

STORE CAPACITY OPTIMISATION

In this report we address the problem that was posed to the MISG by Australian Paper. The problem is one of increasing the efficiency of distributing paper rolls from the manufacturing plants to the customers. A related problem is one of utilising the available capacity at the customer stores in an effective manner.

During the MISG, several approaches to the above problems were proposed. In this report we describe the problem and several methods for solving it. Preliminary results are provided for some of these.

1. Introduction

Australian Paper (AP) is the paper making subsidiary of AMCOR, the Australian owned multi-national paper/packaging company. One of the most significant activities of AP is to provide liner board and corrugating paper for the production of corrugated boxes by AMCOR Fibre Packaging (AFP), a 'sister' subsidiary. The magnitude of this activity can be judged by the fact that AFP sells approximately \$500 million worth of boxes to the Australian market. Worldwide, AMCOR has corrugated box sales of 2 billion dollars.

Representatives of AP's central logistics department approached the MISG with the request to investigate a major decision making problem which they face in managing their inventory of these packaging papers. The problem arises because particular paper grades and sizes are made on a monthly or six-weekly cycle and the management of stock in between makings is particularly difficult as customer requirements are extremely variable. On the surface, the problem appeared to be relatively well defined, although complicated and challenging, due to the multi-product, multi-plant and multi-warehouse nature of the problem.

In several lead-up meetings between the co-moderators and company representatives the view was formed that a simpler sub-problem, covering only a part of the complete system, should be presented and tackled initially. This view of the problem, like many other initial assumptions, was to be sorely tested throughout the week.

1.1 Problem description

The paper mills of AP produce a range of packaging paper to different specifications and, because of high set-up costs, work most efficiently in a cycle where

each product type is made every two to six weeks. On the other hand, mill customers, predominantly AFP, are forced to switch between papers of different specifications at very short intervals, due to a rapidly changing market.

Each customer can hold a limited amount of a range of products (defined as paper of a specific type and size) at a warehouse on their own site. This immediate storage is inadequate, however, to guarantee that there is no stock-out. So further stock is held in intermediate warehouses which are directly under AP's control. Obviously the use of these intermediate stores increases holding and handling costs and also increases the risk of damage to paper reels.

In essence AP asked the MISG to devise a methodological allocation tool for deciding, on a daily basis, which reels should be shifted from mill to customer, mill to intermediate warehouse and intermediate warehouse to customer so that the expected cost of supplying the customer is minimised. They also asked whether the current warehouse layout, of a herringbone type, could be changed to provide better utilisation of the warehouse floor space.

For the packaging paper of concern, AP operates three major mills in Melbourne, Gippsland and Sydney which produce reels in about 20 different widths at four different weights. Customers are based primarily in Sydney, Melbourne and Brisbane. A schema of the logistics system is given in Figure 1.

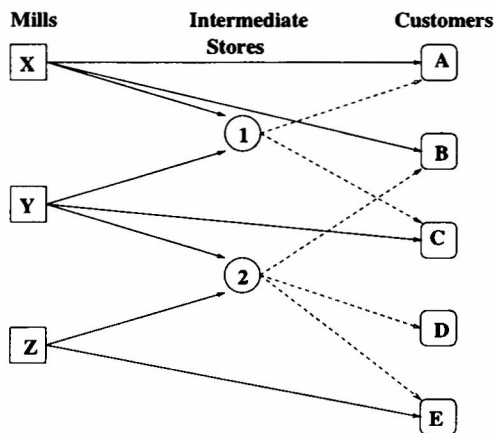


Figure 1: Schema of the logistics system.

To simplify the problem, the company representatives proposed that the model be limited to supplying the needs of one customer. Further they proposed a set of objectives which they felt would accurately measure the success of the project. These included goals such as: reels should be handled at most twice; reels should be used within six months of manufacture; distribution costs should

be minimised; and customer warehouse occupancy should be maximised (as this would reduce the risk of running out of stock).

The layout of the warehouses is currently of a herringbone style (as shown in Figure 6). It was noted that various constraints on fork-lift truck movements would provide serious limitations on re-arranging the lay-out. However the most urgent question according to the company was whether the current fixed allocation of bays to products should and could be replaced by a more flexible arrangement with bays being dynamically allocated.

Finally company representatives were at pains to point out that at the moment they had little or no control over customers or production staff. Solutions involving joint decisions being made with the customer or with the production staff was unlikely to be readily acceptable.

2. Mathematical formulation

2.1 Definitions

In this section we describe a mathematical formulation of the problem as set out in the introduction. In order to do this we make some simplifying assumptions. We also need to define sets, variables and parameters.

$I = \{i : i = 1, 2, \dots, \bar{i}\}$ the set of mills (sources).

$J = \{j : j = 1, 2, \dots, \bar{j}\}$ the set of intermediate stores.

