OPTIMIZATION OF COMMODITY PORTFOLIO MANAGEMENT

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1. The problem statement

The problem we consider is introduced by Uljarice Bačka, LLC. The core business activities of the company are trade of agriculture commodities, warehousing and distribution and crops production. The main traded goods are: corn, wheat, barely, sunflower, soybean, soybean meal and raw material for crops production: fertilizers, plant protection products, seeds and other. Since a large part of company's activities relays on corn, predicting the price of that good is of the main interest. In order to make a reasonable predictions, models which incorporate the crucial factors influencing the corn prices are needed. Of course, the important issue is which data are available. Within the data that we obtained, the correlation analysis is performed in Section 2 to point out the relevant parameters. In Section 3 we introduce different methods for obtaining the predictions and provide some numerical results. According to the current results, some conclusion remarks are given in the last Section.

2. Identification of relevant parameters

In order to identify relevant parameters which influence the price of corn, we analyzed some basic statistic indicators. The following Table 1 shows the correlation coefficients between the corn price in Serbia and the price of wheat, the price of corn in Budapest, the price of corn in Chicago and the exchange rate of euro/dinar:

There is a high correlation between the corn prices at Serbian and Hungarian market, as well as between the corn and wheat prices. There is also a significant correlation between the corn price in Serbia and global market

Year	Wheat	Budapest	Chicago	Euro
2011	0.59	0.94	0.53	-0.14
2012	0.90	0.97	0.83	0.63
2013	0.92	0.95	0.90	-0.84

TABLE 1. Correlations

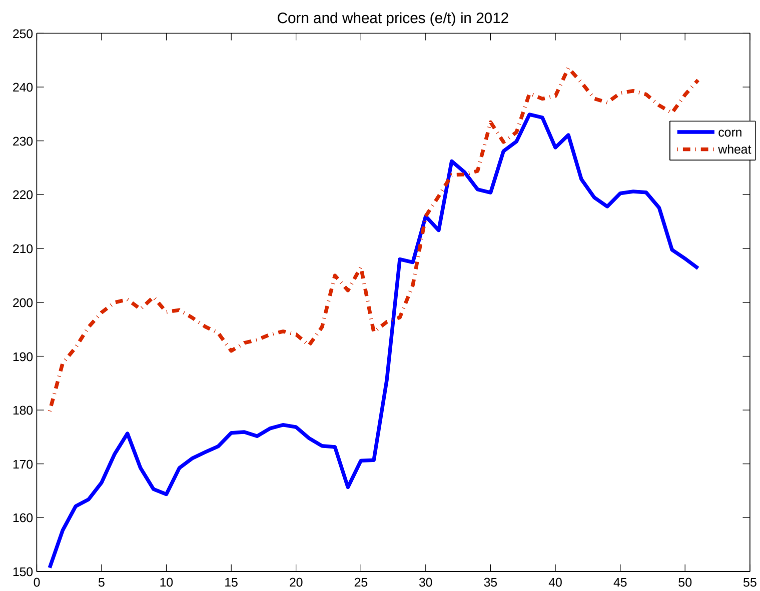

FIGURE 1. Corn and wheat prices 2012 in Serbia

trends expressed through CBOT. These observations are illustrated in the following Graphs.

There is a very good agreement in the movement of corn and wheat price, but there is also high seasonal price movement. Namely, from October until July corn and wheat prices move almost parallel, but after July there is no pattern. Next Graphs show wheat and corn prices in the period Jun-October 2012 and Jun-October 2013 and show the corn price movement at Serbian, Hungarian and Chicago market.

