NUCLEAR ELECTRIC
BERKELEY NUCLEAR LABORATORIES
Stability of the crust on a spreading pool

In order to obtain a licence to run its nuclear power stations, Nuclear Electric is obliged to demonstrate to an independent body (the Nuclear Installations Inspectorate) that an adequate safety margin exists even under severe accident conditions. This requires that the physical processes involved in any situation, however extreme or unlikely, are sufficiently understood that corrective actions can be implemented effectively.

The problem of interest is a hypothetical situation in an advanced gas-cooled reactor (AGR). It perversely assumes that the shut-down systems have failed to arrest some event which has caused the fuel in a particular channel to overheat and that the entire fuel inventory of the channel has melted and poured onto the steel floor below the fuel channel. The fuel will spread quickly over the floor and freeze into a solidified mass. On a much longer time-scale, the solidified fuel will release its nuclear decay heat, partly to the gaseous environment (by radiative heat transfer) and partly to the steel floor (by conduction). The issue of industrial interest is whether or not the floor could melt during the period of release of decay heat, and one of the determining factors is the extent of spreading of the original puddle.

Experiments performed at Berkeley Nuclear Laboratories and elsewhere indicate that the extent of spreading depends on a number of physical parameters, notably the pouring rate, the thermo-physical properties and the superheat of the pouring fluid. The extreme pouring regimes are not of concern because (a) for slow pours, the fluid would be broken into small droplets and would be dispersed over a wide area, and (b) for fast pours, the puddle would spread to a very thin crust 'pizza' which would be too thin to melt the floor. The case of interest is the intermediate regime in which the fluid pours at a moderate rate onto the floor. In this regime, the extent of spreading of the puddle appears to be strongly determined by the stability of a thin solidifying crust on the free surface. In some cases, the crust appears to be strong enough to act as a restraint on the spreading of the puddle, but in others the crust breaks open and fluid is able to seep out (either through radial cracks or at the advancing front) and form lobes. Lobes are seen as desirable because the complex lobe morphology would have greater surface area than an axisymmetric morphology and the peak temperatures would therefore be lower. A series of experiments is underway in the Geology Department at Bristol University (Stephen Sparks, Mark Stasiuk) to investigate the behaviour of the crust in a well-controlled environment and some results my be available at the time of the Study Group. In addition, there is a CASE project in the Mathematics Department at Bristol University (Pauline Bennett, David Riley) to investigate the heat transfer and two-phase phenomena, but not the question of crust stability.

To summarise, the problem of industrial interest is to find the conditions for the stability of the crust on the free surface.

The Nuclear Electric Problem

The Nuclear Electric problem attracted the attention of many of the Study Group participants. This was due in part to the multi-faceted nature of the problem, in part to the wealth of experimental observations (cerrobend in air and polyethylene glycol 600 in a solution of water and ethylene glycol), and in part to the fascinating overview of analogous geological fluid mechanical problems (volcanic lava in either water or air) given by Steve Sparks. The net result was that subgroups worked on several related aspects of the problem, leading to a breadth of coverage rather than depth. Steve Wilson, Tim Pedley and Louise Terrill worked on a simple static model; Julie Moriarty and colleagues from Bristol, Leeds and Nottingham considered a simple lubrication model; Tony Green was very active with his bladder problem — a nonlinear elastic model; Alexander Movchan concerned himself with linear elastic models; meanwhile Howell Peregrine was up a ladder pouring buckets of water over paving slabs — an experimental study of hydraulic jumps. Since the Meeting, Mark Stasiuk, Steve Sparks and David Riley have made significant progress with some simple visco-elastico-gravity current models.

Given the complexity of the Nuclear Electric problem (featuring, for example, radial spreading centres, undermining, overwelling, hydraulic jumps, substrate erosion, etc.), it is not surprising that the group's models did not mature as far as others at the Meeting. In fact, much of the group's time was spent discussing the relationship between the actual problem involving UO_2 in air and the simulant experiments. Several promising ideas, however, emerged from the Meeting and Nuclear Electric are keen to realise the potential in these.