$K = \{k : k = 1, 2, \dots, \bar{k}\}$ the set of customers.

$T = \{t : t = 1, 2, \dots, \bar{t}\}$ the set of time periods.

$P = \{p : p = 1, 2, \dots, \bar{p}\}$ the set of products.

We are given the following problem data:

S_{itp} The supply at paper mill i of product p at time t .

D_{ktp} The demand at customer k for product p at time t .

C_{ij}^{σ} The transport cost per reel from source i to store j .

C_{ik}^{χ} The transport cost per reel from source i to customer k .

C_{jk}^{τ} The transport cost per reel from store j to customer k .

Q_j^{σ} The capacity at the intermediate store j .

Q_k^{χ} The capacity at the customer k .

I_{kp}^0 The initial inventory of product p at customer k .

I_{jp}^0 The initial inventory of product p at intermediate store j .

Finally, the following decision variables will be used in the model:

X_{ikt}^x Number of reels of product p transported from source i to customer k during period t .

X_{ijtp}^σ Number of reels of product p transported from source i to store j during period t .

X_{jkt}^τ Number of reels of product p transported from store j to customer k during period t .

Y_{jtp}^σ The inventory level of product p at time t in intermediate store j .

Y_{ktp}^x The inventory level of product p at time t at customer k .

2.2 Mathematical programming formulation

Using the above decision variables we can now write down a linear programming formulation for the problem of deciding the optimal distribution strategy, given the production schedule and assuming we have *full knowledge* of the anticipated future demand.

$$\text{Min} \quad \sum_{t \in T} \sum_{p \in P} \left(\sum_{i \in I} \sum_{j \in J} C_{ij}^\sigma X_{ijtp}^\sigma + \sum_{i \in I} \sum_{k \in K} C_{ik}^x X_{ikt}^x + \sum_{j \in J} \sum_{k \in K} C_{jk}^\tau X_{jkt}^\tau \right) \quad (1)$$

$$\text{S. t.} \quad \sum_{j \in J} X_{ijtp}^\sigma + \sum_{k \in K} X_{ikt}^x = S_{itp} \quad \forall i \in I, t \in T, p \in P, \quad (2)$$

$$Y_{jtp}^\sigma + \sum_{i \in I} X_{ijtp}^\sigma - \sum_{k \in K} X_{jkt}^\tau = Y_{j,t+1,p}^\sigma \quad \forall j \in J, t \neq \bar{t} \in T, p \in P, \quad (3)$$

$$I_{jp}^0 + \sum_{i \in I} X_{ij1p}^\sigma - \sum_{k \in K} X_{jk1p}^\tau = Y_{j1p}^\sigma \quad \forall j \in J, p \in P, \quad (4)$$

$$Y_{ktp}^x + \sum_{i \in I} X_{ikt}^x + \sum_{j \in J} X_{jkt}^\tau - D_{ktp} = Y_{k,t+1,p}^x \quad \forall k \in K, t \neq \bar{t} \in T, p \in P, \quad (5)$$

$$I_{kp}^0 + \sum_{i \in I} X_{ik1p}^x + \sum_{j \in J} X_{jk1p}^\tau - D_{k1p} = Y_{k1p}^x \quad \forall k \in K, p \in P, \quad (6)$$

$$\sum_{p \in P} Y_{jtp}^\sigma \leq Q_j^\sigma \quad \forall j \in J, t \in T, \quad (7)$$

$$\sum_{p \in P} Y_{ktp}^x \leq Q_k^x \quad \forall k \in K, t \in T, \quad (8)$$

$$\mathbf{X}, \mathbf{Y} \geq 0. \quad (9)$$

Note that in this model, we simply specify a total capacity for each intermediate and customer store. Under the current system however, bays in the stores are assigned to different products in a fixed manner. Hence there are capacity restrictions on individual products. To model this system we simply replace constraints (7) and (8) by the constraints (10) and (11)

$$Y_{jtp}^{\sigma} \leq Q_{jp}^{\sigma} \quad \forall j \in J, p \in P, t \in T, \quad (10)$$

$$Y_{ktp}^{\chi} \leq Q_{kp}^{\chi} \quad \forall k \in K, p \in P, t \in T. \quad (11)$$

If this is done the model decouples by products (i.e. we can solve a separate problem for each product).

We have assumed that demands are known exactly (which is clearly not true in practice), or can at least be forecast with reasonable accuracy (see Section 3 on forecasting). We also assume that the production cycle (and hence supply) is fixed for the entire planning horizon over which decisions are made using the above model. This is not unreasonable since the store managers have little influence on production planning. Moreover, it is our understanding that the production of paper is unlikely to vary from targets set at the beginning of the planning cycle.