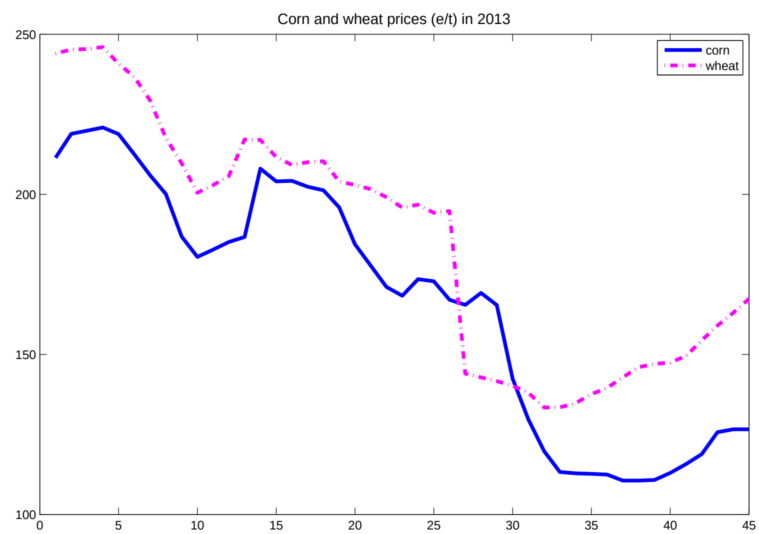

FIGURE 2. Corn and wheat prices 2013 in Serbia

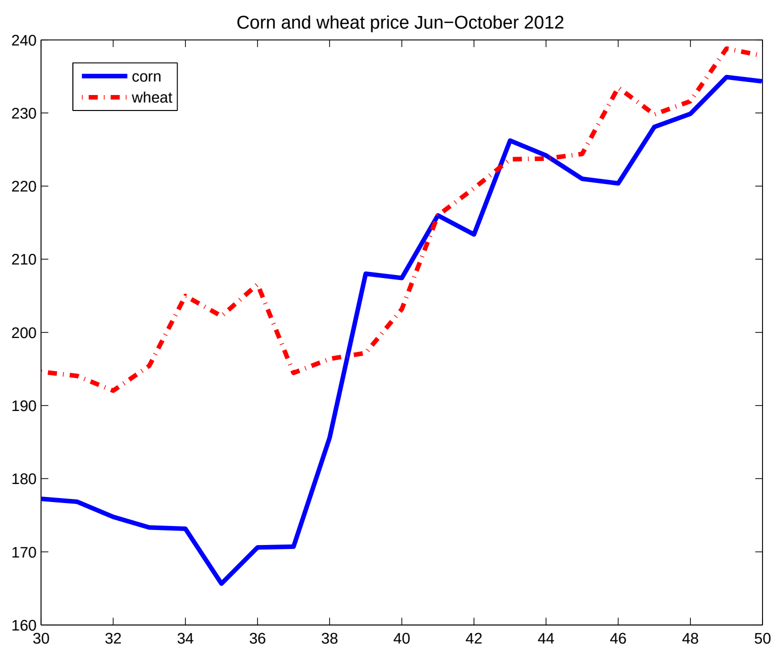

FIGURE 3. Corn and wheat prices Jun-October 2012 in Serbia

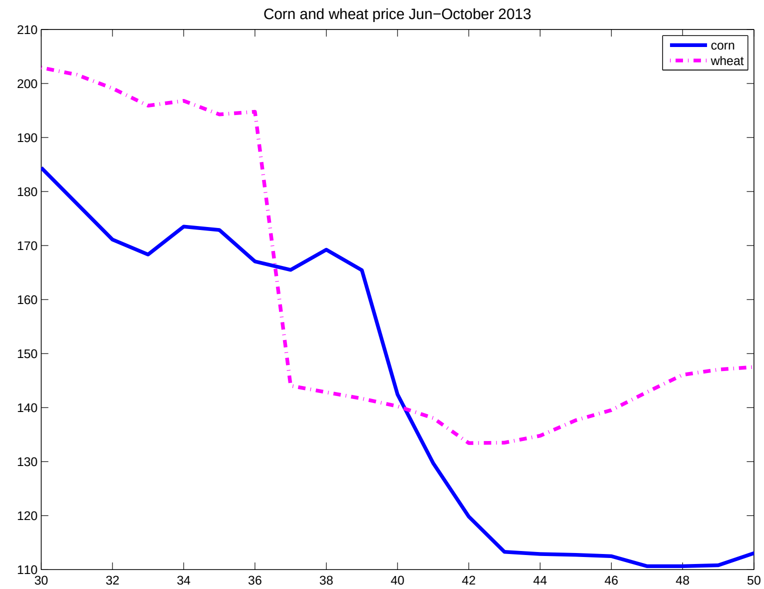

FIGURE 4. Corn and wheat prices Jun-October 2013 in Serbia

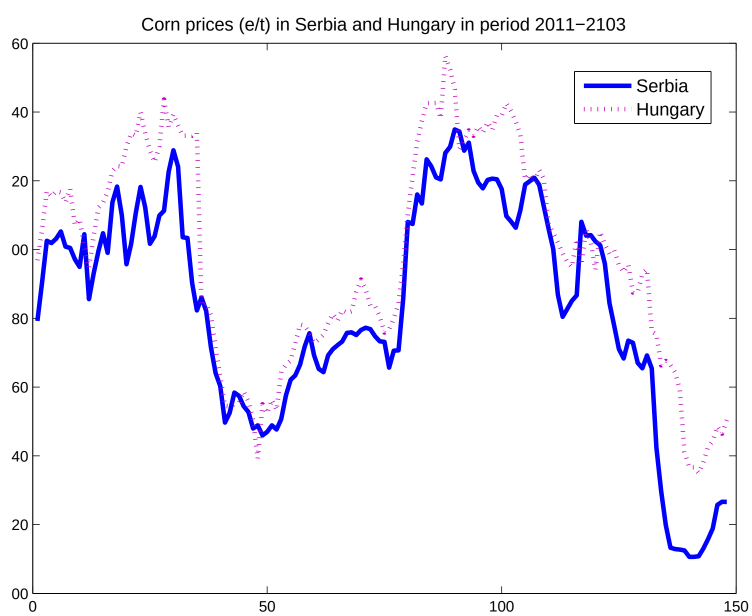

FIGURE 5. Corn prices in Serbia and Hungary 2011-2013

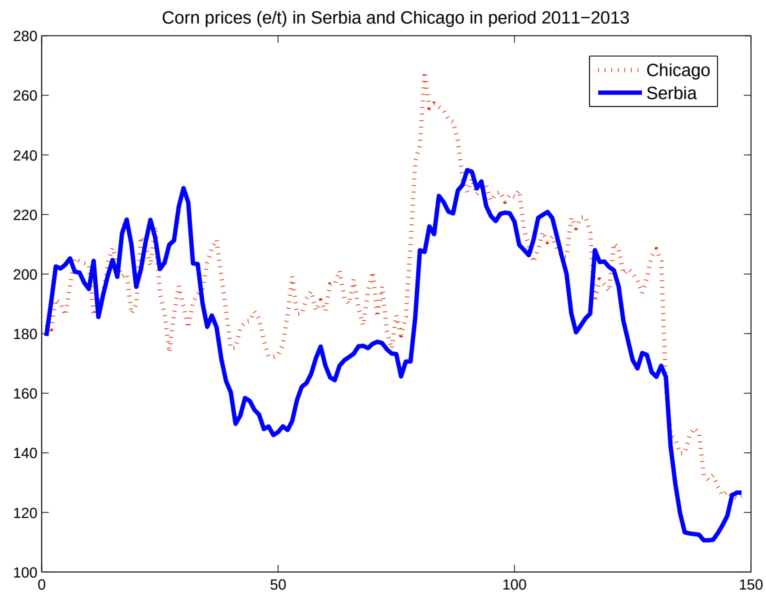

FIGURE 6. Corn prices in Serbia and Chicago 2011-2013

The best model we obtain by linear regression is the following

$$\widehat{Y} = -12,9273 + 0,314666 * x_w + 0,68945 * x_{bc},$$

where $\widehat{Y}$ is the estimated corn price, x_w is the wheat price and x_{bc} is the corn price in Budapest. SEE in this model is 5.01 and R^2 is 96%.

3. PREDICTION METHODS AND NUMERICAL RESULTS

3.1. **CBOT Future contract prices.** Futures is a financial contract obligating the buyer to purchase an asset (or the seller to sell an asset), such as a physical commodity or a financial instrument, at a predetermined future date and price. They were originally designed to allow farmers to hedge against changes in the prices of their crops between planting and when they could be harvested and brought to market. Today producers of corn could use futures to lock in a certain price and reduce risk.

The futures price for a given commodity represents the market's best estimate of what the real price of the commodity will be at the maturity date specified in the futures contract. We investigated the idea of using the corn future prices that are traded on CBOT [1] as an indicator of the real price of corn at the maturity date. Graph shows price movements for the price of corn implied from one future contract assuming the risk free rate as the prevailing interest rate and the discounted maturity price of corn by the risk free rate from the same contract.

