A COMPARISON OF BEARING LIFE IN NEW AND REFURBISHED RAILWAY AXLE BOXES

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A simple linear dynamical model shows that at normal running speeds of freight wagons, forced oscillations due to periodic track compliance are transferred to the overlying unsprung mass and significantly amplified. Due to these oscillations, a small gap opens and closes between the collar of a journal bearing and the axle box many times every second. The forces between these components reach peaks of over 10 tonnes. This is an environment in which wear of the soft spherical graphite iron of the axle box will eventually take place.

Due to repeated unloadings of the weight on the bearing during oscillations, the bearing collar may slowly slip against the axle box wall. Although our calculations show that abrasive wear due to this slippage is negligible, the calculation raises general principles that apply to other possible wear mechanisms. If lifetime is proportional to hardness, we can estimate relative lifetimes of refurbished and new boxes. Although the resleeve material is softer than the original, the cost to lifetime ratio would favour refurbishment under this assumption.

Important unanswered questions are identified and a specific integrated program of field, laboratory, and theoretical study is suggested.

1. Introduction

Wheel axle bearings are critically important to the operation of a railway. Just like the proverbial horseshoe nail, an axle bearing may have a disastrous impact on systemic operational outcomes. A single bearing failure can stop a train at any point on its journey, with potentially catastrophic consequences to

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an entire rail network. Since a single train may carry 1500 bearings, there are millions currently in use. Failure of just one of these may cause overheating and meltdown, bringing transport operations to a halt for a considerable length of time. This is costly in terms of delivery contracts and future loss of goodwill among clients. Therefore, the millions of bearings need to be regularly inspected, serviced and replaced. By necessity, a costly maintenance program must be put in place.

The bearing units are housed in an axle box, environmentally sealed to keep out grit and moisture. A detailed model of the components of the bearings and of their housing is formulated in Section 2. The outer steel collar of the spherical journal bearing is fitted into an unloaded axle box made of Silicon Graphite iron of tensile strength 500 Mpa and 7% critical strain with a tolerance of 0.08 mm freedom. Under normal operating conditions, these rings are held by friction, as each axle of a stationary loaded coal train carries a weight of around 25 tonnes, shared among four bearing rings. The axle boxes are constructed of a softer iron material that can absorb the shock of the bouncing load during motion. However, the softer metal will eventually wear, and eroded fragments will contaminate the bearings. Abrasion and failure of the bearings most commonly results from contamination by these metal fragments.

The life expectancy of the bearing-axle box assembly is between seven and ten years or about two million kilometres. In Australia, there are two approaches to bearing maintenance. Some operators run assemblies for five years without field service, and then strip or replace parts where necessary; other operators carry out regular visual inspections and lubrication until the axle boxes become unserviceable, at which time they are replaced.

In recent times, a Sydney company, Bearing Engineering Solutions, has developed a commercial 'sleeb' process of inserting a new sleeve to replace a layer of the upper wall of a worn axle box. Refurbishment can be effected at only 80% of replacement costs but the new material is softer, having a Brinell hardness index that is lower by 10%. The railway industry and the axle bearing suppliers wish to better understand the mechanisms of wear in the new and refurbished boxes, in order to make more economic maintenance, refurbishment and replacement schedules. In particular, they need to decide whether refurbishment is preferable to replacement. Naturally, this question has important implications on the production rates required of suppliers of axle boxes and bearing units. Hence, this question was brought to the 2001 MISG.

The rate of wear of the axle box will be determined by the dynamics of interaction between the box and the bearing collar, as well as the material compositions of the interacting surfaces. Our preliminary inquiries among professional railway engineers show a high degree of uncertainty on the wear mechanism and on some aspects of axle-box dynamics. Therefore, the group considered a simple one-dimensional linear model, the simplest model that is capable of providing insight on the forced vertical dynamics of the system. This model, presented in Section 2, already serves to illustrate the impulsive dynamic environment that promotes wear of the colliding metallic parts.