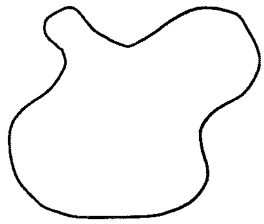
D.S. Riley 26.9.92

Spreading of melts (Nuclear Electric)

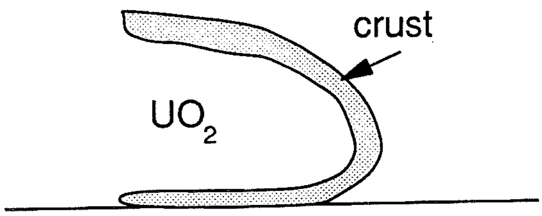
Description of the mathematical model

The whole problem may be divided into several steps:

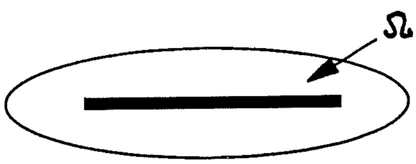
i. Spreading of a viscous fluid on the flat surface (formation of 'lobes').



ii. Formation of a crust (solidification problem).



iii. The problem of the fracture mechanics (longitudinal thermocracks in an elastic layer.)



Our objective is to study the last part. We can formulate this problem in the following way.

Equations

$$\begin{aligned} \mu\Delta u(x) + (\lambda+\mu)\nabla\nabla\cdot u(x) + f(x) + \rho\frac{\partial^2 u}{\partial t^2} - \gamma\nabla T &= 0 \\ \frac{\partial T}{\partial t} - \kappa\Delta T &= \mathcal{F}, \qquad x\in\Omega, t>0 \end{aligned}$$

Boundary conditions

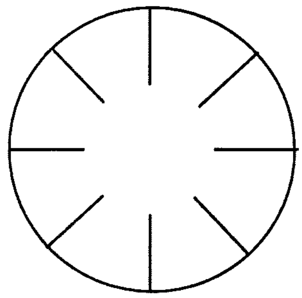
$$\begin{aligned}
\left\{\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\lambda\delta_{ij}\nabla\cdot u\right\}n_j &= \gamma T n_i + p(x), \qquad x\in\partial\Omega_\sigma,\\
u(x) &= \phi(x), \qquad x\in\partial\Omega_u\\
T(x)+A\frac{\partial T}{\partial n}(x) &= \Phi(x), \qquad x\in\partial\Omega
\end{aligned}$$

Initial conditions

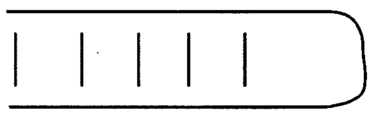
$$T|_{t=0}=T_0, \qquad u|_{t=0}=u_0(x), \qquad u_t|_{t=0}=0, \qquad x\in\Omega$$

There are three important types of cracks:

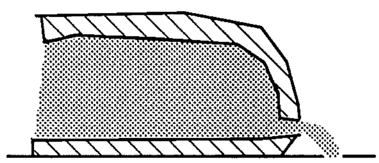
1. Radial cracks



2. Cracks on the surface of a long 'lobe'



3. Cracks on the bottom of the crust

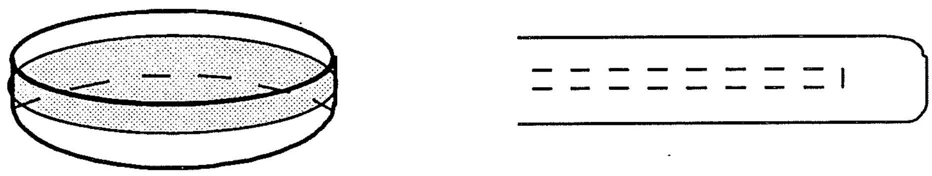


Here we have the usual 'angular' singularity for stresses

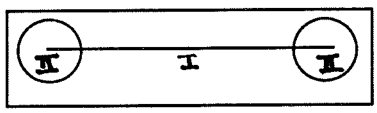
$$\sigma_{ij}\sim C\tau^{-\alpha}\Phi_{ij}(\theta), \qquad \alpha>0$$

Simplified Model

We consider a thin cylindrical layer with a **longitudinal cut.**



For a long 'lobe' and for the axisymmetric formulation one can use a 2D approximation on the cross-section.



The top of the crust (region I) behaves like a 'beam' (or thin plate) and the displacement vector satisfies the fourth order equation,

$$D_4\left(\frac{\partial}{\partial x}\right)u = \mathbf{D}(x)$$

Boundary conditions are derived due to the boundary layer effect in the vicinity of the crack tips.

In a neighbourhood of the tips of the crack (region II) the stress components have a singular behaviour

$$\sigma_{ij} \sim \tau^{-1/2}\sum_{k=1}^{3} C^{(k)}\Phi_{ij}^{(k)}(\theta)$$

Let us emphasise that region I is related to the cracks of first two types (1,2), and region II corresponds to the last kind of fracture (3).