[1]In 2007 CBOT and CME merged creating the largest derivatives market ever

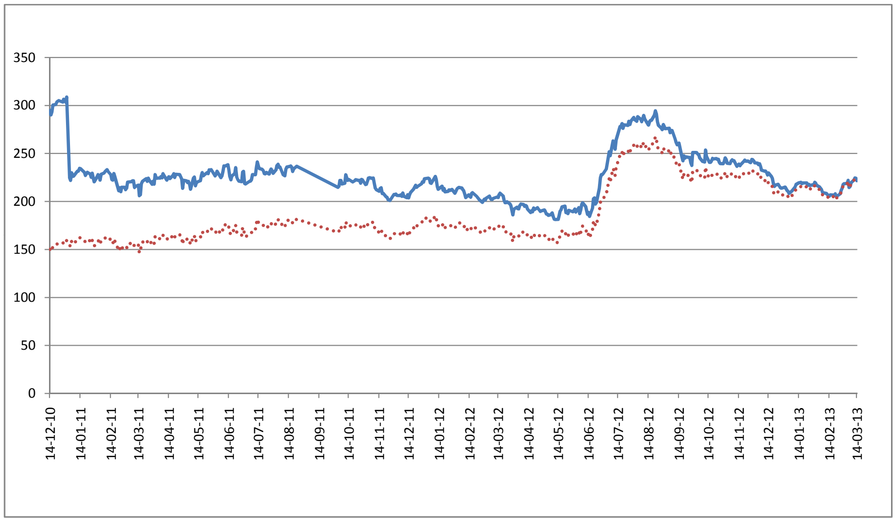

FIGURE 7. Implied and discounted corn prices

We could see on Figure 7 that these two prices are converging to each other as expected. The open question here is the question of the prevailing interest rate and that question should be investigated further. Also, while the futures price provides a point forecast of the cash price at contract maturity, it says nothing about the potential range of prices within which the real price may fall. But the future implied price could be used as a boundary condition for a models such as GARCH (and we will do it in Subsection 3.3).

3.2. **Binomial tree.** The binomial tree model assumes that at each time step price will change to one of two possible values. We begin with an initial price S_0 and we determine two positive numbers d and u such that $0 < d < u$. Then, at the next period the price will be either $d\,S_0$ or $u\,S_0$. Typically, d and u are chosen to satisfy $0 < d < 1 < u$, so change of the price from S_0 to $d\,S_0$ represents a downward movement, and change of the price from S_0 to $u\,S_0$ represents an upward movement. So at each time step two probabilities for the price change are needed, probability p_u that at the next period price will follow an upward movement with the coefficient u and the opposite movement with the coefficient d. Then $1 - p_u$ represents probability of price decrease.

We suppose that time step is one week. Let S_t^y denote corn price at the week t of the year y. Then we define

$$p = \frac{-S_{t-1}^y + 200}{100}$$

and we choose probabilities up p_u and down p_d in the following manner

$$p_u = \begin{cases} 1, & p > 1 \\ p, & 0 \le p \le 1 \\ 0, & p < 0 \end{cases}, \qquad p_d = 1 - p_u.$$

Furthermore, we define

$$U = 1 + \mu - 0.5\sigma^2$$
$$D = 1 - (\mu - 0.5\sigma^2).$$

Parameter σ is supposed to take the following value $\sigma = 0,02049$, while μ is estimated as an average value:

$$\mu = \frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4},$$

where μ_i stands for

$$\mu_i = \frac{S_t^{y-i} - S_{t-1}^{y-i}}{S_{t-1}^{y-i}}.$$

Finally, price prediction for one week is calculated with help of the following formula

$$S_t^y = S_{t-1}^y\,(U\,p_u + D\,p_d)$$

The results are shown at Figure 8.

The prediction is rather good up to July with the average error of 0.266, while the period July - October proves to be difficult for prediction as expected.

Another possibility is to make price prediction within four weeks. In that aim, quantities above are defined in the same manner with back movement of four weeks. We represent these results with Figure 9.