The model as presented above may not always produce a feasible solution. This is because we assume mass conservation constraints apply. In other words we assume that *all* of the supply will be either consumed or stored somewhere (subject to capacity restrictions). Furthermore we stipulate that all of the demand has to be met from the initial stock plus production during the period. It may not be possible to satisfy both of these requirements. For this reason we introduce a dummy source and dummy sink. These are fictional nodes from and to which an arbitrary amount of any product can be received or sent respectively. To ensure that these are only used as a measure of last resort, the costs for using them are made very large. We define

X_{ktp}^{γ} The amount of product p that customer k gets from the dummy source during time period t .

X_{itp}^{π} The amount of product p that mill i puts into the dummy sink at time t .

C^{δ} A large penalty cost for using the dummy source or sink.

X^{γ} and X^{π} are added to the left hand sides of the constraints (5) and (2) respectively and included in the objective function with the coefficient C^{δ} .

2.3 Numerical results

We provide some preliminary results for the above model obtained using the GAMS modelling language (see Brooke *et al.*, 1988) with the OSL MILP

solver. We consider a simplified subproblem involving two customers (E & F), two products each in three sizes (hence we consider 6 different commodities), one intermediate store and two sources. The two sources are derived from despatch data from mill X to customers E and F. The model is run over a planning horizon of 120 days using historical data starting from January 20, 1995.

We compared the effects of using joined versus separate capacity constraints. In other words, we ran the model using either constraints (7) and (8), or (10) and (11). The models were each solved in approximately 137 seconds of CPU time on a SUN SparcStation IPX. For the test problem, the difference in objective value between the two models was \$2.09, approximately 0.004%. This indicates that, at least in this simplified scenario, using fixed bay capacities for each product and size at the customer store does not have a significant effect on the operating costs.

Figures 2 to 4 present more detailed information about the problem for product A size 6. In Figure 2 the supply and demand data are plotted. Figure 3 shows the inventory and bay capacities at the two customer stores when separate capacity constraints are applied. The optimal inventory levels with joined capacities are shown in Figure 4.

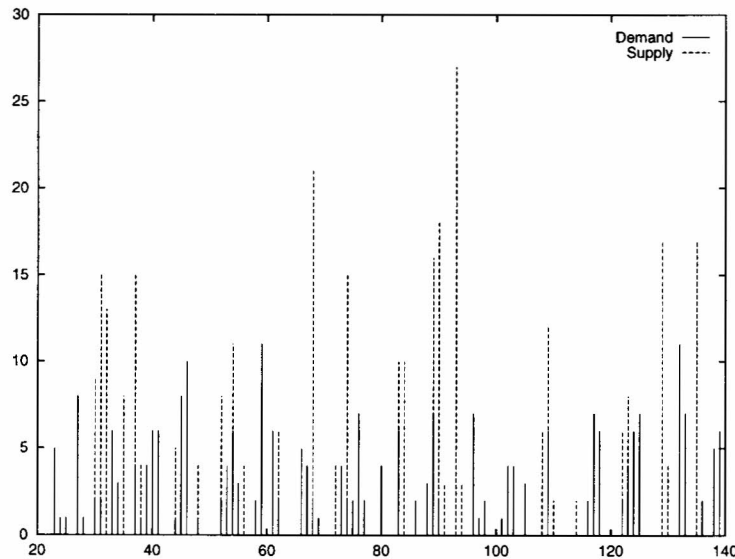


Figure 2: Supply and demand data.

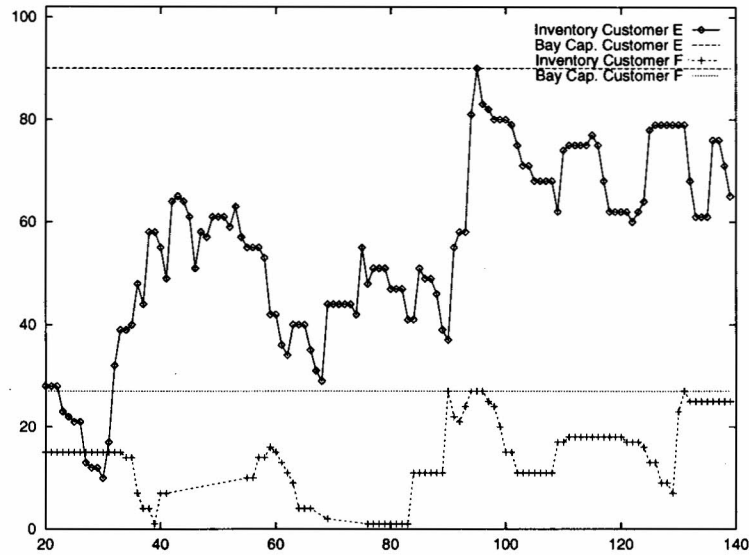


Figure 3: Inventories with separate bay capacities.