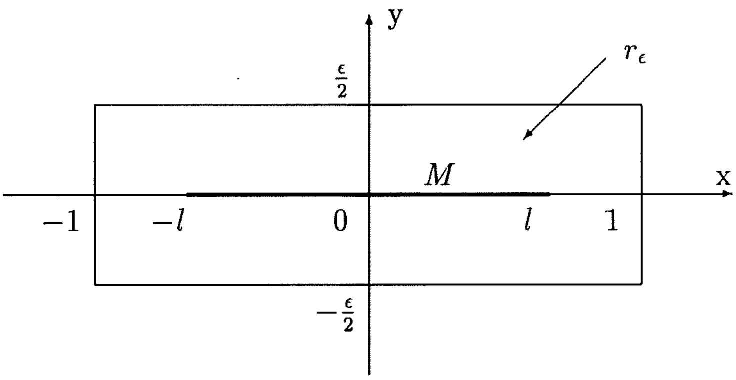
To show the idea of the asymptotic study of the problem for a longitudinal cut in a thin layer, we present some results related to the problem of anti-plane shear of a thin rectangle with a longitudinal cut. We should emphasise that we have to deal with the effect of a boundary layer in the vicinity of the tips of a cut. The problem of a boundary layer is described by means of solutions of the homogeneous problem in a semi-infinite strip.

Finally, we should say that the present approach is based on the results of the following papers

1. N.Kh. Arutyunyan, A.B. Movchan and S.A. Nazarov, *Advances in Mechanics*, Vol. 10, N4, 3-91, 1987.

2. S.A. Nazarov, *Izv. AN Arm.SSr*, XL, N5, 24-34, 1987.

Formulation of the problem.



$$
\begin{aligned}
M &= \{(x,y) : |x| < l, y = 0\}; \\
r_\epsilon &= \left\{(-1,1) \times \left(-\frac{\epsilon}{2}, \frac{\epsilon}{2}\right)\right\} / M \\
\xi &= \frac{y}{\epsilon} \\
-\Delta u(\epsilon, x, y) &= \epsilon^{-2} f(x,\xi), \qquad (x,y) \in r_\epsilon \\
\frac{\partial u}{\partial y}\left(\epsilon, x, \pm\frac{\epsilon}{2}\right) &= \frac{1}{\epsilon} p_\pm, \qquad x \in (-1,1) \\
\frac{\partial u}{\partial y}(\epsilon, x, \pm 0) &= \frac{1}{\epsilon} q_\pm, \qquad x \in (-l,l) \\
u(\pm 1, y) &= 0, \qquad |y| < \frac{\epsilon}{2}
\end{aligned}
$$

Statement 1.i) The principal term of the function u admits the following form ($|x \pm l| > \delta$).

$$
\begin{aligned}
u &\sim \frac{1}{\epsilon^2} w(x), \qquad l < |x| < 1 \\
u &\sim \frac{1}{\epsilon^2} w^\pm(x), \qquad |x| < l, \pm y > 0
\end{aligned}
$$

ii) Functions w and $w^\pm$ satisfy the equations

$$
\begin{aligned}
\frac{\partial^2 w}{\partial x^2}(x) &= p_-(x) - p_+(x) - \int_{-1/2}^{1/2} f(x,\xi) d\xi, \qquad l < |x| < 1 \\
\frac{\partial^2 w^+}{\partial x^2}(x) &= 2(q_+(x) - p_+(x)) - 2\int_0^{1/2} f(x,\xi) d\xi, \qquad |x| < l, y > 0 \\
\frac{\partial^2 w^-}{\partial x^2}(x) &= 2(p_-(x) - q_-(x)) - 2\int_{-1/2}^{0} f(x,\xi) d\xi, \qquad |x| < l, y < 0
\end{aligned}
$$

and the following conditions at the points $x = \pm 1, x = \pm l$

$$\begin{aligned}
w(\pm 1) &= 0, \\
w(l) &= w^+(l) = w^-(l), \\
2\frac{\partial w}{\partial x}(l) &= \frac{\partial w^+}{\partial x}(l) + \frac{\partial w^-}{\partial x}(l), \\
w(-l) &= w^+(-l) = w^-(-l), \\
2\frac{\partial w}{\partial x}(-l) &= \frac{\partial w^+}{\partial x}(-l) + \frac{\partial w^-}{\partial x}(-l).
\end{aligned}$$

Statement 2. In the vicinity of the ends $x = \pm l$ of the cut the function u has the behaviour

$$u(\epsilon, x, y) \underset{\rho \to 0}{\sim} \epsilon^{-2} \left(\text{const} + \sqrt{\epsilon} C \rho^{1/2} \cos\left(\frac{\phi}{2}\right)\right) + O(\rho),$$

where (ρ, ϕ) is a polar coordinates system; $\phi = 0$ corresponds to the direction along the cut. In a neighbourhood of $x = -l$ the following equality holds

$$C = \frac{1}{\sqrt{2\pi}} \left(\frac{\partial w_+}{\partial x}(-l) - \frac{\partial w_-}{\partial x}(-l)\right).$$

We have a similar relationship near $x = l$.