Given some understanding of the dynamic environment, we then require a model for the rate of wear. There are many possible mechanisms, both at the microscopic level and at the macroscopic level, for wear of interacting solid surfaces. Each of these has associated semi-empirical formulae relating wear to stress, hardness coefficients and distance of slip. We refer to Halling [1], Kragelskii and Alisin [5] and Peterson and Winer [8]. During the meeting, the group had time to concentrate on only one of these mechanisms, namely slip abrasion. This came to our attention because of anecdotal evidence of bearing collars slowly rotating against the walls of the axle boxes. Rudimentary calculations of the rate of wear by this mechanism are carried out in Section 3. Although this calculation was made for only one wear mechanism, it illustrates general principles that might apply for all types of wear. From this, we already have a strong indication that resleeving should be a viable economic option. Finally in the Conclusions and Recommendations, we recommend a program of laboratory and field measurements that would throw considerable light on this problem.

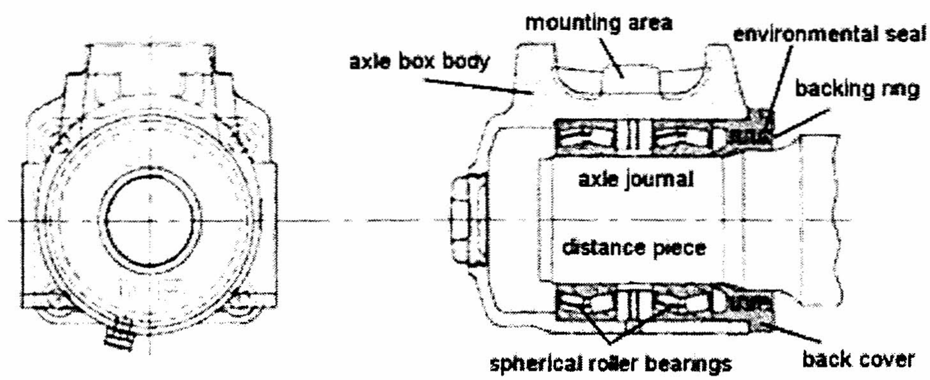

Figure 1: Components of the axle box, with journal bearing.

2. Dynamical modelling

An extensive literature search produced very little research that directly addresses axle box wear. However, in a study of wheel-rail vertical dynamic forces, Jenkins *et al.* [4] used data from accelerometers attached to axle boxes 'as a good guide to track forces'. Fortuitously, this data can be used directly to validate predicted characteristics of axle box motion during transport operations. Some of these characteristics were already recovered in linear models and in nonlinear elastic models used by Jenkins *et al.* Although complete details of the models are not known the values of various useful parameters such as force constants, unsprung masses etc. were given. These include:

k_1	Track stiffness (average value)	1.25×10^8 N/m
k_2	Axle end motor suspension vertical stiffness	3.14×10^8 N/m
m_1	Wheelset mass	3.2×10^3 kg
m_2	Motor assembly mass	3.0×10^3 kg
β	Track damping coefficient	1.29×10^2 N.s/m

These values were chosen to represent British Rail's Class 86 locomotive. The dynamic response of the vehicle's load at low frequencies, up to 10 Hz, is largely affected by the particular design of the suspension system. The patterns of axle box accelerations would vary from one type of vehicle to another. However, Jenkins *et al.* [4] note that 'at intermediate frequencies (20–100 Hz), the track system plays a significant part in determining force levels whilst usually only the unsprung parts of the vehicle and its primary suspension have some influence'.

