MANAGING WAITING LISTS AND THEATRE SCHEDULING FOR SURGICAL PROCEDURES

> "The issue of waiting lists for surgical procedures is not only highly political but of significant concern to the general public. In addition, in acute-care hospitals, large amounts of money are consumed by the theatre units, which is a key feature in the surgical waiting lists problem, so there are important financial issues associated with waiting lists." Watson and Landman (1994).

1. Introduction

One of the problems facing hospitals is scheduling planned operations to provide effective utilisation of the hospital's facilities. This problem involves trying to meet a number of different goals such as maximising the occupancy of beds (and concurrently reducing the variability in the use of beds) and minimising overtime payments to hospital staff. It is also desirable to reduce the need to change operating schedules once patients have been notified of a day for their operation.

The problem is complicated by the fact that there is uncertainty in the duration of operations and also in the recovery times following operations. In addition, the hospital needs to allow for unplanned emergency operations.

Planning the operating schedule over a week, for example, is constrained by the way in which the surgeons draw up their patient lists. Typically, a surgeon will have a weekly booking for a specified number of hours in one of the operating theatres. Patients are initially referred to a surgeon who prepares a list of operations. As they add patients to their lists, the surgeons notify the hospital of the associated operations planned for their booked theatre time. The hospital receives this information weeks or perhaps months in advance.

The scheduling problem of interest here begins when the hospital starts to plan the allocation of staff, to support the operations and provide post operative care for the patients, for the coming week. At this stage, the hospital may negotiate with the surgeons to change their supplied operating lists.

As well as catering for planned operations, the hospital will also have to carry out emergency operations. These emergencies require the use of the operating theatres and of beds in the hospital wards.

A major problem associated with scheduling operations is the availability of beds for the patients. If all the beds are occupied by recovering patients then operations have to be postponed. Patients often receive only very short notice that their operation has been cancelled. It is also possible (though perhaps unlikely) that more patients than expected will be discharged over a few days and that more operations than planned could be carried out. One of the goals of scheduling operations is to achieve a consistently high utilisation of hospital beds.

Hospitals are also constrained to working within a budget. A further goal is to schedule the operations so that overtime payments to staff are kept as small as possible.

2. Statistical methods for data prediction

2.1 A probability distribution for planned surgery

In this section we look at methods for determining appropriate distributions for operating times based on data for operations by different surgeons.

Suppose that we have data for the times that different surgeons require for different operations[1], and that we have a proposed list of patients scheduled for surgery.

If we take X_i as the length of time required for an operation on patient i, with $i = 1, \ldots, N$, then the total theatre time associated with the list is $W = \sum_{i=1}^{N} X_i$. The data gives us an empirical distribution of the X_i, however our real requirement is for a distribution for W so that it is possible to calculate quantities such as

$$\text{Prob(list exceeds available time)} = \text{Prob}(W > \text{available time}), \tag{1}$$

where 'available time' is some value such as the total duration of a surgical block (for example, 4 hours).

These probabilities could be used by:

- the doctor, for example the chief anaesthetist in some hospitals, responsible for fitting the lists submitted by surgeons to available theatres,
- surgeons when compiling their lists for submission to the hospital,
- hospital staff charged with developing improved theatre scheduling methods.

Basically, in order to improve theatre schedules, we need to model and determine theatre times appropriately, and establish unacceptable probability levels for time overruns[2].

[1] There may also be useful covariate information, but trying to include it may subdivide the data too thinly.

[2] We could possibly consider standby lists here, though at most hospitals, given current scheduling methods, it appears that limited bed availability is the critical factor.

Calculating the distribution of W

While it is possible to fit distributions to the data for each operation/surgeon combination, it is unlikely these will convolve nicely, which is what we require for a simple calculation of the distribution of W. Since each $X_i \geq 0$, either Gamma or Lognormal distributions are the likely candidates for the distributions. If X_1, X_2 are (independent) lognormals, then $W = X_1 + X_2$ does not have a nice distributional form. If X_1, X_2 are (independent) gamma, then W has a reasonable form only if they have the same scale parameters[3]. However, it would be unusual to obtain such tractable forms and it seems unlikely that exact analytical methods will play a significant role. Given this situation, there are two possible approaches we could take.