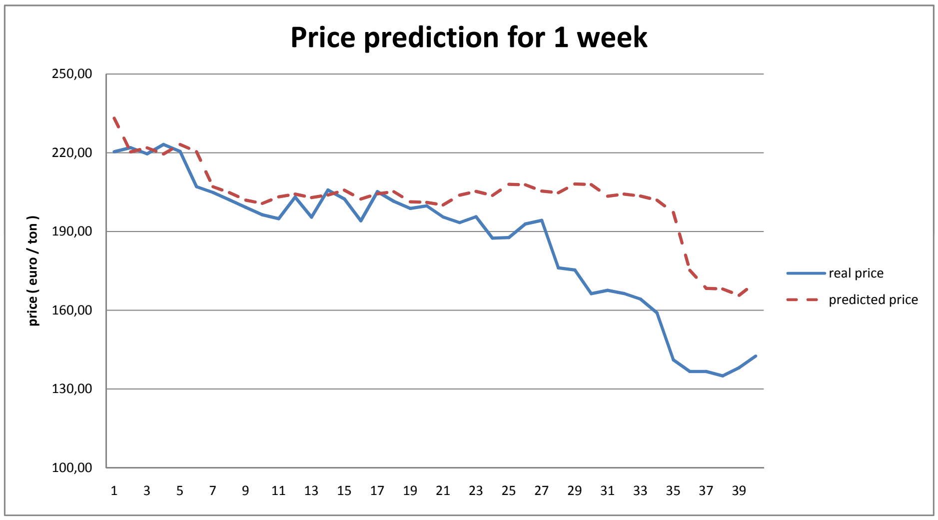

FIGURE 8. Binomial tree prediction - one week

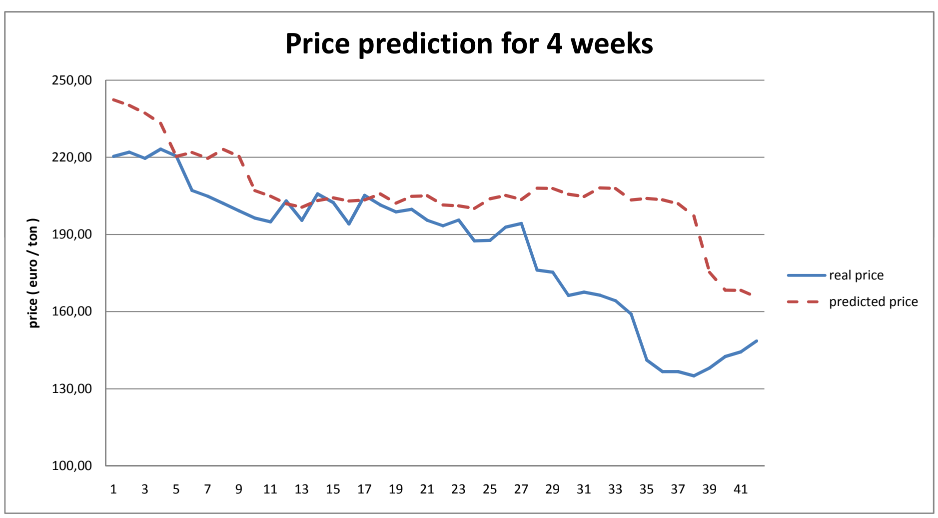

FIGURE 9. Binomial tree prediction - four weeks

3.3. **GARCH model.** In general, GARCH(p,q) model is used for obtaining the volatility estimates. In this model, the assumption of a constant volatility is abandoned and therefore the volatility is allowed to be time-dependent. It is updated in discrete time moments t_i - weekly in this particular case. One of the main ideas of this model is to allow us to put more weight (significance) to more recent data. For more information on this topic one can see [1] for example.

Given the time series of prices for the corn $S_{t_i} := S_i$, we calculate the returns by the following formula

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}}.$$

The volatilities σ_n, or more precisely the variances σ_n^2, are updated in the following manner

$$\sigma_n^2 = \alpha_0 V + \sum_{i=1}^{p} \alpha_i u_{n-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{n-i}^2. \tag{1}$$

Here, $\alpha_i, i = 0, ..., p$ and $\beta_i, i = 1, ..., q$ are positive coefficients which sum up to one. Usually, V represents some long-run variance estimate and the coefficients are obtained by solving the maximum likelihood problem. Assuming that the returns are normally distributed with mean zero and variance σ_t^2, i.e. $u_i : \mathcal{N}(0, \sigma_i^2)$, and that the variance is updated by (1), we obtain the problem of maximizing the function

$$\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{u_i^2}{2\sigma_i^2}}$$

under the suitable constraints. Equivalently, we aim to solve

$$\min_{\alpha_i, \beta_i} \sum_{i=1}^{m} \ln \sigma_i^2 + \frac{u_i^2}{\sigma_i^2} \tag{2}$$

subject to

$$\alpha_i, \beta_i > 0, \quad \sum_{i=0}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i = 1.$$

Furthermore, using the simulations we obtain the trajectories of predicted prices.

The simplest GARCH model is GARCH(1,1). It aims to update the volatilities only according to the most recent data and the long-run variance estimate which leaves us with only 3 parameters to be estimated by (2). However, varying p and q has a big influence on the resulting forecast. Moreover, we saw in Section 3.1 that prices induced from the future contracts can be used as relatively good approximations for prices at some future points. Therefore, we used this information to tune the parameters p and q. To be more precise, assume that we are at 1st October in year 2012 and we have one year long time series. Suppose that we know the price that

