that makes it attractive for the second hurricane. Conversely, two hurricanes may follow each other's path, because the first hurricane does *not* influence the pressure landscape around its track so that the second hurricane finds a similar pressure landscape and is drawn to this path by the same proceses as the first hurricane.

(4.1.8) Finally, here are some ideas we were not able to elaborate, because an algorithm to detect the *meeting point* and *breaking point* is yet to be developed. Are the pressure landscapes of two close hurricanes particularly similar at the *meeting point*, that is, the point from which we first consider their tracks to be close? Conversely, do the pressure landscapes of two close hurricanes differ significantly at the *breaking point*, that is, the point from which the respective tracks of the evolving storms are not considered close?

4.2 The point vortex model

(4.2.1) The simplest PDE model of large scale atmospheric motion is the barotropic potential vorticity equation on the β-plane. The β-plane assumes that the Earth can be modelled as flat with Cartesian coordinates, x and y, representing the longitudinal and latitudinal coordinates respectively. β denotes the northwards variation of the Coriolis parameter. The equation describes the time evolution of a two-dimensional field, $\psi(x, y, t)$, which should be thought of as modelling the geo-potential averaged in some way in the vertical direction.

(4.2.2) The equation reads as follows:

$$\frac{D}{D\,t}\left[\zeta - F\psi - \beta\,y\right] = 0 \tag{7}$$

(4.2.3) $\frac{D}{D\,t}$ represents the advective derivative:

$$\frac{D}{D\,t} = \frac{\partial}{\partial t} + u_x\,\frac{\partial}{\partial x} + u_y\,\frac{\partial}{\partial y}$$

where

$$\mathbf{u}(x,y,t) = (u_x, u_y, 0) = \left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}, 0\right) = \nabla\times\psi(x,y,t)\,\mathbf{z} \tag{8}$$

is the geostrophic velocity. This is two-dimensional since all vertical motions are assumed to have been averaged out in this model.

(4.2.4) $\zeta = -\nabla^2\,\psi$ is the relative vorticity.

(4.2.5) F controls the amount of vertical vortex stretching due to variations in the geo-potential. Since the model does not contain vertical motions, vortex stretching enters the model only through this term which acts as a source for the ageostrophic velocity which imparts a weak apparent compressibility to the two-dimensional velocity.

(4.2.6) If we neglect the Earth's rotation and vortex stretching due to variations in the geopotential ($\beta = 0$ and $F = 0$) then (7) and (8) become

$$\begin{aligned} \frac{D\zeta}{D\,t} &= 0 \\ \mathbf{u}(x,y,t) &= \nabla\times\psi(x,y,t)\,\mathbf{z} \end{aligned} \tag{9}$$

(4.2.7) This is the stream-function formulation of the two-dimensional Euler equation. The equation says that vorticity is conserved along streamlines of the fluid flow. If we further assume that the vorticity is entirely supported on a set of N point vortices located at positions $\mathbf{x}_i(t) = (x_i(t), y_i(t))$ having strengths (circulations), κ_i, $i = 1\ldots N$:

$$\zeta(\mathbf{x},t) = \sum_{i=1}^{N}\kappa_i\delta(\mathbf{x} = \mathbf{x}_i(t)), \tag{10}$$

then it turns out that (9) are equivalent a set of ordinary differential equations for the positions of these vortices. These differential equations describe a Hamiltonian system

$$\kappa_i\frac{dx_i}{dt} = -\frac{dH}{dy_i} \tag{11}$$

$$\kappa_i\frac{dy_i}{dt} = \frac{dH}{dx_i} \tag{12}$$

$$\tag{13}$$

where the Hamiltonian is

$$H = -\frac{1}{4\pi} \sum_{i\neq j} \kappa_i \kappa_j \ln |\mathbf{x}_i(t) - \mathbf{x}_j(t)| \tag{14}$$

(4.2.8) Since we do not have time to make detailed studies of the questions of whether vortices in the Barotropic Potential Vorticity equation follow each other we may at least use these equations to study how usual hydrodynamic vortices interact in the presence of a large scale steering flow.

(4.2.9) We used the point vortices to study how the interactions between pairs of point vortices can influence their paths in the presence of a dominant large scale steering flow. From now on we take $N = 2$. There is no steering flow in (11) so we impose one by adding some terms to the right hand side.