Let us first estimate a typical flex in the rail segment midway between adjacent sleepers. With a train passing at speed 80 km/h (22 m/s), the rail segment may be loaded at a frequency of $\nu = 10$ Hz by successive wheels. By the standard formula for amplitude response of a periodically forced damped spring,

$$a = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + \beta^2\omega^2}},$$

where F_0 is the force amplitude, m is the supported mass and $\omega = 2\pi\nu$ is the angular frequency. With a 12 tonne weight supported at each axle end, and the effective track mass quoted by Taylor of 125 kg, this predicts a track displacement amplitude of around 1 mm. A track distortion of this size would produce enormous vertical accelerations on a vehicle moving at high speed. For example, for a sleeper separation of 0.77 m, a train axle moving with speed 100 km/h experiences vertical track variations at a frequency of $\nu = 36$ Hz, leading to peak vertical accelerations of approximately $(2\pi\nu)^2 a \approx 5.2\ g$, where g is the acceleration due to gravity. This agrees with Figure 19 of Jenkins *et al.* [4] which depicts significant detection frequencies of axlebox accelerations between $-6\ g$ and $6\ g$.

The periodic track distortion provides a periodic external force at the point of support of the overlying load, in much the same way as the point of suppport of a Hooke spring is commonly oscillated in undergraduate physical science demonstrations to generate a periodic external force. This description is not merely an analogy. The overlying bogie and the mountings and primary suspension system experience large weights and dynamic stresses that may be sufficient to cause compressions of the order of a few millimetres. Since this still corresponds to small strain, we may in the first instance consider a linear damped forced spring model for the vertical dynamical response. Figure 20 of Jenkins *et al.* [4] is a power spectral density taken from actual acceleration data from an accelerometer attached to an axle box on a locomotive travelling on straight welded track at 160 km/h. The sharpest peak in the spectrum occurs at 57 Hz, which at that speed is simply the sleeper frequency. Hence it is instructive to represent periodic external forcing on the unsprung mass by a single principal Fourier component which is unsprung mass times acceleration of the wheel contact, $-m\omega^2 asin(\omega t)$, where $\omega = 2\pi\nu$. Here, ν is the sleeper frequency $\nu = v/\lambda$, where v is the train speed and λ is the sleeper spacing.

An important second largest peak in the acceleration power spectral density occurs at 34 Hz, which Jenkins *et al.* [4] interpret as the resonant frequency for unsprung mass displacement. One thing that they did not point out is that at a speed of 94 km/h the sleeper frequency coincides with this resonant frequency for unsprung mass displacement. Their Figure 3 depicts a displacement transfer function with a peak value of 3.2 at a resonant frequency of 34 Hz for the unsprung mass. This means that with a track variation amplitude of $a = 1$ mm, we could expect amplitudes of around $A = 3$ mm for displacement of the unsprung mass, so that there will be peak vertical accelerations of around 15 g at some points higher on the bogie.

The value of 3.2 for the peak transfer function gives us additional information on the parameters of a descriptive linear model. Let $y_0(t)$ be the vertical upward coordinate of the centre of contact of a changing track surface that is in contact with a given rolling wheel. Let $y(t)$ be the coordinate of a fixed point on the supported unsprung mass. As an approximation, we represent flexing at the track contact point by its principal Fourier sinusoid component,

$$y_0(t) = asin(\omega t),$$

where $\nu = \omega/2\pi$ is the sleeper frequency. The unsprung mass system must have internal damping as the displacement does not grow without bound. In a linear model, we have

$$\begin{aligned} my''(t) &= -\beta(y' - y_0') - k(y - y_0) - mg \\ &= -\beta y' - ky + \beta a\omega cos(\omega t) + kasin(\omega t) - mg. \end{aligned}$$

If we shift the coordinate to $\bar{y} = y - y_e$ with origin at the static equilibrium point $y_e = -mg/k$, then we have

$$m\bar{y}''(t) = -\beta(\bar{y}' - y_0') - k(\bar{y} - y_0).$$

This equation is considered on p. 307 of the text by Ogata [7]. By constructing the steady oscillatory solution, we obtain the transfer function

$$f = A/a = \frac{\sqrt{1 + (2B\Omega)^2}}{\sqrt{(1 - \Omega^2)^2 + (2B\Omega)^2}},$$

where $\Omega = \omega\sqrt{m/k}$, the ratio of ω to the natural frequency $\omega_n = \sqrt{k/m}$, and $B = \beta/2\sqrt{mk}$ is the damping factor, the ratio of the damping coefficient to its value at critical damping.

