USING FRACTALS AND POWER LAWS TO PREDICT THE LOCATION OF MINERAL DEPOSITS

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Around the world the mineral exploration industry is interested in getting that small increase in probability measure on the earth's surface of where the next large undiscovered deposit might be found. In particular WMC Resources Ltd has operations world wide looking for just that edge in the detection of very large deposits of, for example, gold. Since the pioneering work of Mandelbrot, geologists have been familiar with the concept of fractals and self similarity over a few orders of magnitude for geological features. This includes the location and size of deposits within a particular mineral province. Fractal dimensions have been computed for such provinces and similarities of these aggregated measures between provinces have been noted. This paper explores the possibility of making use of known information to attempt the inverse process. That is, from lesser dimensional measures of a mineral province, for example, fractal dimension or more generally multi-fractal measures, is it possible to infer, even with small increase in probability, where the unknown (preferably large) deposits might be located.

1. Introduction

The business of mineral exploration is the essential first stage of the mineral supply process. At its most basic, mineral exploration is the process of predicting the spatial location of undiscovered mineral deposits. The Exploration Division of WMC Resources Ltd bought the problem to the attention of MISG organisers.

It has long been recognised in the exploration industry that the spatial distribution of mineral deposits is not random but in fact clustered on many different

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scales. Fractal distributions were first used to model ore-deposit spatial distributions by Mandlebrot (1983). However little additional work appears to have been done and the available literature is very limited. Carlson (1991) looked at the spatial distribution of gold and silver deposits in the south-west USA mineral province, and Blenkinsop (1994, 1995) considered the spatial distribution of gold deposits in Zimbabwe. Both workers concluded that fractal distributions could be used to model the spatial distribution of deposits in their study areas. Carlson employed the *radial density* method to measure the fractal distribution whereas Blenkinsop used both this method and the *box-counting* method.

Despite the above work, the current reality is that formal mathematical concepts relating to the spatial distribution of mineral deposits have had very little impact on the practical business of mineral exploration or even on the academic economic geology sector. Nonetheless, the basic implications of the clustering characteristics of mineral deposits are understood intuitively by most explorationists. People know, for example, that a good place to look for a mineral deposit is close to where one has already been found. In many mineral provinces, geologists have noted an approximately constant spacing (e.g. 30 km) between major mineral deposits. This is commonly referred to as *periodicity*, although in strict mathematical terms it is more likely to be a manifestation of some fractal distribution of faults (see Bour and Davy, 1999).

Given the above, it would seem a reasonable possibility that a more rigorous application of mathematical spatial distribution concepts has the potential to aid in the prediction of the location of undiscovered mineral deposits. We are particularly interested to see if we can characterise the spatial distribution of known mineral deposits in a particular mineral province in terms of a fractal distribution. We ask the following specific questions.

- Does knowledge of known mineral deposits enable us to make any predictions about the locations of undiscovered mineral deposits in this province? Put another way, can we use this information to model a probability distribution for mineral occurrence in the province that honours the known distribution of deposits but extends into areas that so far may have not been well explored?

- Can we use a fractal distribution defined in Province X, where the spatial distribution of ore deposits is well-known, to model a probability distribution for ore deposit occurrence in some Province Y (with comparable geological, and by inference, similar ore distribution characteristics) where there has been little exploration and little is yet known about the distribution of mineral deposits?

Now it should be emphasized that what we are trying to do is somewhat similar to the problem: given the mean of a finite set of numbers, what are the numbers, or what is the probable distribution of the numbers? Of course we need other information about the set of numbers in order to make progress. If we have an upper and lower bound on the set, then a uniform distribution between these bounds is a possible answer. To distinguish more than this we need more information.

Geologists have many extra data dimensions of information about a mineral province, albeit at some cost. There are the surface faults by size, position, direction; the chemical composition of surface rocks by position; rock type and age by position. The correlations of known deposits with features of these extra dimensions are what guide the working geologist. The data of many of these extra dimensions have a fractal distribution, as in, for example, Pelletier (1999) and Turcotte (1989). Is it possible that the fractal nature of much of this data can be used to improve the predictions of where undiscovered deposits might be?

2. The fractal dimension computation

Given a large mineral province, for example, the Yilgarn craton of Western Australia, the fractal dimension for a particular mineral (say gold) can be computed in a number of different ways. We refer, for example, to Blenkinsop (1995) for a description of the radial and box counting methods and Roberts and Cronin (1996). These computations can be extended to multi-fractals as outlined in Cheng (1999). There is a concept of local and global fractal dimension in these computation methods, especially for the radial method, where the global fractal dimension is some average of the local fractal dimensions. Given that an average tends to obscure detail, that in this case we are looking for, some statistical attention needs to be paid to the computation of fractal dimension (d, say) and the associated power law

$$Q(r) = kr^d,$$

where r is a one dimensional distance (size of square or radius of circle), k is a constant of proportionality and $Q(r)$ is the quantity being measured, for example, the number of deposits of size greater than some threshold within a radius r of some point, or, the number of squares of size r needed to cover all deposits of size greater than some threshold over the entire province.