(4.2.10) Furthermore, we take both vortices to have the same circulation, $\kappa_1 = \kappa_2 = \kappa$. The equations which we solved are then written in complex coordinates, $z_i(t) = x_i(t) + i\, y_i(t)$ $(i = 1, 2)$ for brevity,

$$\frac{dz_1^*}{dt} = -\frac{i\,\kappa}{z_1 - z_2} - i\, V_{\text{ext}}(z_1) \tag{15}$$

$$\frac{dz_2^*}{dt} = -\frac{i\,\kappa}{z_2 - z_1} - i\, V_{\text{ext}}(z_2) \tag{16}$$

where the external steering flow is

$$V_{\text{ext}}(z) = \frac{1}{z - a_1} + \frac{1}{z - a_2}. \tag{17}$$

(4.2.11) Here a_1 and a_2 are the complex coordinates of the centres of a pair of steering vortices. We took $a_1 = 2$ and $a_2 = -2$ to produce the large scale steering flow plotted in Figure 10. The interesting feature is that the flow has a hyperbolic point at $(0, 0)$. With $\kappa = 0$ the point vortices simply follow the streamlines shown in Figure 10.

(4.2.12) We performed a number of numerical experiments to show how $\kappa \neq 0$ affects this passive advection of vortices. Figure 11 shows the tracks of two vortices having $\kappa = 1 \times 10^{-4}$ which start from $(1.84923, -0.992499)$ separated in time by 2.2 time units. In order to gauge the scale, the circulation of the steering vortices is 1 and the initial spatial separation corresponding to a delay of 2.2 time units is about 1.4 spatial units. From the figure, it is clear that both vortices follow essentially the same path (imposed by the large scale steering flow).

(4.2.13) Figure 12 shows the same situation with the intensity of the point vortices increased to $\kappa = 5 \times 10^{-4}$. In this case, the two vortices follow each other for a while but undergo rapid separation as they approach the hyperbolic point. This is because hyperbolic points can produce large amplification of small perturbations in trajectories.

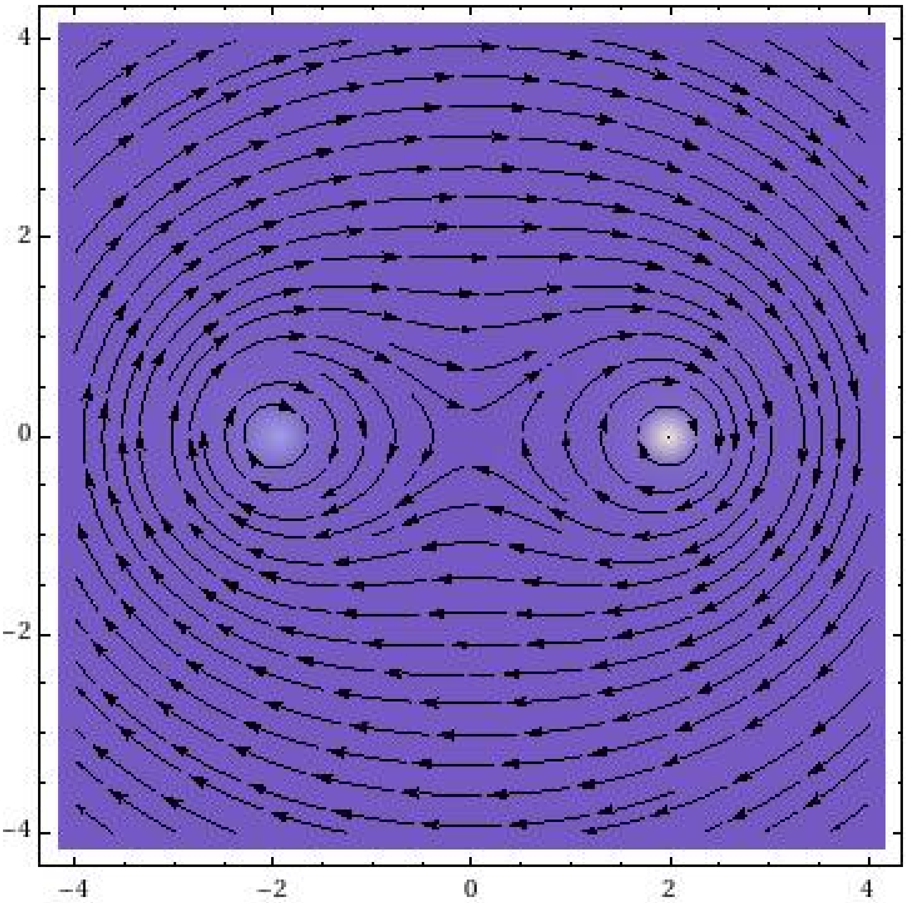

Figure 10: Large scale steering flow within which our simulated vortices will move.

(4.2.14) The conclusion to be drawn from this simple model is, as one might expect, that the question of whether vortices follow each other is not straightforward even in this simplest case. Trajectories which come close to hyperbolic points of the steering flow are very difficult to predict.