Corollary. In a particular case of a symmetric 'deformation' $\int_{-1/2}^{1/2} f(x, \xi) d\xi = 0$, $p_- = p_+, q_- = q_+$ we have

i)

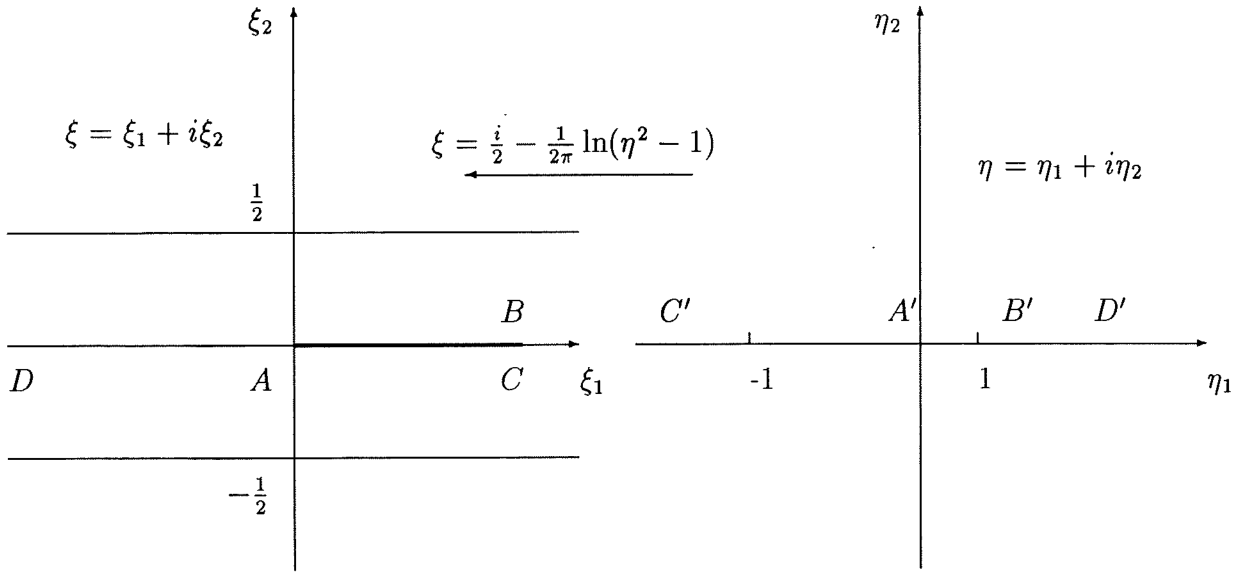
$$\begin{aligned}
w &= 0, \\
w^+ &= -w^- \\
\frac{\partial^2 w_-}{\partial x^2}(x) &= 2(q - p) \\
w^+(-l) &= w^+(l) = 0
\end{aligned}$$

ii) $u(\epsilon, x, y) \sim \epsilon^{-2}(\text{const} + \sqrt{\epsilon} C \rho^{1/2} \cos(\phi/2))$, where

$$C = \sqrt{2\pi} \frac{\partial^2 w_+}{\partial x^2}(-l)$$

.

Appendix
The conformal mapping



$O\eta_1\eta_2$:

$$
\begin{aligned}
\Delta_{\eta_1\eta_2}\Gamma(\eta_1,\eta_2) &= 0, \qquad \eta_2 > 0 \\
\frac{\partial \Gamma}{\partial \eta_2}(\eta_1, +0) &= 0, \qquad \eta_1 \in \mathbb{R} \\
\Gamma(\eta_1,\eta_2) &\underset{|\eta|\to\infty}{\longrightarrow} 0, \\
\Gamma(\eta(\xi)) &\sim \pm\xi_1, \qquad \xi_1 \to +\infty, \qquad \pm\xi_2 > 0 \\
\Gamma(\eta) &= \frac{1}{2\pi}\ln\left|\frac{\eta-1}{\eta+1}\right|
\end{aligned}
$$

A. Movchan 25.6.92

A simple dynamic model

Goal :

- To study the spreading of molten UO_2

Assumptions

- Axisymmetric

Observations from experiments indicate that the flow is unlikely to be truly axisymmetric, but here we are looking at the simplest possible case, and restrict attention to radial flow.