Given that fault lines may have a preferred direction in a province (it is well known that mineralization is related to fault lines) some thought was given to using direction in computing local fractal dimension, to see if this gave a more uniform local fractal dimension, and hence a better estimate of the global fractal dimension. Suggestions were to use a rectangle (box counting) or ellipse (radial) with a major axis inclined at one of a finite set of angles to the north-south axis, and see if this made any statistical difference to the estimate of the (multi-) fractal dimension. Are they all as good as each other (no discriminatory power) or is there a statistically discernable pattern? There is already literature questioning these power law aggregating measures of geological phenomenon, in for example, Clark *et al.* (1999) and Nicol *et al.* (1996). These questions and concerns have an effect on the next section.

3. Optimization, but to what objective?

The question of where is the next big deposit can be expressed as where to place concentrations of probability measure so as to better approximate or maintain some measure of fractality. This would be a straight forward computation in terms of Monte Carlo simulation in 2D, but the stumbling block is to know the objective. Should we try for a uniform local fractal dimension? It appears we need to know more about the relationships (correlations) of the various measured geological data dimensions of a province in order to be able to use this idea.

An idea appearing in Emmerson and Roberts (1996) may be useful. Given some data about a mineral province, build an ensemble of possible basis probability distributions for that province, using a binary multiplicative multi-fractal (self-similarity) scheme. Convex linear combinations of these basis distributions might then be used to fit to the data to give an overall probability distribution. More detail can be found in Roberts and Cronin (1996). Similar work can be found in Xie and Sun (1997) and Feng and Xie (1998) and an algorithm by Sommerlade (2000).

4. Fractal tiling

Related to the optimization schemes are the fractal tilings. The region is subdivided into rectangles. Use n adjacent rectangles and their geological properties to write down a prediction function for some quantity of a close neighbouring rectangle. This gives a model equation for the quantity of the $(n+1)$-st rectangle in terms of parameters and various quantities of the other n rectangles. The parameters of the model can be estimated using the data of a known province and statistical goodness of fits should be used so as to test the appropriateness of the model. This can be done in a self similar manner by taking rectangles which are twice the size of the smallest, four times the smallest etc.

We may or may not fit the same model parameters to all levels of sizes. Some writers have used the power law data for a mineral province to estimate the total amount available, Blenkinsop (1995), Caers and Rombouts (1996), McCammon and Kork (1992). Using this knowledge and the model parameters computed for a province, the extra endowment of mineral could be distributed spatially using the same self similarity geometry as used for the model estimation. This might be extended to a relatively unknown province of similar geological features.

The variations possible here are large in number and worthy of numerical experimentation. Firstly there is the template of rectangles to be chosen and secondly which geological features should be included in the modelling. For any feature there is the choice of using discrete variables (present, not present) or using the amount or concentration within the rectangle. Careful statistical analysis may be required for this modelling exercise. At a meeting of interested researchers in Perth after the MISG, some work associated with Professor David Groves (Cassidy *et al.*, 1998, and Neumayr *et al.*, 1998) indicates that some thought has already gone into this line of research. At this gathering the terms *fuzzy*, *kriging*, *radial basis functions* among others were mentioned by non-mathematicians. It would appear that collaboration between statisticians, mathematicians and geologists should prove fruitful.

5. Multi-fractals

It became clear that simple fractals were not enough to understand the questions and ideas posed. The concept of multi-fractals is needed and the literature on the computation and use of this subject can be traced back from, for example, Agterberg (1997), Barton and La Pointe (1995), Cheng (1999), Roberts and Cronin (1996). The statistical moments of a finite data set reveal the number of numbers in the set (zeroth order), the mean (order one), the variance (order two) and so on. Multi-fractals give a continous curve of measures of positive orders. Mineral provinces seem to have characteristic multi-fractal dimensions, subject to the caveats of Section 2. This opens an exciting avenue for new research in trying to solve the inverse problem posed in this project. Remember that for an unknown finite data set, if enough moments of the set are known a good estimate of the actual values can be obtained.

6. Conclusions and recommendations

The major recommendation from the group's efforts is that further research is needed. The several avenues outlined above need to have literature searches in the areas of geology, geophysics, statistics and multi-fractal analysis. Some combination of all these areas is more likely to yield results, given the need to use as much information as is available. The extra information available needs to be evaluated for its effectiveness in prediction. One way this can be done is to run numerical experiments on a good data set, for example the gold deposits of the Yilgarn province of Western Australia. Fabio Boschetti will lead this research among a large group of interested researchers in WA.

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