(4.2.15) Finally, it is worth pointing out that if the intensity of the vortices is comparable to the intensity of the steering flow, a situation which is closer to reality, the point vortex model exhibits very complicated trajectories and is probably chaotic. See Figure 13.

5 Conclusions and next steps

5.1 Conclusions

(5.1.1) In this report, we define two metrics used to assess the closeness of a pair of curves. We applied these metrics to analysing the hurricane track data

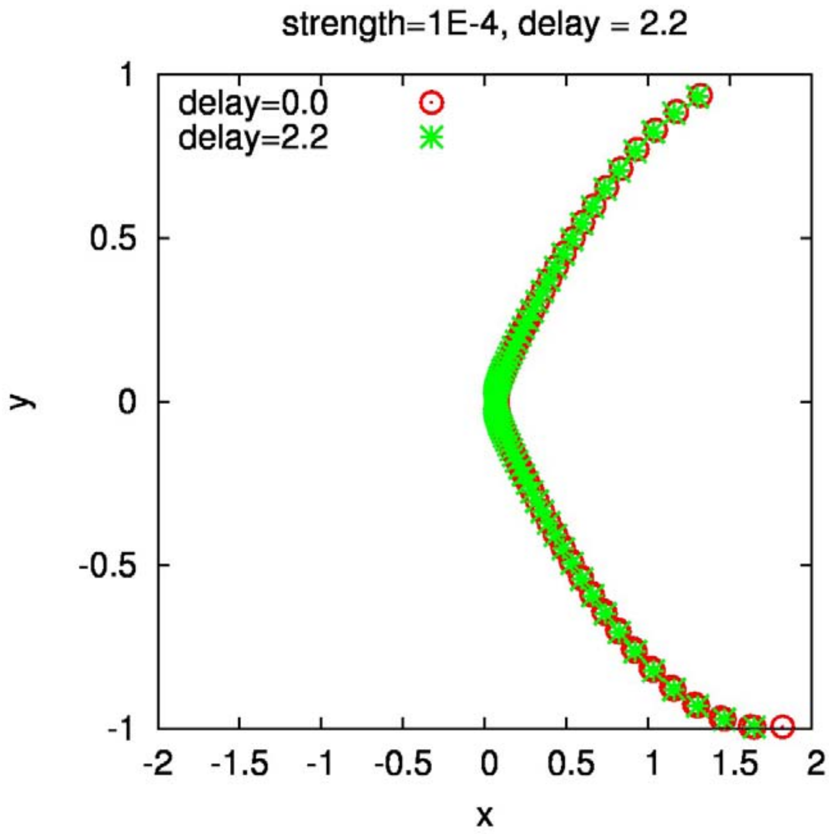

Figure 11: Two relatively weak vortices ($\kappa = 1e - 4$). Both follow the same track.

in HURDAT and while one metric suggested that sequential hurricane tracks were much more likely to be closer than non-sequential tracks, this was not confirmed by the other metric.

(5.1.2) A point vortex model in the presence of a large scale steering field was developed. This was simulated for the case of two time-sequential vortices. When the vortices were very weak compared to the steering pattern, they followed each other. When the vortex strength was increased slightly, but still significantly below the strength of the steering flow, they followed each other initially until they reached the hyperbolic point of the steering flow field, at which stage they diverged. When the vortex strength was comparable with the strength of the steering flow field (as would be the case with hurricanes) the trajectories of the vortices within the flow field appeared to be chaotic.

