Long Term behaviour of Geothermal Reservoirs
(Camborne School of Mines Geothermal Energy Project)

A number of different aspects of geothermal reservoir behaviour were studied during the week. These ranged from the correct formulation of the thermo-elastic deformation equations for granite, and the possible practical methods of implementing the thermo-elastic effects within the existing large computer codes, to the generation of simple models describing the long term thermal effects.

We shall describe the results of the simple modelling. The approach was to consider first how the rock cooled as a result of water moving through the network of cracks (Rock Cooling Model) and then to consider second how this cooling might change the rock stresses and water viscosity, and hence the effective permeability of the cracks, thereby altering the flow pattern from inlet to outlet bore-hole (Water Flow/Cooling Interactions).

1. Rock Cooling Model

Models of how the rock cools must allow for heat conduction within the rock and for heat transport by the water flow through the cracks. It was decided that a general model would be too complicated so two extreme situations were considered.

1.1 Thermally isolated cracks.

Two cases were considered; first, the case of cracks that are separated by a distance greater than the thermal diffusion length scale in the rock and, second, the case where the thermal diffusion length is much larger than the crack spacing. These models have been called model A and model B respectively in earlier literature. The physical problem, unfortunately, is probably in the grey area between these (thermal diffusion lengths of 20 metres over the 25 year lifetime of the reservoir and crack spacings of around 10 metres). Given the uncertainties about the crack spacing however, these simple models should indicate the qualitative behaviour.

The case of widely separated cracks has been considered by Camborne before. The model considers a single crack, on $y = 0$ say. The heat diffusion equation is solved in the rock, neglecting heat diffusion in the direction parallel to the crack. In the crack, the temperature of the water is taken to be uniform across the width of the crack and heat is advected with

the water flow. The conditions relating temperatures in the rock to that in the water are that the heat flux and temperature are continuous at the rock/crack surface. In the model we take t as time, x as the distance along the crack, y as distance into the rock normal to the crack, $T_r(x, y, t)$ as the rock temperature, $T_w(x, t)$ as the water temperature, ρ as the water density, k, κ and c as the usual thermal constants, V as the water velocity in the crack and w the width of the crack; see Figure 1.
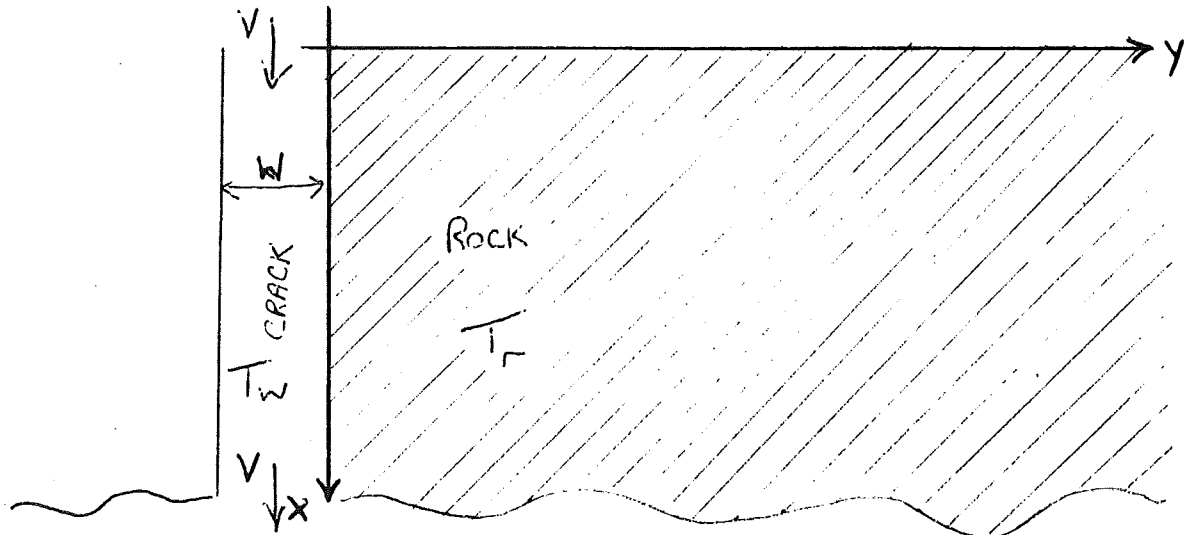

Figure 1

The heat flow equation in the rock is

$$\frac{\partial T_r}{\partial t} = \kappa_r \frac{\partial^2 T_r}{\partial y^2},$$

and boundary conditions on $y = 0$ and water temperature equations are

$$\rho c_w V \frac{\partial T_w}{\partial x} = -k_r \frac{\partial T_r}{\partial y}(x, 0, t), \quad T_w(x, t) = T_r(x, 0, t), \quad \frac{\partial(wV)}{\partial x} = 0.$$

The rock temperature is assumed to tend to some constant at infinity. In addition, initial conditions for the rock temperature distribution and the temperature of the water at the crack mouth are required. If the rock is initially at a uniform temperature and the water entering the crack is at a constant temperature then an analytic solution can be found using standard methods.