The damped resonant frequency is found by locating the local maximum in the transfer function

$$\Omega_r = \frac{\omega_r}{\omega_n} = \sqrt{\frac{\sqrt{1 + 8B^2} - 1}{4B^2}}.$$

By substituting this in the expression for the transfer function, we obtain a relation between damping ratio and maximum transfer coefficient,

$$(8B^4 - 4B^2 - 1)f_m^2 - 8B^4 = -\sqrt{8B^2 + 1}f_m^2.$$

The symbolic manipulation package REDUCE [3] has been used to solve this equation for B. The relevant solution is

$$B = \left[\frac{f_m^2 - f_m^2\sqrt{1 - 1/f_m^2}}{2(f_m^2 - 1)}\right]^{1/2}.$$

From the value $f_m = 3.2$, we deduce $B = 0.16$. From the resonant frequency $\nu = 34$ Hz, we then deduce the natural (angular) frequency

$$\sqrt{k/m} = \omega_n = \omega_r/0.976 = 219 \text{ rad/s}.$$

Hence, we are able to reduce the number of independent model parameters from three to one. If we consider an unsprung mass of 2000 kg, then this model has force constant $k = 9.59 \times 10^7$ N/m and damping coefficient $\beta = 1.40 \times 10^5$ N.s/m. This forced oscillator model may be used to indicate time-dependent stresses transmitted through the unsprung mass and this will indicate typical stresses between the axle box and the bearing collar. However, an important additional ingredient in the dynamics of the axle box is the free play of around 0.1 mm between the inner wall of the box and the collar of the bearing when the load is removed. Since, over short intervals of time during the force cycle, the axle actually accelerates downwards at a rate higher than gravity, the two surfaces may separate and have zero stress.

Let the rest length of the oscillating system be l when the axle box is barely touching the bearing collar. Because of the free play tolerance w, no elastic restoring forces will be experienced if $0 \leq y - y_0 - l \leq w$. Hence we adopt the piecewise linear model

$$my'' = -mg - \beta y' + F_e,$$

where

$$F_e = \begin{cases} 0, & 0 \leq y - y_0 - l \leq w \\ -k(y - y_0 - l - w), & y - y_0 - l > w \\ -k(y - y_0 - l), & y - y_0 - l < 0. \end{cases}$$

In terms of the extension $z = y - y_0 - l$, this is a periodically externally forced oscillator with discontinuous internal elastic force function,

$$mz'' = -mg + F_e - \beta z' - \beta a\omega cos(\omega t) + ma\omega^2 sin(\omega t)$$

where

$$F_e = \begin{cases} 0, & 0 \leq z \leq w \\ -k(z - w), & z > w \\ -kz, & z < 0. \end{cases}$$

This system has been solved numerically by the ODE45 function of MATLAB [2], which uses a 4th–5th order Runge-Kutta method. Figure 2 shows the output of the elastic force F_e as a function of time, given initial conditions at the elastic/gravity equilibrium position. During each second, there are many collisions between the axle box and the bearing collar. These collisions are signified by the stress level rapidly changing from a zero value. There are many oscillations in the force, and they may have peaks as large as 20 times the weight of the unsprung mass. Since there are two bearings at each axle end, each bearing collar may experience short-term forces of 20 tonne weight.

3. Mechanisms for wear

In the lifetime of an axle box, the unloaded gap between the metal box typically increases from 0.08 mm to 0.14 mm, due to wear of the softer spherical graphite iron surface. We have seen in the previous section that frequent energetic collisions take place between the axle box and the bearing collar. In these circumstances, there are several possible mechanisms for wear that might be considered. It has been observed during maintenance inspections that the bearing collar has been sliding slowly against the axle box wall. Since there are heavy supported loads, the group questioned whether some wear could take