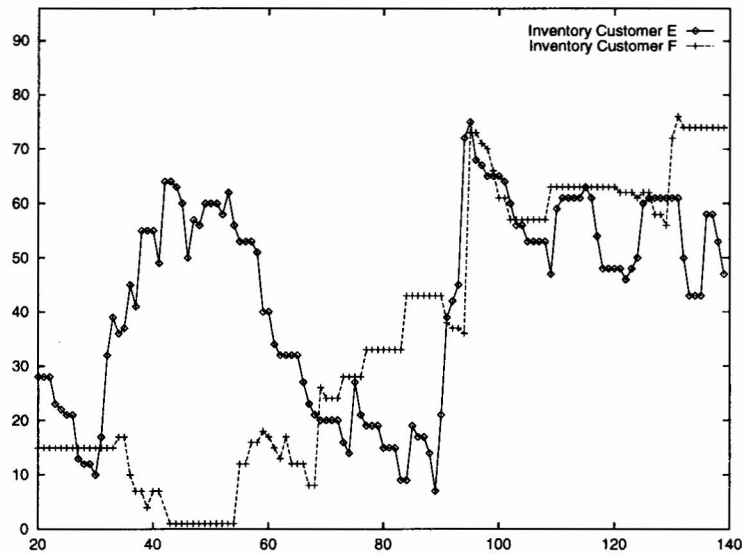


Figure 4: Inventories with total capacities only.

Since we have full knowledge of future demand, the MILP model copes well with the fluctuations by managing inventories. An inherent disadvantage of

using a linear model for this type of application is that it will always fill one of the customer stores in preference to another rather than spreading the stock evenly. Another disadvantage is that the MILP model only solves optimally for the current planning horizon with no consideration given to the future. One way to overcome this difficulty is to specify appropriate ending conditions (e.g. cyclical constraints).

Despite its weaknesses, this approach provides a useful basis for further work. At the very least it gives a useful lower bound on how well we could do if we had perfect information. In situations where perfect information is available, the LP model could provide a good decision support tool to inventory managers.

3. Data modelling and forecasting

In this section daily usage data for products at a single customer are analysed. For this customer, no product exhibited seasonal demand and the mean level for most products did not appear to change markedly over 1995.

3.1 Forecast of daily demand

The demand for a high usage product (Product A supplied by Mill X) was tracked using exponential smoothing (as described in Section 3.3, omitting the seasonal term) using the first 180 working days as an initialisation period. The forecast of the next day's demand is 8.8 reels. An approximate 80% confidence interval for the demand on the next working day is (4, 14) reels. One might then argue that the predicted demand for the next planning period of, say, 10 days would be $10 \times 8.8 = 88$ reels. However a more accurate forecast can be made.

3.2 Forecast of demand for the planning cycle

If our objective is to forecast the total demand in a planning period, then it is not appropriate to add the individual daily forecasts because actual demands within the period are correlated with each other. The total demand for the same product (Product A from Mill X) was forecast using exponential smoothing (ignoring seasonal variation) applied to the total demand on groups of 10 consecutive working days (approximating fortnights), using the first 18 fortnights as an initialisation period. The predicted demand for the next planning period is 92 reels with an 80% confidence interval of (73,111) reels (a much smaller variation than would have been estimated from adding the forecasts for 10 successive days).

For a low usage product (for example Product M from Mill Z) we could develop a decision rule based on the relative costs involved, such as: deliver

sufficient reels from each production cycle to make up the total number in store to 8 reels (since demand never exceeded 8 reels) or only supply from the central store (as this will occur relatively rarely).

3.3 Holt-Winters' method

This method for tracking past usage and forecasting future values is based on classical decomposition (the observations are assumed to be the sum of a trend, possible long term cyclic variation, seasonal variation and a disturbance term). For more details of this method refer to Chatfield (1978). It uses exponential smoothing as follows:

$$\text{MODEL: } D_t = \mu + r_t t + \sigma_t + e_t,$$

where D_t is the observed value for period t ,
 μ is the mean value at the start of the series ($t = 0$),
 r_t is the change in trend at time period t ,
 σ_t is the additive seasonal factor for period t ,
 e_t is an error term.

SMOOTHING OF DATA, D_t

Mean: $m_t = a(D_t - \sigma_{t-s}) + (1-a)(m_{t-1} + r_{t-1})$ where σ_{t-s} is the seasonal factor for period t and there are s seasons in a year,

Trend: $r_t = b(m_t - m_{t-1}) + (1-b)r_{t-1}$,

Seasonal factor: $\sigma_t = c(D_t - m_t) + (1-c)\sigma_{t-s}$,

where the starting values of r , m and S and the smoothing parameters a , b and c are estimated from an initial period of data (commonly by using the criterion of minimising the mean square of the one step ahead forecast errors).

Forecasts h steps ahead at any time t are given by

$$D_{t+h} = m_t + hr_t + \sigma_{t+h-s}.$$

For the non-seasonal case, c and the seasonal factors σ_t are omitted and/or if the trend is not changing over time, b and the trend rate r_t may be omitted.