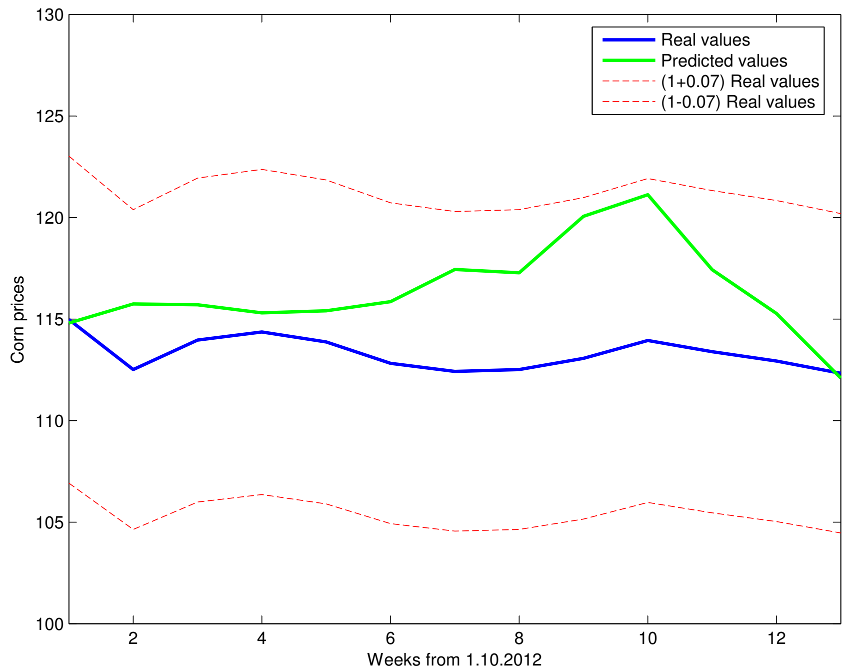

FIGURE 10. GARCH(p,q) forecast

is going to occur on 1st of January 2013. We can view this as some kind of boundary condition. In other words, we can choose parameters p, q such that the predicted price on 1st of January obtained by the GARCH(p,q) model is the closest to the one that we got from another (more reliable) source. We present the resulting forecast.

Besides the predictions, this graph represents the real Serbian market prices of corn stated in euros per ton and adjusted to include the exchange rate influence. The results show that the relative error does not exceed 7% in the observed three month prediction. The average relative error for that period is around 2.8% while the relevant number for one month prediction is 1.8%.

3.4. **Stochastic approach.** From the historical data one can easily see that the period from corn harvest in mid October until wheat harvest at the beginning of July is rather stable regarding corn price. Therefore, Black-Scholes formula can be used in order to model dynamic of the price for this period. The idea is to estimate the parameters that govern the price process, drift and volatility using the historical data from different sources.

3.4.1. *Mathematical model.* Let S_t denote the corn price at the moment t. We assume that the variability of the return in a short period of time Δt is the same regardless of the corn price. This suggests that the standard deviation of the change in a short period of time Δt should be proportional to the corn price and leads to the model

$$\mathrm{d}S_t = \mu\, S_t \mathrm{d}t + \sigma\, S_t\, \mathrm{d}W_t,$$

where W_t has the normal distribution $\mathcal{N}(0,t)$. The parameter μ is the expected rate of return (the drift).

In more general case, we assume that S_t satisfies a stochastic differential equation

$$\frac{\mathrm{d}S_t}{\mathrm{d}t} = \mu\, S_t + \sigma\, S_t\, \xi_t,$$

where μ and σ are supposed constant on each time step (to be specified later) and ξ_t represents the white noise.

The white noise ξ_t can be seen as derivative of Brownian motion W_t in the sense of distributions i.e. $\xi_t = \mathrm{d}W_t/\mathrm{d}t$. Therefore, equation (3) can be formally written in differential form

$$\mathrm{d}S_t = \mu\, S_t\, \mathrm{d}t + \sigma\, S_t\, \mathrm{d}W_t, \tag{3}$$

where W_t has the normal distribution $\mathcal{N}(0,t)$, as before.

The solution of SDE (3) is a stochastic process

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\, W_t^*}, \tag{4}$$

where W_t^* has standardized normal $\mathcal{N}(0,1)$ distribution.

Numerical results are presented in the next Section. We use Monte Carlo simulation [2] of this stochastic process as a way of developing some understanding of the nature of the corn price process in equation (3).

3.4.2. *Numerical results.* In order to test numerically the mathematical model described above, drift μ and volatility σ should be estimated. In fact, it turns out that the crucial point is good estimation of drift μ. Let us describe first the methodology that we use.

It has been noted that prices in Budapest, Chicago and Serbia from the last season have impact at the price in Serbia in the current season. Also, the price strongly depends on the state of the global market in the current

[2] A Monte Carlo simulation of a stochastic process is a procedure for sampling random outcomes for the process.

season. We may say that prices in Budapest and Chicago reflect the behavior of global market. Also, market in Serbia at the current season should be taken into account in order to capture local phenomenon, that may be different from the global one. A reasonable idea is to update the parameters whenever possible. So we decided to take into account the prices in Budapest, Chicago and Serbia in the past months of the current season.