1.2 Thermally coupled cracks.

The time scale of interest, 25 years, is such that the cooling effects of water in neighbouring cracks may be coupled. The temperature distribution normal to the crack is therefore of less importance in the model. Instead, we average across the region, in a manner similar to porous medium models. The variables of interest are then the spatial coordinates x, y, the average rock temperature T_r and the average water temperature T_w. In averaging we allow for the fact that the heat flux from the rock is into the water but neglect the condition that water temperature and rock temperature are continuous across a crack. This is necessary as the rock temperature is an average value, taken over an entire block of rock. The detailed temperature structure within the blocks is absorbed into a single heat transfer coefficient h. This parameter should account for the thermal properties of the rock and the shape of the rock blocks. A crude estimate for h is

$$h = \frac{\kappa_r}{L^2}$$

where L is a typical length scale of a rock block and κ_r is the thermal diffusivity. The averaged model for the temperature fields is

$$\rho_r c_r \frac{\partial T_r}{\partial t} = h(T_w - T_r), \quad \rho_w c_w K \boldsymbol{v} \cdot \nabla T_w = -h(T_w - T_r),$$

where K is the permeability of the rock, and $\boldsymbol{v}$ is an average water velocity. If we assume Darcy's law for the average water velocity, and conservation of water (zero divergence of the mass flow rate), then we can close the system with the equations

$$\nabla \cdot (K\boldsymbol{v}) = 0, \quad \boldsymbol{v} = -\nabla \phi,$$

for the flow field. This system of equations consists of an elliptic equation for the flow and a telegraph equation for the heat flux. The problem was solved with the rock initially at a uniform temperature with the injected water flowing at a constant rate through uniformly permeable rock: the solution can then be derived using Laplace transforms. A numerical solution was also undertaken and the results are enclosed.

Water flow/Cooling interaction

Here an attempt was made to try to understand how the variations in temperature might influence the flow, and possibly lead to instabilities. Instabilities in the flow pattern

might result in the geothermal reservoir being only partially mined of its heat when the water flowing out is no longer sufficiently hot for commercial use. The worst case scenario is similar to the fingering problem encountered when pumping water into an oil reservoir to displace the oil; the water pushes through to the production well before all the oil is removed, leaving large pockets of unrecovered oil.

Various suggestions were made as to how this interaction of heat and flow could be incorporated into a model. The main discussion centered on the inclusion of flow/cooling effects in the averaged porous medium model given above.

The simplest suggested model was based on the assumption that the porous medium type model, given above, is adequate for describing the heat and water flow and the further assumption that the flow/cooling interaction could be included by allowing the permeability K of the porous medium to depend on the temperature. There are two main effects to consider. Firstly, the changes in the water temperature over the operating temperature range of the reservior cause the viscosity to vary by up to a factor of four. Hence the permeability is a function of the local water temperature T_w. Secondly, the temperature variations cause shrinkage of the rock which can more than double the expected crack width (and hence alter permeabilities by a factor of eight assuming that the flow is laminar). This shinkage should of course be included by considering the stress distribution within the rock. Camborne have a large computer code that considers this redistribution; however, it takes a substantial amount of time to run so some simple model may give additional insight.

A possible model for the shrinkage was suggested that involved treating the permeability as a non-local function of the rock temperature. A possibility was to have a convolution of the temperature with some kernel of the form

$$K(T) = \int \mathcal{K}(\boldsymbol{x}, \boldsymbol{y}) T(\boldsymbol{y}) \, dV_y,$$

that indicated the redistribution of displacements due to the stresses created. There was discussion on the possible functional form of the kernel $\mathcal{K}(\boldsymbol{x}, \boldsymbol{y})$. To accommodate the fact that the stress redistributes anisotropically, due to the shearing that occurs along existing cracks, the permeability would have to be anisotropic aligned to the dominant shear direction. One suggested simplification was that the redistribution may be on a length scale much smaller that those of interest and therefore it may be possible to consider the shrinkage as a function of the local rock temperature (so that $\mathcal{K}$ would be essentially a delta function),

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although numerical results using the Camborne code seem to indicate this may not be a very good approximation.

There is currently on going work to consider the stability of a two dimensional flow with either shrinkage or viscosity changes included. The analysis is similar to the oil/water stability work but has some interesting additional features. In order to make the problem tractable, the changes due to shrinkage or viscosity are assumed to occur rapidly at a particular temperature. This then makes it possible in each case to break the solution into two distinct regions separated by a sharp interface and consider the linear stability of this interface.