4. Deciding on safety stock levels using dynamic programming

4.1 A restricted subproblem

In this section we will model a restricted subproblem of the complete inventory problem. This model will provide answers on what stock levels should be

maintained at the customers, and when (and how much) stock should be transferred from the intermediate store to the customer. In particular, this approach is designed to deal with the uncertainty in demand. We will make the following assumptions/restrictions:

1. The bay capacities are fixed so that the problem decomposes by product/size combinations. (Some comments on how to deal with variable bay capacities will be made at the end of the section).
2. We restrict ourselves to a single source, intermediate store and customer. Hence no trade off will be made between different customers and stores.
3. Production is according to a fixed schedule. That is we assume that we know precisely when and how much of a given product/size is produced.
4. Demand (usage) is uncertain but a probability distribution $p(d)$ is known which gives the probability of using d rolls on any given day. This assumes that usage on any given day is independent of what happens on other days. As indicated in the previous section, this is not true. However for the purposes of this model this is not an unreasonable assumption.

It is obvious from the above assumptions that the optimal dispatch policy is to always put as much into the customer's store as possible, with the remainder being placed into the intermediate store. The only question is when and how much should be transferred from the intermediate store to the customer. The aim of this section is to develop lookup tables based on the known production schedule and the demand probabilities which will tell a person managing the inventory how much to transfer each day from intermediate stores, given the inventory at the beginning of the day. The results of this model could also be used in the model described in Section 2.

4.2 Parameters and data

To decide on transfers we require several pieces of information. Below we list all of the inputs for our algorithm.

- C^r Delivery cost per reel from the intermediate store to the customer. It includes the cost of double handling the reel. This was set at \$15.
- C^s Delivery cost per reel from the source to either the customer or the intermediate store. Since both of these destinations are in the same city, the cost is the same. Also all reels have to be moved immediately from the source. Hence the actual value does not affect the optimal solution.

- C^π Penalty cost of supplying a reel if it is not available in the customer's store on the day that it is needed. The magnitude of this cost is crucial in determining the level of safety stock. We set this cost at \$45.
- C^o An overstock penalty per reel that has to be shipped to the intermediate store rather than to the customer due to excess inventory at the customer store. We made this cost equal the intermediate store delivery cost.
- C^n Holding cost per night per reel for inventory at the customer store. This was set at zero as there seems to be no quantifiable cost associated with holding stock.
- $p(d)$ The probability of a demand of d reels occurring on any given day. As a first approximation we used the frequency distribution obtained from the usage data of 1995 to approximate $p(d)$.
- $s(\bar{t})$ The supply at the last time period in our planning horizon (*i.e.* \bar{t}), which is the number of reels arriving from the source at the city containing the intermediate store and customer on day \bar{t} . This has been taken from historical data for 1995.
- Q The number of reels of the product that could be stored at the customer. Obtained from the data supplied to us of current bay capacities.
- I^0 Initial inventory. No real data was available so we have 'guessed' some reasonable values to use as a starting point.

4.3 The dynamic program

Dynamic programming is an effective method for solving optimisation problems, and is described in detail in Sniedovich (1992). The state of the system is defined by the current inventory level y , as well as the day t (which determines the number of days $(\bar{t} - t)$ until the next shipment of size $s(\bar{t})$ arrives from the source). We want to minimise the expected cost $f(y, t)$ of deliveries, stock outs and so on, by transferring $x(y, t)$ reels from the intermediate store to the customer. For any $0 \leq y \leq Q$ and $0 \leq t \leq T$, the cost is defined recursively as follows:

$$f(y, t) = \min_{0 \leq \xi \leq Q-y} \left\{ (C^n + C^r) \xi + \sum_d p(d) f(y + \xi - d, t + 1) \right\}. \quad (12)$$

This recursive relationship defines the minimum expected cost at time t for inventory level y , assuming that we know the value of $f(y', t + 1)$ for any inventory level y' in the next time period $t + 1$. Hence, in order to calculate the value of

$f(y, t)$ for all y and t we just need to know the value of f at time \bar{t} and for $y < 0$ (since $y + \xi - d$ may be less than zero). These boundary conditions are given by:

$$f(y, \bar{t}) = \begin{cases} 0 & \text{if } y + s(\bar{t}) \leq Q \\ C^o * (y + s(\bar{t}) - Q) & \text{otherwise} \end{cases}$$

$$f(y + \xi - d, t + 1) = C^\pi * (f(0, t + 1) + |y + \xi - d|) \quad \text{if } y + \xi - d < 0.$$

We define $x(y, t)$ as the value of ξ in (12) at which the minimum is attained. We can now solve the problem using the above recursion equation by calculating $f(0, t)$, $f(1, t)$, \dots , $f(Q, t)$ for $t = \bar{t} - 1$, then for $t = \bar{t} - 2$ and so on.

