Void spaces under tubes in fluidized beds

A: Work prior to Study Group

From videos of two-dimensional fluidized beds, it is seen that, at low fluidization velocities, lens-shaped voids develop under tubes and lead to bubble formation from the sides of the tubes, as in fig.1

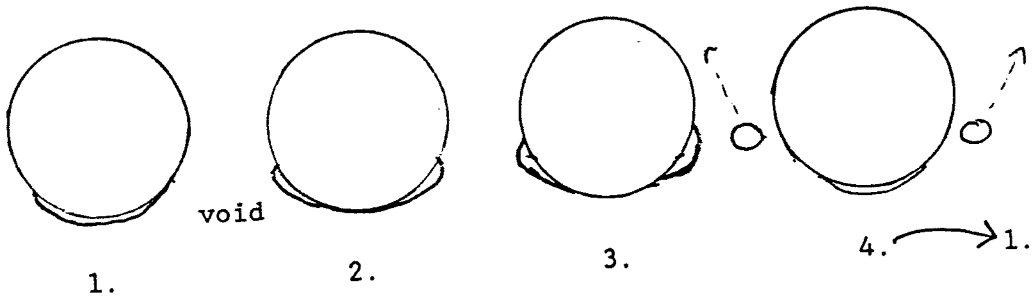


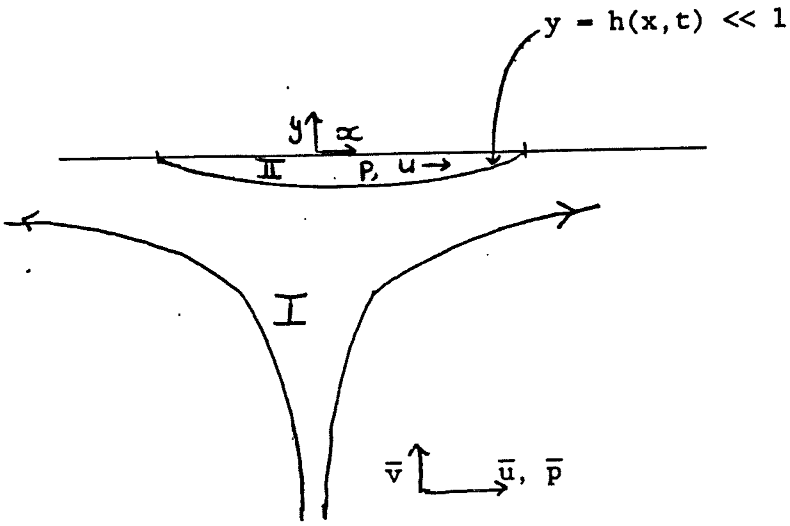
Fig.1 Periodic bubble formation under tubes

The gas flow around the pipe has a stagnation point on the upstream side. Vertical velocities close to the stagnation point are below minimum fluidization velocity and this, we believe, leads to local defluidization of a region which 'slumps' to closest packing, initiating a thin void space under the tube.

A model was proposed for void development based on the ad hoc assumptions that

1) the void is long and thin so flow is approximately one-dimensional;
2) flow through the void boundary is negligible;
3) gas viscosity is negligible in the void ($Re_{void} \geq 10^3$, estimated from video);
4) gas flow in the porous medium is given by Darcy's law;
5) the pressure in the void is imposed by the Darcy flow in the porous medium.

Fig.2 shows the simplified geometry used.



Stagnation point flow

Fig.2 Unsteady, stagnation point flow problem (dimensionless)

In region I, $\bar{p} = \frac{y^2}{2} - \frac{x^2}{2}$

since $\bar{v} = 0$ on $y \simeq 0$.

The pressure is continuous across the void boundary, so $p_{void} = -x^2/2$ to lowest order.

In region II $(0>y>h)$

$$h_t + (uh)_x = 0 \qquad \text{continuity}$$

$$\rho(u_t + uu_x) = -p_x \qquad \text{x-momentum,}$$

Since $p = p(x)$ $= -x^2/2$, then $u = u(x)$ and the void thickness is of the form

$$h = \frac{1}{u} F\left(\int \frac{dx}{u} - t\right)$$

As a crude model for flow round a pipe (rather than just stagnation point flow), u was chosen to be $1 - e^{-x}$. The void thickness could then be found explicitly and, as shown in Fig.3, after an initial transient, a travelling wave was observed (cf Fig.1).