We could rely on the Central Limit Theorem, that is, assume W is approximately normal with, from data on surgeon operation times,

$$\text{mean} = \sum_{1}^{N} \mathrm{E}(X_i) \quad \text{and} \quad \mathrm{Var} = \sum_{1}^{N} \mathrm{Var}(X_i).$$

This approach should be investigated, but our guess is that N will be too small and the individual distributions too heavy-tailed for this to work reliably. Furthermore, any attempts at corrections (using Edgeworth series for instance) require estimations of higher-order moments which are notoriously unreliable.

Alternatively, we could use an empirical approach. For each X_i, use the data to calculate $f_{ij} = \mathrm{Prob}[X_i < t_j]$, where the time sequence $\{t_j\}$ increases monotonically to ∞, say each $t_{j+1} = t_j + 15$ minutes. Since $t_j \to \infty$, $\sum_j f_{ij} = 1$ for each i. Then we have

$$\mathrm{Prob}[W \leq w] = \sum_{j_1,\ldots,j_N : \sum_i j_i \leq w} f_{i_1 j_1} \cdots f_{i_N j_N} .$$

These calculations appear very practical for small N, although a large amount of data storage (for the f_{ij}) may be required.

2.2 A probability distribution for emergency surgery

Suppose that E patients are admitted as urgent emergency cases needing operating theatre time. We assume that E is a random variable with $\mathrm{E}[E] = \mu_E$, $\mathrm{Var}[E] = \sigma_E^2$, and that typically $E \sim \mathrm{Poisson}[\mu_E]$. We further assume that the E patients can be regarded as a random sample from a population of patients requiring random operating times X with distribution $f_X(x)$ [this entails no loss of generality], with $\mathrm{E}[X] = \mu_X$ and $\mathrm{Var}[X] = \sigma_X^2$.

[3] For example, if $X_i \sim \mathrm{Gamma}(\alpha_i, \lambda_i)$ $(i = 1, 2)$, and if $\lambda_1 = \lambda_2 = \lambda$ say, then W will have the form: $W \sim \mathrm{Gamma}(\alpha_1 + \alpha_2, \lambda)$.

Then the total operation time required is the random sum, including the possibility that $E = 0$ and hence $T = 0$, given by

$$T = \sum_{i=1}^{E} X_i .$$

If we let $\pi_n = \text{Prob}[E = n]$, then

$$\begin{aligned} \text{E}[T] &= \mu_X \mu_E, && (2) \\ \text{E}[T^2] &= \mu_E \sigma_X^2 + \mu_X^2 \text{Var}[E] + \mu_X^2 \mu_E^2 , \\ \text{giving} \quad \text{Var}[T] &= \text{E}[T^2] - \text{E}[T]^2 = \text{E}[E]\text{Var}[X] + \text{E}[X]\text{Var}[E] && (3) \end{aligned}$$

which is a well known result.

2.3 A probability distribution for all surgical procedures

If we add W, the operating times for planned surgery, and T the operating times for emergency surgery, to obtain a *total* time Z, then assuming independence of W and T, we have

$$\text{E}[Z] = \text{E}[W] + \text{E}[T] \quad \text{and} \quad \text{Var}[Z] = \text{Var}[W] + \text{Var}[T] .$$

$\text{E}[W]$ and $\text{Var}[W]$ can be determined from historical data relating to the categories of the scheduled patients and a sensible criterion would be to ensure that

$$\text{Prob}[Z > \text{available time}] < \alpha \qquad (4)$$

where $\alpha = 0.05$, say, to allow for gaps between operations and so on. The patients to be rescheduled are then selected, as far as possible, to ensure that this constraint is satisfied. Essentially, (4) is an extension of (1) to include both scheduled and emergency operations.

In the above calculations we have estimated only the mean and variance of (T and) Z and not the entire distribution, which would be needed to evaluate (4). The issue of finding such a distribution is looked at in the next section. In the meantime we note a heuristic, based on the normal distribution, that says 95% of the time, the observed data lies within $\pm 2\sigma_Z$ of μ_Z.

2.4 Predicting bed occupancy t days ahead

Data (See Table 1) suggest that the number of emergency patients, E, arriving each day is a Poisson process with an approximately constant arrival rate μ_E (Sunday is the exception).