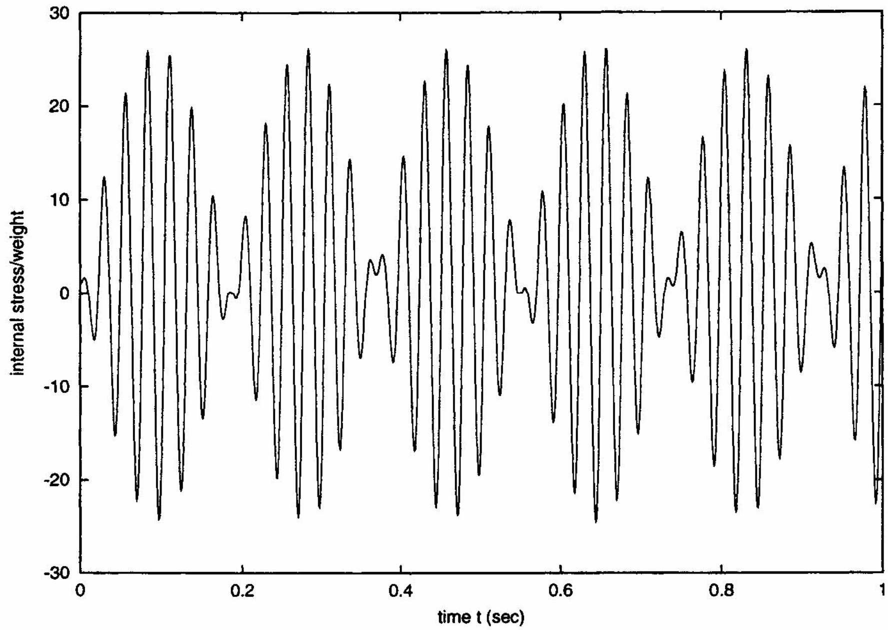

Figure 2: Elastic stress versus time in a one-dimensional forced oscillator model with a limited gap in connectivity.

place by abrasive sliding. For abrasive sliding, the amount of material removed from the softer surface is usually given by the well-known formula, sometimes referred to as Archard's formula (see Norton [6]):

$$\begin{aligned} V &= KPs/H, \\ \text{equivalently} \quad y &= KPs/HA \\ \text{or} \quad \frac{dy}{dt} &= KPv/HA, \end{aligned}$$

where V, y and $\frac{dy}{dt}$ are respectively the volume, depth and rate of material removed, P is the normal stress, s and v are respectively the distance and rate of slippage, A is the macroscopic area of contact, H is the hardness of the softer material and K is the wear coefficient. H is a characteristic stress, usually measured by a standard indentation test. For example, the Brinell hardness test measures the indentation diameter left by a harder 10 mm-diameter sphere under a weight of 3000 kg. This gives an area and a hardness measured in units of kg_f/mm^2, where kg_f is kg force. The wear coefficient K is dimensionless, a product of a distance ratio by a stress ratio, (*wear depth/slip distance*) $\times$ (*hardness/normal stress*). Repeated measurements of K on the same material may vary by up to 100%, so that it matters little whether the Brinell hardness or the related Vickers hardness (which gives values about 5% higher, Norton [6]) is used in the definition of K.

At this point, we make a simple but important observation. For all common mechanisms of wear, the macroscopic rate of wear is inversely proportional to hardness of the wearing surface. Since the resleeving material has a Brinell hardness index that is 10% less than that of the original soft iron of the axle box, its lifetime should be 10% shorter than that of the original axle box. However, the cost of resleeving is 20% less than that of replacement. *Prima facie*, resleeving seems to offer some savings. We have not taken into account associated order costs, more frequent maintenance and down-time. However, since axle boxes are removed from bogies and trucked to the supplier, bogie down-time and storage need not increase significantly.