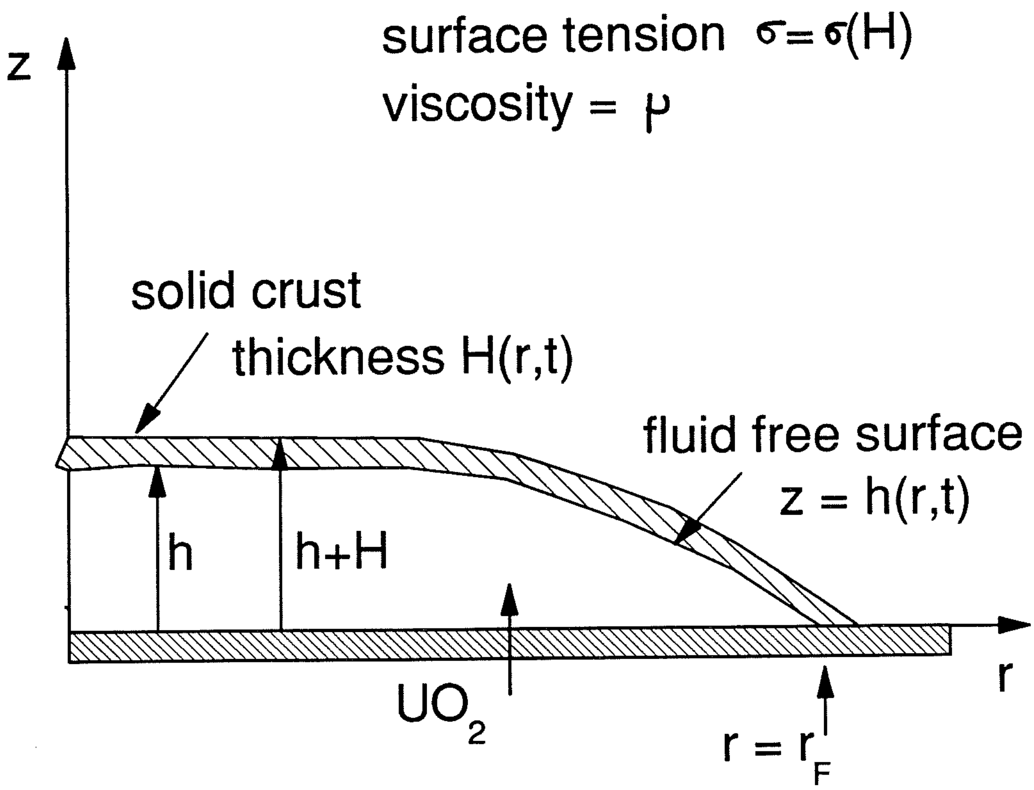
- Slowly moving

This is an asymptotic study of the flow away from the hydraulic jump (if present), where the fluid velocity is small, and viscous forces dominate. Quoted figures of characteristic flow rate $Q \sim 1ls^{-1}$, characteristic radial length scale $L \sim 1m$ give a Reynolds number of 330. The characteristic depth of fluid H is on the order of centimetres, giving a reduced Reynolds number $Re(H/L)^2$, which is small compared to unity, thus justifying neglecting inertial terms in the momentum equations.

- Conduction dominated

The Prandtl number, $Pr \sim 0.35$ so that the product of the Peclet number and $(H/L)^2$ is much less than unity. In this circumstance, the temperature field is conduction dominated.

The physical model is shown below.



Since the fluid layer is typically thin, we use the lubrication approximations. This yields the velocity flow field,

$$u(r,z,t) = \frac{\partial p}{\partial r}\left[\frac{z^2 - 2hz}{2\mu}\right] + \frac{\partial \sigma}{\partial r}\frac{z}{\mu}, \tag{1}$$

where the pressure field is given by

$$p = \rho g(h+H) - \sigma \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right), \tag{2}$$

r and z are the usual polar coordinates, t denotes time and g is the acceleration due to gravity. In writing down the above equations we have assumed that variations in viscosity due to thermal effects are negligible. The second term on the right-hand side of (1) is a surface shear stress due to the surface tension gradient of the crust. The structural properties of the crust were unknown to us, but after some discussion we decided it was reasonable to assume the tensile strength of the crust was a function of its thickness H. Thus we model the surface tension of the molten UO_2 at the crust interface as being $\sigma = \sigma(H)$.