4.4 Results

We apply the method described in this section to customer E, product B, size 1 over the period January 20 to March 31 in 1995 (70 days). To assess the performance of this method we simulate its operation using the historical usage data. That is, after calculating the optimal $x(y, t)$ we start with inventory I^0 . Then on each day t of the period considered we look at the inventory y and transfer $x(y, t)$ reels from the intermediate store. Then the supply and demand given in the historical data are added and subtracted respectively to give a new inventory. The cost is incremented by the appropriate delivery costs. If the new inventory is negative, it is reset to zero and the stock-out penalty is added to the cost. While there are some obvious problems with this approach (such as the fact that the usage may not reflect actual demand at times when stock outs occurred), it provides at least some measure of the effectiveness of the algorithm described in this section.

The probability mass function for the demand (based on the data from the remaining days in 1995) is given by:

Number of reels d	0	1	2	3	4	5	6
probability $p(d)$	0.28	0.34	0.23	0.08	0.04	0.015	0.015

The total cost for this example is \$1380. The inventory is shown as a continuous line in Figure 5. This shows that there is only one day on which the algorithm gets caught out — right at the end of the time interval considered, when it allows the inventory to run too low in expectation of the next delivery. Figure 5 also shows the transfers from the intermediate store that were made to achieve the inventory level and the safety stock levels, which are given by $x(0, t)$ or equivalently, by $\min\{y : x(y, t) = 0\}$ for each t .

4.5 Conclusions

The method described in this section appears to work well on all the examples tested. The dynamic programming approach has several advantages: it is simple

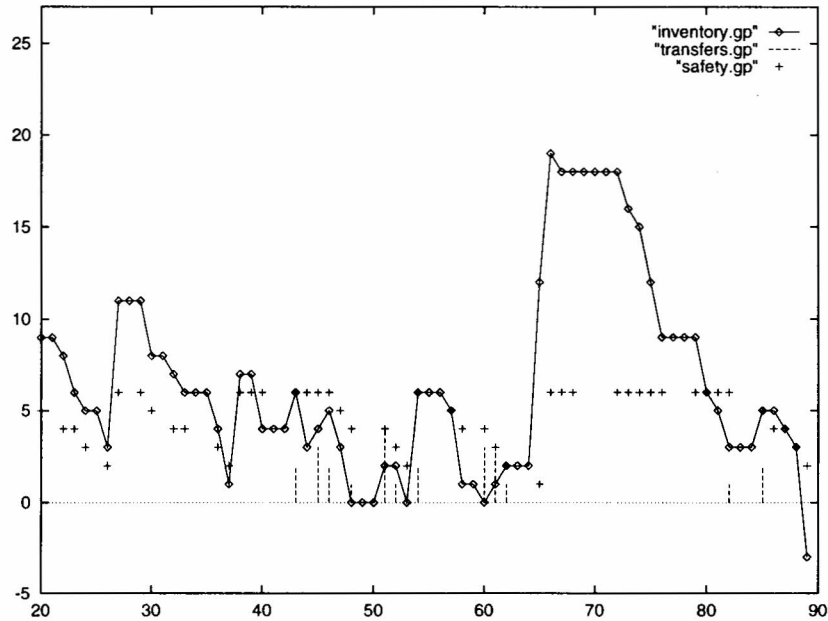


Figure 5: Inventory, transfers and safety stock levels for product B size 1 at customer E.

to implement, and takes the demand uncertainty into account. While it does not allow a tradeoff between different customers or products to be made, the most significant result of the optimization, namely the safety stock levels, can easily be incorporated into other models, such as that in Section 2, that consider a bigger picture.

5. A rule based approach to inventory management

This approach attempts to ensure that stocks of paper are available at customer stores in time to meet their production needs. Also it aims to minimise handling and the use of intermediate stores. We use a 'rule of thumb' approach to solve the problem. Stocks at the customer store can be divided into two categories: *feed-stock* required for next r days of production, and *inactive-stock* needed some time in the future. A (prioritised) preliminary specification of the rules for allocating production is as follows.

1. Ensure that feed-stocks are at their r day target level.
2. Fill all available customer space (except space for feed-stocks) with customer orders.

3. If there is not enough space in the customer store for the entire order then send the residual to the closest intermediate store with available space.

It is anticipated that these rules will have to be modified and expanded. One possibility is to have safety stock in the customers' stores to accommodate unanticipated changes of plan.

The above could be implemented as a computer program. As inputs it requires the customer's store capacity, current inventory level at the customer store and production level for each product. The outputs would describe location and quantity of deliveries of each product from the source.

6. A stochastic network flow approach

This approach is prompted by the success of the stochastic network approaches used successfully by Powell (1987) for solving vehicle despatch problems. In these problems the decision problem is to allocate certain vehicles to trips originating in particular locations and to position idle trucks so that they are in the 'best' position given the expectations for likely tasks originating sometime in the future.