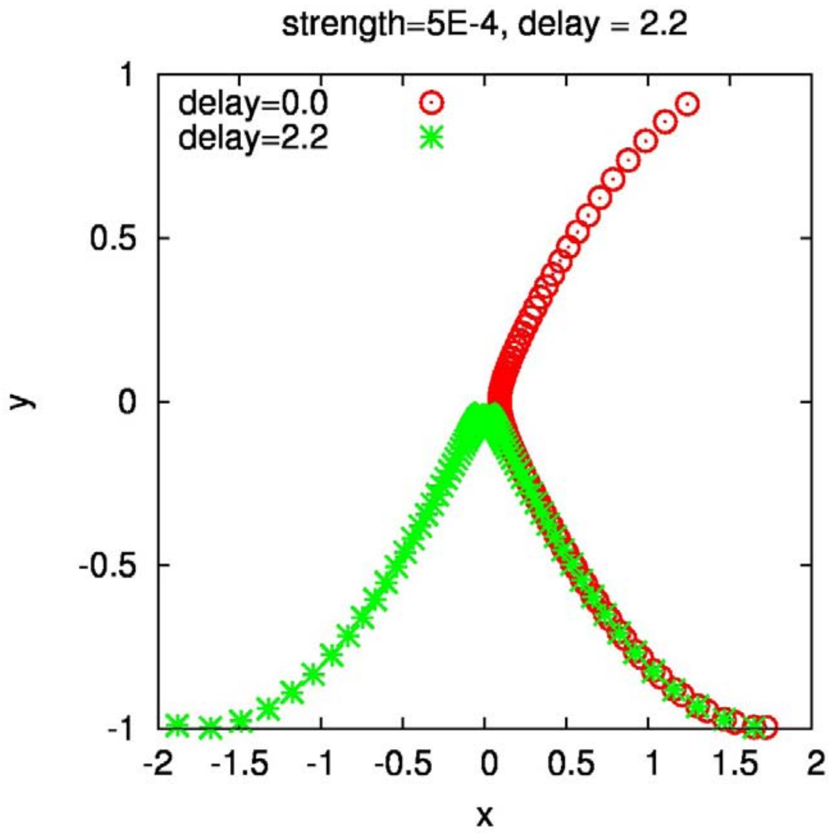

Figure 12: Two slightly stronger vortices ($\kappa = 5E - 4$). Both follow the same track, initially, but note deflection near a hyperbolic point.

5.2 Next steps

(5.2.1) We have two metric that disagree about the likelihood of the closest storms within a single season also being sequential storms. This can be investigated further, and probably resolved quite easily.

(5.2.2) Both curve distance metrics introduced various simplifications in order that they could be coded and run in the short time available. These simplifications would need to be examined more carefully, to ensure that the metrics are behaving in a sensible way.

(5.2.3) Some thought should be given to how we assess confidence in any results. If we appear to show that track memory effects are real, then how confident are we in that statement? Given the political nature of this area this becomes quite important.

(5.2.4) In some cases it is not subsequent storms that follow each other, but

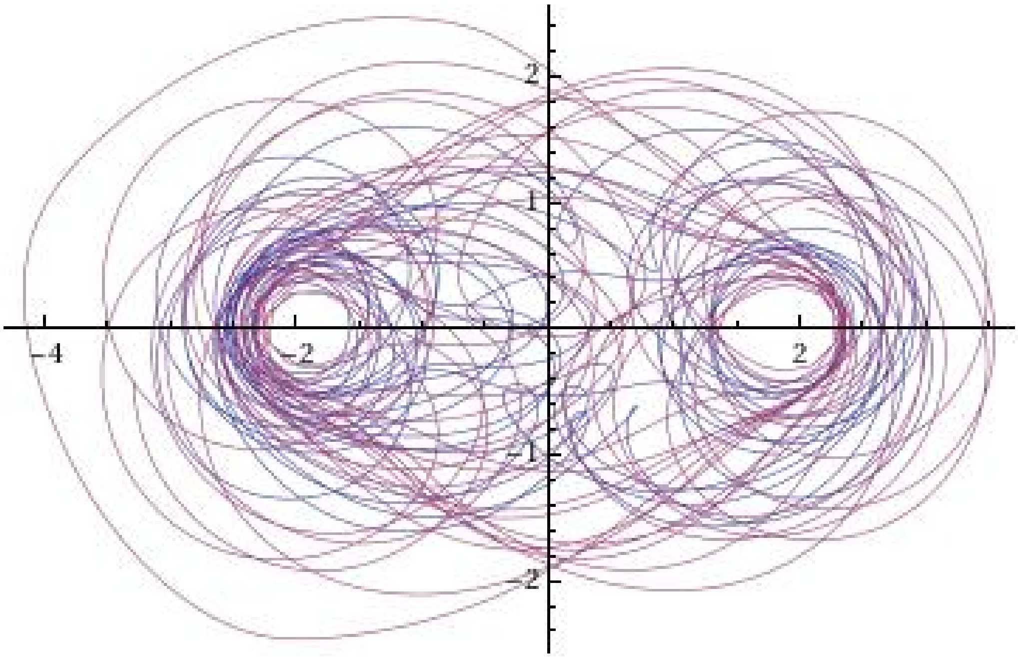

Figure 13: Trajectories of a pair of vortices having strengths comparable to the steering flow. When followed over a long period of time, it can be seen that the point vortex model exhibits complicated trajectories and is probably chaotic.

storms that are separated by 2 or 3 others *e.g.* Katrina and Rita (2005) were separated, as were Frances and Jeanne (2004). In both cases, they followed similar paths for a section of their track (in the 2004 case, the similar trajectory was over land, causing more damage to already weakened structures). The distance metrics and subsequent hypothesis testing could be modified to include time separation and time dependence of any observed effect. They could also be modified to include genesis location.

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[8] *http://www.aoml.noaa.gov/hrd/hurdat/*