The evolution equation for the height of the fluid film and solid crust is then

$$3\mu\frac{\partial h}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left[rh^3\frac{\partial}{\partial r}\left(\sigma(H)\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial h}{\partial r})\right) - \rho g(h+H) + \frac{3}{2h}\sigma(H)\right)\right]. \tag{3}$$

This equation can be modified to include the fact that there is a source of molten UO_2 at $r = 0$. This entails adding an extra flux to the left-hand side of the flow domain. The boundary conditions at the right end of the flow domain are $q = 0$ and $h = 0$ at $r = r_F$. Some stress criterion could be added to this model to allow for crust rupture. The boundary conditions at the left end of the flow domain are less straightforward and will be left for future consideration.

Temperature Distribution

We assume that the temperature of the molten UO_2 remains at the melting temperature $T = T_m$ across the molten layer. The temperature variation across the crust is given by

$$\frac{\partial^2 T}{\partial z^2} = 0 \tag{4}$$

subject to the boundary conditions

$$T = T_m \quad \text{at} \quad z = h \tag{4a}$$

and

$$-k\frac{\partial T}{\partial z} = \sigma_B T^4 \quad \text{at} \quad z = h + H \tag{4b}$$

where σ_B is the Stefan-Boltzmann constant.

Heat balance across the solid crust

The heat flux across the solid crust/molten fluid interface, must equal the change in thermal energy of the crust. Thus

$$\rho L \left[\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial r} + \frac{H}{r} \frac{\partial (ru)}{\partial r} \right] = -k \frac{\partial T}{\partial z} \tag{5}$$

evaluated at $z = h$, with $u(r, z, t)$ as defined in (1). In the above equation, ρ is the crust density per unit width, and L is the latent heat of the crust.

Thus (3) and (5) are two coupled equations for h and H which can be solved at each time step. One method of solving these equations is to use a finite difference time marching scheme.

J. Moriarty 26.5.92

The Crust and 'Surface Tension'

An addendum to J. Moriarty's report entitled: **A Simple Dynamic Model**

When the UO_2 is entirely liquid, as at initial meltdown, then it seems reasonable to give it the usual properties of a viscous liquid, including a surface tension. However, as it cools from the upper surface, the liquid first forms a flexible film or membrane and then this develops into a crust (a 'thick' film?) at which stage the boundary condition must be modified.

A film might still be interpreted as providing a tensile capability, at the surface, which is analogous to a surface tension but much larger than the 'real' surface tension of the molten liquid. It is reasonable to expect that this 'effective surface tension' increases with the thickness of the film (but difficult to envisage how it may be characterised by experiment).

Once a crust is formed and prior to rupture, the problem changes to an essentially radial flow between two coaxial plates, with the surface tension theory possible governing the 'leading edge'. The lower plate is a flat, rigid base which is an excellent conductor of heat, with little cooling of the liquid at the interface. The upper plate (the crust) is a circular elastic shell of varying thickness (dependent on the temperature field) with a circular hole where the molten liquid enters. The outer radius of the shell increases with time at a rate to be determined.

Lubrication theory still seems a reasonable model, since the fluid thickness is still typically 'small'. However, its upper surface condition is now one of 'no slip', i.e.

$$u(r,0) = u(r,h) = 0.$$

The relevant theory would require amendment of (1), (2) and hence (3) - refer main report - which together with (5) would then provide the flow solution. This solution would then give the shear traction condition applied to the crust.

The analysis of the crust at each time t is as for an elastic plate of given varying thickness being deformed by the action of radial surface shear traction

$$S(r) = \mu\frac{\partial u}{\partial z} \qquad at\ z = h,$$

applied to its lower surface. The deformation and stress analysis (cf. Timoshenko and Woinowsky-Krieger, Ch.9, Section 67 etc.) would provide stress magnitudes – σ_{rr} and $\sigma_{\theta\theta}$ in particular – which can be used in a stress criterion for fracture (circumferential cracking if σ_{rr} reaches a critical value, radial rupture if $\sigma_{\theta\theta}$ reaches the same critical value); alternatively the displacement solution would allow a critical-maximum-displacement criterion to be used.

Improvements to this crust/liquid model would be to include thermoelastic deformation of the crust and a temperature-dependent viscosity of the liquid, with the viscosity becoming infinite at the solidification temperature.

T. Rogers 4.9.92

The Bladder Problem

In this approach, the skin formed by the cooling of the outer material is assumed to behave like a flexible elastic membrane. The membrane will have no bending stiffness and the tensions will be some nonlinear function of the extension. The simple example is the rubber balloon where it is known that the nonlinear nature of the material allows the possibility of two different inflated configurations at the same internal pressure. This phenomenon is well known to anyone who has inflated a bicycle inner tube. Initially the tube inflates symmetrically to form a torus but, as the volume of air contained in the torus is further increased, there comes a stage at which one section of the tube suffers a much larger extension than the remainder. At first sight this might be regarded as a weak area of the tube, but it is easy to show that this is not the case since the area of larger inflation can be manipulated to some other section of the tube and will remain there.