In order to perform a numerical test, let us assume that we are in November 2012 and we want to make a prediction of prices for the period November 2012–June 2013 at the Serbia market.

Let us first estimate the drift μ, which amounts to estimate the returns in the current season 2012/13, since the drift measures the average rate of growth of price. The weekly price list from the past season – from October 2011 until July 2012 – in Budapest, Chicago and Serbia are available. Therefore, week returns of the past season can be calculated. Averaging over each month we obtain returns per month for market in Budapest $\mu^{1,2}_{\mathrm{BP}}$, Chicago $\mu^{1,2}_{\mathrm{Chi}}$ and Serbia $\mu^{1,2}_{\mathrm{Srb}}$:

$$\mu^{1,2}_{\mathrm{BP}} = \begin{bmatrix} 0.002192 \\ -0.026584... \\ 0.025769 \\ 0.030596 \\ 0.000397 \\ 0.012006 \\ 0.007089 \\ -0.005355 \\ -0.004859 \end{bmatrix}, \quad \mu^{1,2}_{\mathrm{Chi}} = \begin{bmatrix} 0,013596 \\ -0,013915 \\ 0,019623 \\ 0,008102 \\ -0,004059 \\ 0,005653 \\ 0,002829 \\ -0,014883 \\ 0,005351 \end{bmatrix}, \quad \mu^{1,2}_{\mathrm{Srb}} = \begin{bmatrix} 0,017100 \\ -0,013792 \\ -0,001927 \\ 0,025699 \\ 0,009220 \\ 0,002810 \\ 0,004795 \\ -0,000531 \\ -0,005671 \end{bmatrix}.$$

We suppose that the return in one month depends on the return in the same month of the last season at global markets (Budapest and Chicago), and in Serbia. Also, we assume that the returns in one month before in all three markets have some impact. In other words, the target return μ is assumed to be a convex combination of the returns in Budapest $\mu^{1,2}_{\mathrm{BP}}$, Chicago $\mu^{1,2}_{\mathrm{Chi}}$, Serbia $\mu^{1,2}_{\mathrm{Srb}}$ in the past season 2011/12 and of returns in past month at all three places $\mu^{2,3}_{\mathrm{BP}}$, $\mu^{2,3}_{\mathrm{Chi}}$, $\mu^{2,3}_{\mathrm{Srb}}$. Therefore, we search for $\alpha_1, \ldots, \alpha_6$ such that

$$\begin{aligned} [\mu]_i = {} & \alpha_1 \left[\mu^{1,2}_{\mathrm{BP}}\right]_i + \alpha_2 \left[\mu^{1,2}_{\mathrm{Chi}}\right]_i + \alpha_3 \left[\mu^{1,2}_{\mathrm{Srb}}\right]_i \\ & + \alpha_4 \left[\mu^{2,3}_{\mathrm{BP}}\right]_{i-1} + \alpha_5 \left[\mu^{2,3}_{\mathrm{Chi}}\right]_{i-1} + \alpha_6 \left[\mu^{2,3}_{\mathrm{Srb}}\right]_{i-1} \end{aligned} \tag{5}$$

is the closest to the actual value $\left[\mu^{2,3}_{\mathrm{Srb}}\right]_i$ for $i = 1, \ldots, 8$. We allow α_k to be dynamic i.e. we assume $\alpha_k = \alpha_k(i)$, for $k = 1, \ldots, 6$. One set of parameter estimations is given in Table 2.

month and year	α_1	α_2	α_3	α_4	α_5	α_6
November 2012 ($i = 2$)	0.1	0.2	0.2	0.4	0.05	0.05
December 2012 ($i = 3$)	0	0	1	0	0	0
January 2013 ($i = 4$)	0.3	0.2	0.2	0.1	0.1	0.1
February 2013 ($i = 5$)	0	0.9	0	0.1	0	0
March 2013 ($i = 6$)	0	0	0	0	0	1
April 2013 ($i = 7$)	0	0	0	0	1	0
May 2013 ($i = 7$)	0	0	0	0	1	0
June 2013 ($i = 8$)	0.3	0	0.2	0	0	0.5

TABLE 2. Values of weighted coefficients in different months

Choosing coefficients α_k from the Table 2 and using (5) one obtains the following prediction of monthly returns in the current season 2012/13:

$$\mu = \begin{bmatrix} -0,006202 \\ -0.001927 \\ 0,012125 \\ -0,017391 \\ -0,024348 \\ 0,017883 \\ -0,023685 \\ -0,018426 \end{bmatrix} \tag{6}$$