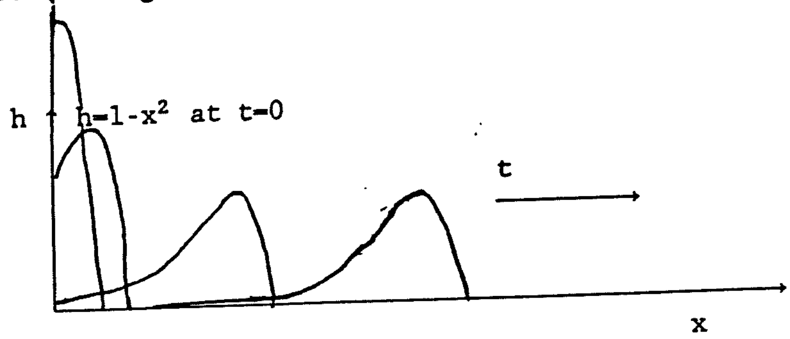


Fig.3 Evolution of void space

B: Work during Study Group

The assumption of no flow across the void boundary was modified, as it is clear that gas flow is easier through a void than through a porous medium. Flow across the boundary into the void would make the void flow rotational. Thus different equations would have to be used which reflect this nature of through-flow. In order to deal with a simpler problem, an 'immobilized' bed was thought of (i.e. h was prescribed) to obtain a steady (if unrealistic) problem.

For the flow in the regions shown in Fig.4, the following dimensional equations are written

Region (1): $$\left.\begin{aligned} \rho(uu_x+vu_y) &= -p_x \\ u_x+v_y &= 0 \end{aligned}\right\} \quad h_0 h < y < 0$$

Region (2): $$\left.\begin{aligned} \bar{u} &= -\frac{\kappa}{\mu}\bar{p}_x \\ \bar{v} &= -\frac{\kappa}{\mu}\bar{p}_y \end{aligned}\right\} \quad -H < y < h_0 h$$

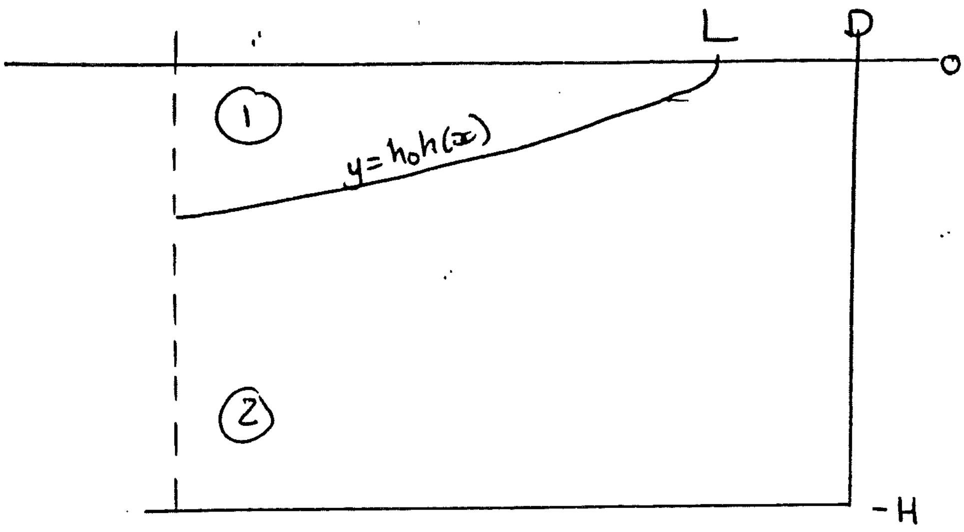


Fig.4 Flow regions for immobilized bed

with boundary conditions:

On $y = 0,\ h \neq 0$	$v = 0$
$y = 0,\ h = 0$	$\bar{v} = 0$
$y = -H$	$\bar{p} = p_0$
$x = \pm D$	$\frac{\partial \bar{p}}{\partial x} = 0$

on $y = h_0 h$: $p = \bar{p}$,

$$[(u,v) - \alpha(\bar{u},\bar{v})].\underline{n} = 0,$$ $\underline{n}$ - normal to h.

and, assuming inflow into the void, either

$$[(u,v) - \alpha(\bar{u},\bar{v})].\underline{t} = 0 \quad \text{(a)}$$

or $$(u,v).\underline{t} = 0 \quad \text{(b)}$$ $\underline{t}$ - tangent.

Here α denotes the voidage at $y = h_0\ h$.

Boundary condition (a) is used for fully developed bubbles in fluidized beds [ref.1], but here the void is in a defluidized region of particles and it might be more appropriate to use the boundary condition (b) which is used for air flowing over sand dunes [ref.2].