It is possible to envisage a similar sort of phenomenon here. The molten material forms an outer flexible membrane on cooling, but, since more material is being supplied, the membrane must extend in order to accommodate the increase in volume. There then exists the possibility of the membrane being extended to a state which would allow some portion to undergo a much larger inflation than the remainder with a consequent thinning of the membrane in that region. Continued expansion due to a further supply of fluid material could then result in fracture of the membrane, allowing the fluid to flow out.

Consider an initially spherical membrane of initial radius R_0 which is filled with with a volume $V > \frac{4}{3}\pi R_0^3$ of weightless fluid, causing it to inflate to a new radius $R = (3V/4\pi)^{1/3}$. Now imagine this membrane allowed to rest on a horizontal table and switch on gravity so that the body takes on the configuration shown in Figure 1c. The point initially at P_0 goes to P_0' in the inflated configuration and to P in the deformed configuration. The tangent at P makes an angle $\psi(\theta)$ with the upward vertical (Oz), $s(\theta)$ denotes the arc length OP, $z(\theta) = ON$ is the height of P above the horizontal table and $NP = r(\theta)$ is the radius. Then the equations of equilibrium for the membrane are

$$\frac{d}{d\theta}\left(\frac{T}{s\prime}\right) = (T - S)\frac{sin\psi}{r} = 0, \tag{1}$$

$$T\frac{d}{d\theta}(cot\psi) + Scosec^2\psi cos\psi s\prime = ps\prime^2, \tag{2}$$

where T is the azimuthal tension at P and S is the hoop tension at P. These tensions will be some function of the deformation at P, the function being a characteristic of the membrane material,

$$T = T(s\prime, \psi), S = S(s\prime, \psi). \tag{3}$$

The prime denotes differentiation with respect to θ and $p = p(\theta)$ is the fluid pressure within the membrane at P. This is given by

$$p = p_A + \rho g\{z(\pi) - z(\theta)\}, \tag{4}$$

where p_A is the pressure at A, ρ is the fluid density and g the acceleration of gravity. For specified functions T and S in equations (3), equations (1–2) form a pair of coupled O.D.E.'s for $s(\theta), \psi(\theta), r(\theta), z(\theta)$. These, together with the equations

$$\frac{dr}{d\theta} = sin\psi(\theta)\frac{ds}{d\theta}$$

$$\frac{dz}{d\theta} = cos\psi(\theta)\frac{ds}{d\theta}$$

can be integrated numerically to determine the deformed shape and the resulting tensions T and S at each point. The initial conditions are $r(0) = 0, s(0) = 0, z(0) = 0, \psi(0) = 0$ and it is necessary to satisfy the conditions $r(\pi) = 0, \psi(\pi) = 0$. In order to do this it will be necessary to employ a 'shooting' method in which estimated values are given for p_A and $s'(0)$ and these values are modified until a solution is obtained which matches the required conditions at $\theta = \pi$. Alternative conditions can be derived if the sphere is not closed but has a spherical cap removed so that fluid is accumulating within the membrane.

The solution obtained here is based on the assumption of a symmetric deformation. In order to examine the problem of fluid breakout it is then necessary to impose a small perturbation on this solution and to use this to examine the stability of the symmetric configuration. For appropriate forms of the function in equations (2) there will exist symmetric solutions which are such that small perturbations will grow with time indicating the onset of instability. The equations governing these perturbations can be derived from the general equations for membranes in the absence of symmetry but they are not reproduced here.