The reel management problem faced by Australian Paper has certain elements in common: paper reels must be moved from the mills, either to an intermediate warehouse or to a customer; reels from intermediate stores can be moved to one of the customers, or they can be held for a further time period. The dominant stochastic nature of the problem is that if a reel is sent to a customer (never to be repositioned) it will take up a store position until it is used, at a time not known precisely. On the other hand if it is retained at an intermediate store it remains available for more than one customer (in emergencies) but it is not immediately available to any one (customer). A secondary but important stochastic element is introduced if one considers that the production schedule is also subject to uncertainty. Hence in reality both the usage and the supply are subject to stochastic fluctuations. In this situation the intermediate stores act as buffers, whose optimal levels (if they exist at all) need to be calculated.

It may be appropriate to determine, over a long time frame, the expected value of additional reels being sent to the customer store. Hence if the level of product p at customer k is Y_{ktp} and the expected time before the next making of this type of reel is $(T - t)$ days, and if the expected demand is D_{ktp} then an expected value function for the n 'th reel being sent to the k 'th customer is:

$$E_k(n, Y_{ktp}, T - t, D_{ktp}).$$

Note that the expectation obeys recursive relationships such as

$$E_k(1, Y_{ktp}, T-t, D_{ktp}) = E_k(1, Y_{ktp} - 1, T-t, D_{ktp}) + E_k(2, Y_{ktp} - 1, T-t, D_{ktp}).$$

It is proposed to build up these expected values in a way not too dissimilar to that explained in Section 4.

If we take the expected values over the complete time horizon, say a month or six weeks, then we can formulate the decision problem as a simple network problem. This allows us to have a specified storage for each item, in which case there are separate problems to solve for each reel type. Alternatively, we could, through the use of dummy nodes for each reel type, allow for the case of totally flexible store arrangements.

It should be stated that this simple network model can also be modelled with the LP approach, provided the expected value function is monotonically decreasing. By looking at the problem with this approach, it is possible to consider its recourse nature: if we make a decision Δ_t at time t , new information at time $t + 1$ will enable us to remedy any difficulties as a result of Δ_t . Thus if the expected value functions are taken only over a single time period, we could treat the problem as a space-time stochastic network.

It was not possible to carry out an extensive analysis of the above model within the time constraints of the MISG. However, this approach is worth further consideration should a full scale decision tool be developed.

7. Bay capacity modelling

In the storage area at each customer, bays are assigned to a particular product. Bays come in several different sizes. In this section we determine how much capacity should be assigned to each product. The objective is to minimise the total amount of paper which has to be delivered to an intermediate store due to an inappropriate use of storage space at the customer. The problem can be solved independently for each customer.

For a given customer we require D_{tp} and S_{tp} , the demand and supply of product p at time t . Note that S_{tp} is different to X_{iktp}^x used in Section 2. S_{tp} represents the total supply of product p at time t destined for customer k (either directly or via the intermediate store). Also if B is the set of all possible bay capacities, we need N_b and B_b , the number and size of bays of type $b \in B$. The bay capacity may be planned to fit historical data, but may also use forecast data.

Analogous to Section 2 we define X_{tp}^x , X_{tp}^σ and X_{tp}^τ to represent the various possible transfers. Similarly Y_{tp}^x and Y_{tp}^σ represent the inventory at the customer

and intermediate store. In this section, the subscripts i , j and k have been dropped from the earlier definitions for the sake of clarity. This is because we are dealing with only one mill, one intermediate store and one customer. In addition we define Z_{bp} to be the number of bays of type $b \in B$ to be allocated to product p .

For each $p \in P$ and each $t \in T$ we have the inventory balance equations

$$Y_{tp}^\sigma + X_{tp}^\sigma - X_{tp}^\tau = Y_{t+1,p}^\sigma \quad \forall t \in T \quad (13)$$

$$Y_{tp}^\chi + X_{tp}^\chi + X_{tp}^\tau - D_{tp} = Y_{t+1,p}^\chi \quad \forall t \in T, \quad (14)$$

where Y_{0p}^σ and Y_{0p}^χ are the initial inventories. The total amount of product p leaving the source at time t is S_{tp} . Hence

$$X_{tp}^\sigma + X_{tp}^\chi = S_{tp}. \quad (15)$$

In addition to the inventory balance constraints, we must also ensure that the capacity used by a product at the customer does not exceed the capacity that has been assigned to that product:

$$Y_{tp}^\chi \leq \sum_{b \in B} B_b Z_{bp}, \quad \forall t \in T, p \in P. \quad (16)$$

Furthermore, the number of bays of a given type assigned to products cannot exceed the existing number of bays of that type:

$$\sum_{p \in P} Z_{bp} \leq N_b \quad \forall b \in B. \quad (17)$$

Clearly we also require that all variables be non-negative, and that Z_{bp} must be integral for all products p and bay types b .