CPP, ADK, HO, SDH, SPT, EJH, JRL, WAG, PJH.
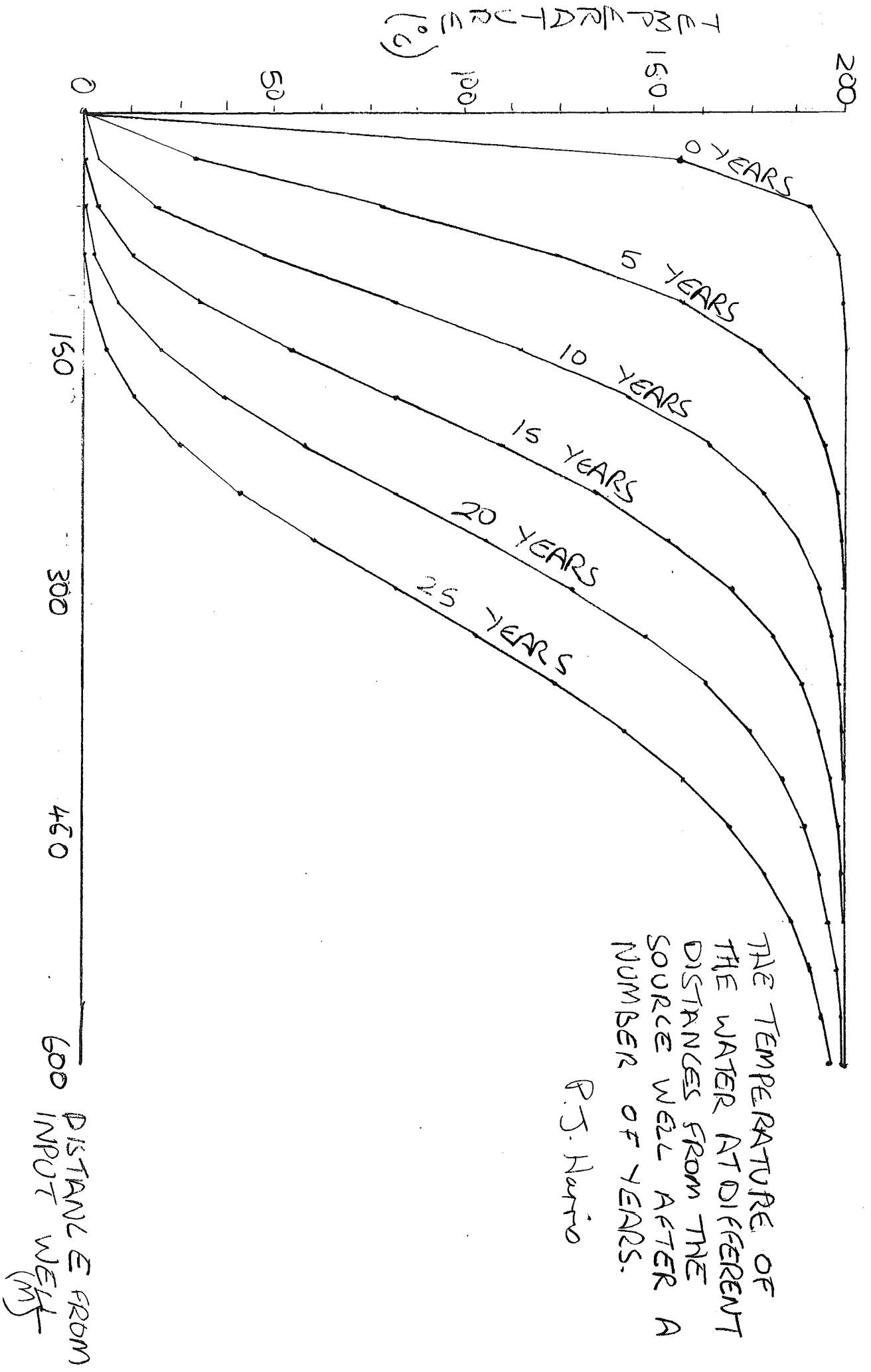
THE TEMPERATURE OF THE WATER AT DIFFERENT DISTANCES FROM THE SOURCE WELL AFTER A NUMBER OF YEARS.

P.J. Harris

TEMPERATURE (°C)

DISTANCE FROM INPUT WELL (M)

0 YEARS
5 YEARS
10 YEARS
15 YEARS
20 YEARS
25 YEARS

THE TEMPERATURE OF
THE ROCKS AT DIFFERENT
DISTANCES FROM THE
SOURCE WELL AFTER
A NUMBER OF YEARS.

P.J. Harris

DISTANCE FROM
INPUT WELL
(M)

600

450

300

150

0

25 YEARS

20 YEARS

15 YEARS

10 YEARS

5 YEARS

0 YEARS

200

150

100

50

0

TEMPERATURE (°C)