Regarding the volatility, it turns out that the standard deviation of returns in each month at the Serbian market is relatively the same. Therefore, we can assume that the volatility is constant for the whole season,

$$\sigma = 0.020497. \tag{7}$$

Once the values of drift μ and volatility σ are estimated, a simulation for the price (4) can be performed. Prediction is done for one month, while the time step is one week. More precisely, inputs are number of simulations, number of weeks (within one month), volatility σ, drift μ estimated for that month and initial price S_0 that is the average of prices in the month before. One simulation outcomes prices in given number of weeks. In this work, 100000 simulations are performed, and the output is the average of all simulations. Results are presented in the Figure 11, where we compared

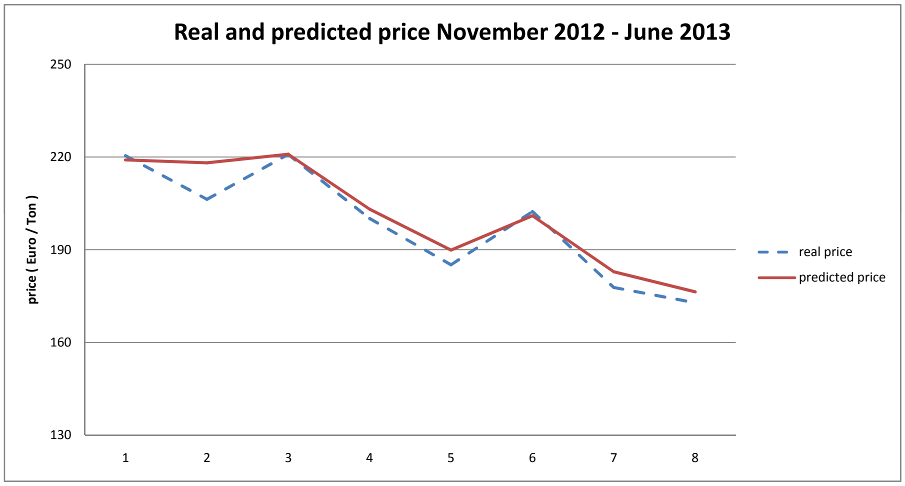

Figure 11. Real and predicted price in the season 2012/13

real prices and predicted ones at the end of each month within the period November 2012–June 2013.

Relative error is the highest in the first month, and it amounts 5.7%. Relative errors for other price predictions are up to 3%.

4. Conclusions and Perspectives

Methods which are applied in order to get corn price prediction resulted in an acceptable relative error between the predicted and real prices. However several improvements seem possible.

First, the influence of the planted area and yield to the corn price should be more investigated. Table 3 shows the planted area, the corn price and the yield of corn in Vojvodina in the last five years.

Year	Planted area ($10^6 ha$)	Total yield ($10^6 t$)	Price (e/t)	Total value ($10^6 e$)
2009	0.6777	4.0003	98.3491	393.4243
2010	0.6987	4.6888	138.2532	648.2388
2011	0.7332	4.4045	188.4003	829.8169
2012	0.7523	2.2834	194.4451	443.9956
2013	0.6841	3.9540	165.4614	654.2410

Table 3. Planted area, yield of corn and corn price in Vojvodina

It could be noticed that planted areas are almost the same. Also, a low yield implicates high price (and vice versa). The common belief that the total value should be nearly constant during the years is not confirmed! A further investigation is needed to determine relationship between the planted area, the yield and the corn price. Given that the data for planted areas is rather unreliable improvement in this direction seems unlikely.

High correlation coefficient and R^2 in regression models indicate that relation between the corn prices at different markets and also with the wheat price could be linear, but deeper statistical analysis might be needed.

The stochastic approach yields good prediction of prices within one month. Nevertheless, some improvements can be reached. First of all, estimation of the drift μ based on existing data can be done in more sophisticated way using mathematical tools such as the Least Squares Method. Also, the estimation of volatility σ can be done on a monthly level by means of the same method. Moreover, one should not be restricted to the period of one month: a season can be divided differently. This requires more careful analysis of the data. However, the most important goal is to include Future contract prices in estimation of drift and volatility, since it can be assumed that price of Future contract already captures all global phenomena related to the current year, such as weather forecast, planted areas, global market trends etc.

References

[1] J. C. Hull 2011, *Options, Futures and Other Derivatives*, 7th ed., Pearson Prentice Hall.
[2] B. Øksendal (1998) *Stochastic differential equations*, 5th ed., Universitext, Springer-Verlag, Berlin.
[3] www.cmegroup.com
[4] www.investopedia.com
[5] www.msu.edu
[6] www.quandl.com