Now we make some estimate of abrasive wear. Since we firstly need to know whether this mechanism can predict the observed order of magnitude of wear, we do not use many features of a detailed dynamical model at this stage. However, the latter provides a guide on the frequency of separation (zero-stress) events, and on the typical normal stress at other times. We adopt a simple two-stage model for the slipping process. During separation, of duration t_1, the axle provides a small torque τ_1 on the bearing collar, since no lubrication system is perfect. Angular acceleration $\dot{\omega}$ occurs during this time. Following separation, the two surfaces contact again. Then friction results in angular deceleration, rapidly bringing the angular velocity to zero in some time interval $t_2 - t_1$. During this time interval, we represent normal force by an average value N_0. The axle-bearing torque will have an average value τ_2, higher than τ_1, because the bearings will be slightly distorted under load. If $\Omega(t)$ is the angular velocity, then we can obtain some estimate of slippage from the approximate dynamical equations

$$I\dot{\Omega} = \begin{cases} \tau_1 & 0 < t < t_1 \\ \tau_2 - \mu a N_0 & t_1 < t < t_2, \end{cases}$$

where μ is the kinetic coefficient of friction between the two surfaces, a is the outer radius of the bearing collar, I is the moment of inertia of the bearing and t_2 is the time at which slipping ceases. Thus,

$$t_2 = t_1 + \frac{\tau_1 t_1}{\mu a N_0 - \tau_2}.$$

Note that the torque τ_2 is given by

$$\tau_2 = \mu_B \, a N_0,$$

where μ_B is the friction coefficient of the working bearing. Integrating, we have

$$\Omega = \begin{cases} \dfrac{\tau_1 t}{I} & 0 < t < t_1 \\ \dfrac{\tau_2 - \mu a N_0}{I}(t - t_1) + \dfrac{\tau_1 \, t_1}{I} & t_1 < t < t_2. \end{cases}$$

Integrating again, we obtain the slip distance

$$s = \int_{t_1}^{t_2} a\Omega \, dt = \frac{r\tau_1^2 t_1^2}{2I(\mu a N_0 - \tau_2)}.$$

Hence, from Archard's formula, the average depth of material removed by abrasion at each slippage event is

$$y = \frac{K\tau_1^2 t_1^2}{2HAI(\mu - \mu_B)}.$$

It is important to note that the depth of wear is independent of the loading N_0. Although this is a simplified two-stage model, it nevertheless indicates that even the high stresses applied to the axle box, in this energetic environment, will not promote abrasive wear.

For approximate values of the relevant parameters, we take:

$K = 10^{-5}$	for lubricated steel on iron [5]
$a = 0.1$ m	(Koyo specifications)
$\tau_1 = 3 \times 10^{-3}$ N.m	(Koyo specifications)
$H = 200$ kg/mm^2	(e.g. Table C-7 of Norton [6])
$= 2 \times 10^9$ N/m^2	
A (contact area)	(from specifications of axle box)
$= 70$ mm $\times$ 30 mm	
$= 2 \times 10^{-3}$ m^2	
$I = 3 \times 10^{-2}$ kg.m^2	(moment of inertia of a ring)
$\mu - \mu_B = 0.2$	(common kinetic coefficient of friction)
$t_1 = 6 \times 10^{-3}$ s	(from dynamical simulation model)

From this data, the predicted mean depth of wear per slippage event is a miniscule 7×10^{-20} m. From the dynamical model, approximately 30 vibration cycles, including slippage events, take place every second. At this rate it would take around 10^5 years to show any appreciable wear of the order of 0.01 mm depth.

Since we do not suspect any order-of-magnitude errors in our data that are large enough to explain a correction factor of order 10^{-4}, we conclude that abrasive wear is not a major contributing mechanism. However, we should not lose sight of the fact, that as predicted by our dynamical model, inter-surface forces of several tonnes over small contact areas occur in thumping cycles around 30 times per second. This is similar to the dynamic environment of a rock crusher. Therefore we should not be surprised that some decay will occur by some mechanism. We quote from Section 7.13 of Norton [6]: 'Repeated, time-varying loads tend to fail parts at lower stress levels than the material can stand in static load applications ... Thus, we should expect that our machine, though carefully designed to be safe against all other forms of failure, will eventually succumb to surface fatigue if so loaded for enough cycles'.