W.A. Green 25.7.92

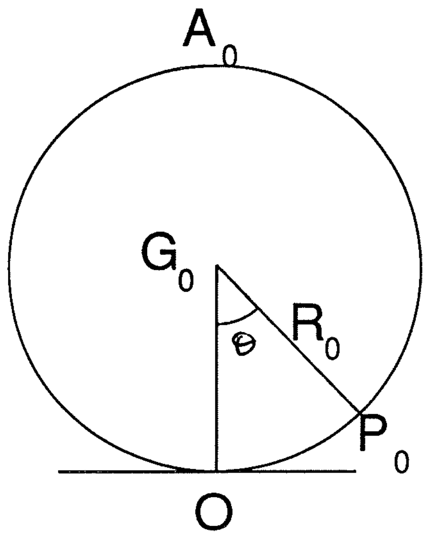


Fig 1a

Initial membrane

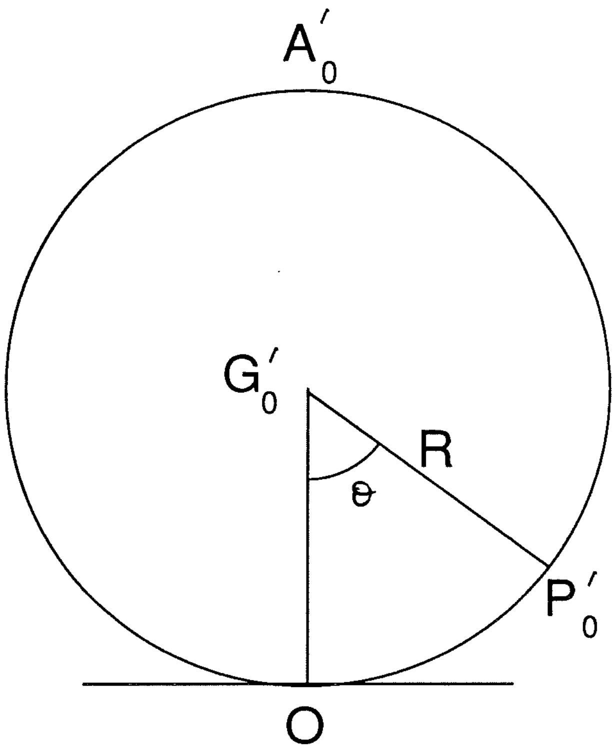


Fig1b

Inflated membrane

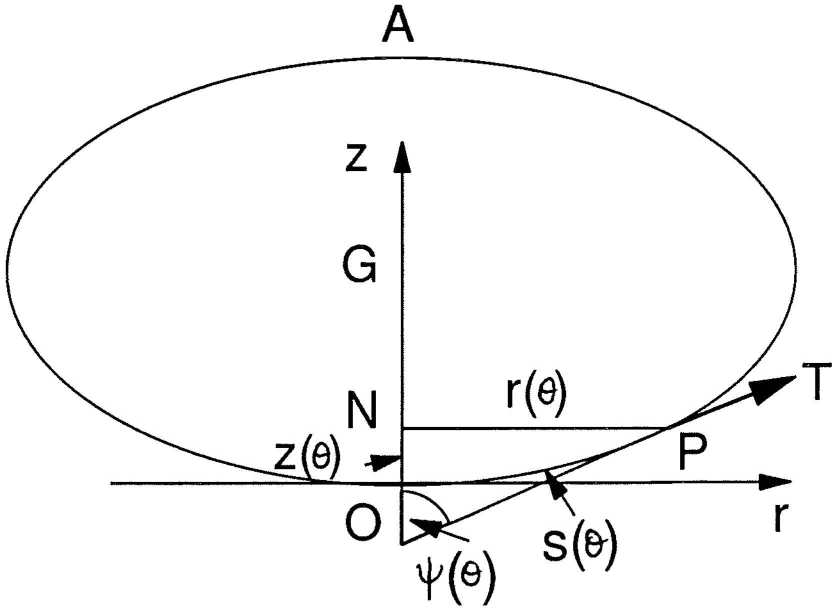


Fig 1c

Inflated membrane containing heavy liquid

Hydraulic jumps

The flow of liquid down onto a flat plate usually results in a rapid outflow brought to subcritical, often near zero, velocities by a hydraulic jump. The jump is usually at a radius defined by the height of the subcritical flow which is itself determined by conditions at the spreading contact line boundary. After the initial, splash-type, spread it is probably quite adequate to treat the supercritical flow and hydraulic jump as quasi-steady, since most fluid particles pass through this area in a time which is short compared with the evolution of the pool of liquid surrounding the jump. As this deepens, it is possible for the jump to become 'submerged'.

In the parameter range of experiments presented at the Study Group, it seemed likely that, at least initially, the jump may be determined by bed friction slowing down the supercritical diverging flow. This problem is treated in some detail by Watson (1964, Journal of Fluid Mechanics **20**, 481–499). The results from a computer program, written to evaluate the effects of the boundary layer growing from the bed on the flow using Watson's equations, indicated that this had an important influence on the flow and that some interesting further study would be worthwhile.

The hydraulic jump could be important for heat transfer in the rapid flow portion of the event. This is because of the greatly enhanced turbulence caused by the energy dissipation in the jump. For strong jumps (Froude number > 4.5), this is well documented in a recent book which also provides a good literature review: W.H. Hager 'Energy dissipators and hydraulic jump', Kluwer Academic Publishers, 1992. Recent work on this topic by Bowles and Smith (1992, Journal of Fluid Mechanics **242**, 145–168) follows Watson (1964) in using boundary-layer theory, but also considers viscous-inviscid interaction.

D.H Peregrine 25.9.92