The objective is to minimise the amount of product that is transferred via the intermediate store. The complete mixed integer linear program is:

$$\min \sum_{p \in P} \sum_{t \in T} X_{tp}^\sigma$$

Subject to (13) – (17)

$$X^\chi, X^\sigma, X^\tau, Y^\chi, Y^\sigma \geq 0,$$

Z non-negative integer.

We do not constrain the reel transfer or inventory variables to be integer. These will be integer if the Z variables as well as the parameters are all integers.

In the discussion above, we do not consider the fact that bays have a pre-assignment, and may not easily be re-assigned. This could be addressed by

simply changing the current state to that determined by the optimization as quickly as possible. Alternatively, if we decide that some bays may not have their pre-assignments changed, we can infer positive lower bounds on Z_{bp} . A third alternative is to allow the optimisation process to choose different capacities for each product in each time period, provided that it is possible in practice to implement the changes. Since the last alternative represents a radical departure from current practice, we do not consider it here.

This model was not tested with data supplied by Australian Paper due to the lack of time at the MISG. Further investigations would be necessary to test the effectiveness of this approach.

8. Efficient physical layout of the customer store

8.1 Description

Another aspect of the overall problem is the question of whether or not the physical layout of the customer store is efficient. Apart from increasing the total storage capacity, it is desirable to be able to have a more flexible set of bay sizes.

The major constraint on the layout of the customer store is the turning circle required by the forklift in the course of loading and unloading storage bays. The present store layout has the floor divided into storage blocks with each block divided into bays or corridors in a herringbone fashion at an angle $\theta = 48^\circ$ to the horizontal. Each storage block is necessarily divided down its length in a fixed position where opposing corridors meet. Under this design the forklift requires a turning circle of 3.0 m. If $\theta = 0$ (that is, if the corridors are set horizontally), the turning circle required by the forklift is 4.5 m.

8.2 Implementation

One approach taken to the customer store design problem was to set $\theta = 0$, require the space between storage blocks to be increased to 4.5 m, and calculate the capacity of the store under such a construction. There are obviously many ways of setting the floor design under these conditions, and there is nothing to suggest that the result is the optimal store layout. However, it was found that under one such design theoretical storage capacity of the store increased by 15%. It is not surprising that, even though more space is required between storage blocks with $\theta = 0$, the total capacity increases, since under the herringbone design so much space is lost at the ends of storage blocks.

This design has the advantage that since there are no opposing corridors the division down the length of a storage block is entirely arbitrary and may vary in

corridors within a given storage block. Hence, storage capacities for individual stock items are variable. Moreover, the total number of storage bays in the store increases, giving greater access to the items stored. The major disadvantage of this approach is that it does not guarantee the *optimal* store layout.

Figure 6 shows the current herringbone layout and the proposed new layout. Other layouts are possible. Extensive simulations will be required to ensure optimality of the chosen design.

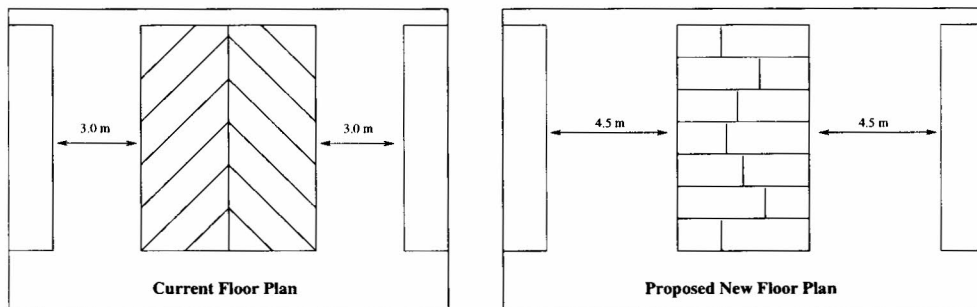


Figure 6: Two layout options.

9. Conclusions

In this report we consider several approaches to solving different problems faced by Australian Paper in the inventory and logistics operations associated with their business of paper manufacture and distribution. We have attempted to outline these approaches as well as some of their strengths and weaknesses. We presented preliminary analyses for a few of these approaches. We conclude that efficiency gains in various aspects of Australian Paper's operations may be obtained through a detailed study based on the above approaches.

Acknowledgements

The moderators, Bob Johnston and Mohan Krishnamoorthy would like to thank the representatives from Australian Paper, Barry Grennan, Rosalyn Malyon and Bruce McComb, for their support and timely assistance. We would also like to thank the following participants who contributed their time and ideas: Natasha Boland, John Cogill, Alexandré Cunéo, Andreas Ernst, Andrew Grenfell, Phil Kilby, Ernest Lim, Kaye Marion, Graham Mills, Brian Murphy, Phil Neame, Roland Shami, Ralph Snyder and Yakov Zinder.

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