Since forces of the order of 36 tonne force are expected to occur between the two surfaces, with contact area of 2×10^{-3} m^2, this corresponds to peak compressive stresses of around 180 MPa. For iron material of tensile strength above 400 MPa, the endurance limit under cycles of compression and tension is around 160 MPa [6]. This means that with an unlimited number of compression-tension loading cycles, fatigue will eventually occur. As the temperature rises, the endurance limit will decrease. In the current situation, tensile stresses are not an issue, and the endurance limit will be much higher. However, the fact that the compressive stress amplitude exceeds the compression-tension endurance limit suggests that the softer surface is indeed under some threat. If sleepers are approximately 0.8 m apart, then a million kilometres of travel corresponds to more than 10^9 cycles of thumping. There is an empirical law that states that the number of cycles N of thumping before the onset of cracking is proportional to P^{-3}, where P is the stress amplitude. We have seen that stress amplitudes are roughly in proportion to the amplitude of track flexing. We presume that over long tracks, shorter spacing between sleepers to decrease rail response is not an economic option. The amplitude of displacement is approximately in inverse proportion to the damping constant. This indicates that if the damping coefficient of the track system was increased by 50%, this could produce three-fold increases of lifetimes.

4. Conclusions and recommendations

In our short exposure to the problem of axle box wear, we have developed a very simple dynamical model to indicate the stress and energy environment of the axle box-bearing system and we have considered wear mechanisms and their quantitative effects. Although the dynamical model predicts amplification of track irregularities to bring frequent high-stress collisions between the steel bearing collar and the iron axle box, our calculations suggest that abrasive wear during slippage would not be a major contributor to the observed damage. Surprisingly, in a simplified three-stage separation-slip-stop model, ultimately the rate of wear is independent of the normal stress. This indicates that the high normal stresses would not be a major accelerant of abrasive wear in more general cases. Hence, in future it might pay to consider other wear mechanisms such as thumping fatigue, that are influenced more strongly by the stress amplitude. Ultimately, a dynamical model should also consider stresses on the axle due to track curvature, wear and misalignment.

The specific question brought to MISG was on the relative economy of resleeving rather than replacing worn axle boxes. We conclude that at current costs, on the basis of lifetime being proportional to hardness, resleeving is likely to be financially advantageous.

The relative costs of the two strategies (replacement and resleeving) will be affected by the shorter lifetimes of the less expensive resleeved boxes, leading to more frequent maintenance. However this is likely to be less of concern in the future for two reasons. Firstly, wayside hot-box detectors are beginning to be installed next to Australian track. These are not yet fail-safe but the technology of infra red and acoustic detectors must surely improve. At that time, a just-in-time minimal maintenance strategy can be adopted with less risk. Secondly, there is a major thrust in the industry, as evidenced by the themes of the Cooperative Research Centre for Railway Engineering and Technology, to make better use of information technology to record vehicle movements. When a system-wide data base system is in place, it will become possible to automatically flag when axles have covered the maximum distance before major servicing is necessary.

One of the most interesting outcomes of this short study was to identify gaps in knowledge of the wear mechanisms and of lifetime statistics for axle boxes. We believe that it would be timely to initiate a major multidisciplinary program of laboratory, field, and computer-modelling studies that:

(1) identified the microscopic wear mechanism by microscopic surface analysis;

(2) related the rate of wear to key features of the foundation-track-wheel dynamics;

(3) used vehicle assignment data, wayside detection data and maintenance reports to approximate the distribution of axle box lifetimes and to ascertain the typical slippage rate of bearing collars.

Such a project would aim to identify the important features of the dynamics of the interaction between the axle box and the bearing collar during train motion, to better understand the microscopic mechanism of axle box abrasion and wear, and thereby to formulate an optimal maintenance program of protection, refurbishment and replacement. This will be a multi-disciplinary project including aspects of solid mechanics, materials science, field observations, statistical risk analysis and maintenance scheduling.

The information could be used more generally to predict wear of other moving parts, and to better control the dynamical interactions between ballast, sleepers, track, wheel and vehicle load.

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