Non-dimensionalizing using:

In porous medium region (2)

$\bar{x} = L\bar{x}'$, $\bar{y} = L\bar{y}'$

$\bar{u},\bar{v} = \frac{\kappa}{\mu}\frac{p_0}{L}(\bar{u}',\bar{v}')$

$\bar{p} = p_0\bar{p}'$

In region (1)

$y = \varepsilon L y'$ where $\varepsilon = \frac{h_0}{L}$

$x = Lx'$

$p = p_0 p'$

$u = \frac{\kappa p_0}{\mu L\varepsilon}u'$

$v = \frac{\kappa}{\mu}\frac{p_0}{L}v'$

We get in (2): $\left.\begin{aligned}\lambda(uu_x+vu_y) &= -p_x\\ u_x+v_y &= 0\end{aligned}\right\}*$ where $\lambda = \frac{\kappa^2}{u^2}\frac{p_0}{\varepsilon^2 L^2}$ and the boundary is given by $y = h(x)$,

and in (1): $\nabla^2\bar{p} = 0$ where the boundary is given by $\bar{y} = \varepsilon h(\bar{x})$.

On the boundary: $p = \bar{p}$

$$\left.\begin{aligned}\text{and (since } \underline{n} = (\varepsilon h',-1)),\ uh' - v &= \varepsilon\bar{u}h' - \bar{v}\\ u + \varepsilon^2 h'v &= \varepsilon(\bar{u}+\varepsilon h'\bar{v})\end{aligned}\right\} \qquad (+)$$

using the continuous tangential velocity condition (a) (both (a) and (b) give u = 0 to lowest order).

It is unlikely that λ will assume large values; indeed, estimates indicate that it is ~ 1/10. For $\lambda \ll 1$, the pressure in the void space would be spatially constant (to within $O(\lambda)$), consistent with Davidson's assumption of constant pressure in fully developed bubbles [ref.1,3]. In this case when D $\to \infty$ the velocity through the boundary, $\bar{v}$, is proportional to $\frac{1}{\sqrt{L^2 - x^2}}$.

For arbitrary λ, we can relate $\bar{p}|_{y=0}$ to $\bar{v}|_{y=0} = \frac{\partial \bar{p}}{\partial y}\Big|_{y=0}$, from the porous medium problem, by

$$\bar{p} = \mathcal{L}_{\lambda}(\bar{v}) = p|_{y=h(x)}.$$

The equation * in the void, together with the boundary conditions (+), can only be solved once a second equation relating $\bar{p}$ and $\bar{v}$ is found. This can be done using the method of Cole & Aroesty [ref.4]

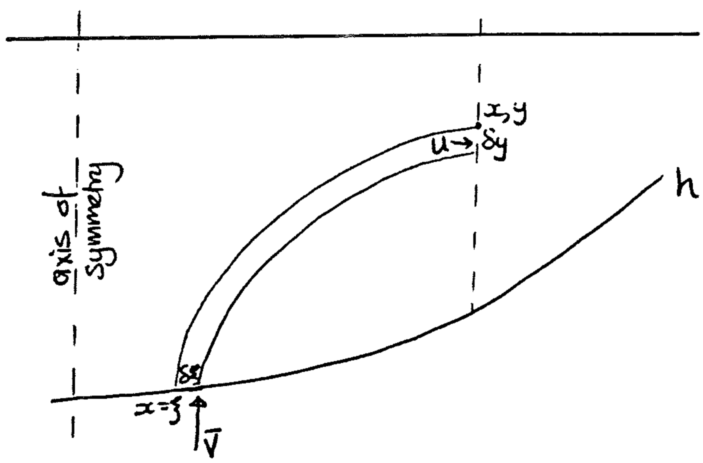


Fig.5 Stream tube in Cole & Aroesty model

From Bernoulli's theorem, along a streamline emerging from the porous medium at ξ (see Fig.5) we have

$$p(x) + \frac{\lambda}{2} u^2(x,y) = p(\xi) .$$

By mass conservation within the stream tube in Fig.5, $\bar{v}(\xi)\delta\xi = u(x,y)\delta y$ and hence

$$\frac{\bar{v}(\xi)}{u(x,y)} = \frac{\partial y}{\partial \xi}\Big|_x$$

The voidspace thickness h(x) can now be obtained by integration:

$$-h(x) = \int_0^x \frac{\partial y}{\partial \xi}\, d\xi = \int_0^x \frac{\bar{v}(\xi)}{u}\, d\xi$$

But $$u = \sqrt{\frac{2}{\lambda}(p(\xi) - p(x))}$$

so

$$-h(x) = \int_0^x \frac{\sqrt{\lambda}\,\bar{v}(\xi)\, d\xi}{\sqrt{2(p(\xi) - p(x))}}$$

A solution for the unsteady problem (i.e. a mobile fluidized bed) is still unclear, as is the mechanism of initial defluidization and slumping.

1) Batchelor, G.K. Archives of Mechanics 26 (3), 1974, pp339-351.

2) Yalin, M.S. Mechanics of Sediment Transport, Pergamon Press, Oxford, 1972.

3) Davidson, J.D & Harrison, D. (eds). Fluidization, Academic Press, London 1971, p91.

4) Cole, J.D. & Aroesty, J. Int. J. Heat Mass Transfer 11 1968 pp1167-1182.

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