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
N	119	119	117	118	121	119	119
Mean	6	7	7	7	7	7	7
Std Dev	3	3	3	3	3	3	3
Sum	730	821	808	846	804	847	811

Table 1: SAS output: emergency operations (John Hunter Hospital).

If we assume that each emergency patient stays in hospital for a random length of time, and that these lengths of stay are independent and identically distributed random variables with means μ_S then the process of emergency arrivals is an $M/G/\infty$ system. This has the nice property that the number in the system (in equilibrium) at any time is a random variable with a Poisson distribution and mean $\mu_E\mu_S$. Thus it is possible to predict, with stochastic error, the number of beds occupied by emergency cases at an *arbitrary time*.

However, we are probably more interested in predicting how many beds will be occupied by emergencies in t days time ($t = 1, 2, \ldots$), given the current state of the system. We mention two possibilities.

We could use clinical information about current (today's) patients to assess their chances of being in the system in t days time and then add an estimate of the number of new arrivals who will still be in hospital in t days time. The latter component is Poisson with mean μ_EProb[length of stay $\geq t$]. Hopefully the former component will be known with a reasonable degree of accuracy.

A second possibility is to use statistical, rather than clinical, information on the current patients. That is, consider the question: 'Given the $M/G/\infty$ model, what is the distribution of bed occupancy in t days given b_n patients here today?'

Variance of predicted bed occupancy

Let B_n be the number of beds occupied on day n by emergency cases and assume that the patients' lengths of stay are given by independent and identical geometric distributions, that is,

$$\text{Prob(length of stay} = k \text{ days)} = q^k p, \quad k = 0, 1, 2, \ldots$$

The mean length of stay is $\mu_S = q/p$. (This results from determining discharge each day by independent Bernoulli trials, where p is the probability of discharge).

If we assume no control, complete independence and Poisson emergency arrivals E_n on day n with mean μ_E and standard deviation σ_E, then

$$B_{n+1} = S^1(B_n) + S^1(E_n),$$

where the operator $S^t(N)$ denotes the number of patients, out of an initial population of size N, who remain in hospital after t days, and

$$\mathrm{E}[B_{n+1}] = qb_n + q\mu_E\,,$$

where the lower case b_n denotes the number of occupants at day n. That is, b_n is known at day $n+1$. Equivalently, we seek $\mathrm{E}[B_{n+1} \mid B_n = b_n]$.

Now, $\mathrm{Var}[B_{n+1}] = pqb_n + q^2\sigma_E^2 + pq\mu_E$ and the probability of surviving t days is q^t $(t = 1, 2, 3, \ldots)$. So $B_{n+2} = S^2(b_n) + S^2(E_n) + S^1(E_{n+1})$, and

$$\mathrm{E}[B_{n+2}] = q^2 b_n + \mu_E(q + q^2)\,.$$

In addition, since $S^2(E_n)$, $S^1(E_{n+1})$ are Poisson distributed,

$$\mathrm{Var}[B_{n+2}] = q^2(1-q^2)b_n + \mu_E(q + q^2)\,.$$

Similarly,

$$B_{n+t} = S^t(b_n) + S^t(E_n) + S^{t-1}(E_{n+1}) + \ldots + S^1(E_{n+t-1})\,.$$

giving

$$\begin{aligned}\mathrm{E}[B_{n+t}] &= q^t b_n + \mu_E(q + q^2 + \ldots + q^t) \\ &= q^t b_n + \mu_E(q/p)(1-q^t) \\ &= q^t b_n + \mu_{B_\infty}(1-q^t),\end{aligned}$$

where μ_{B_∞} is the mean occupancy in the stationary state, namely the product of the emergency arrival rate μ_E and the mean length of stay μ_S.

We therefore have

$$\mathrm{E}[B_{n+t}] \to \mu_{B_\infty} \quad \text{as} \quad t \to \infty.$$

and can also write, $\mathrm{Var}[B_{n+t}] = b_n q^t(1-q^t) + \mu_{B_\infty}(1-q^t)$.

Finally, if $b_n \approx \mu_{B_\infty}$ (averaging over values of B_n), then

$$\mathrm{Var}[B_{n+t} \mid B_n = b_n] \approx \mu_{B_\infty}(q^t - q^{2t} + 1 - q^t) = \mu_{B_\infty}(1 - q^{2t}). \qquad (5)$$

Generalising the length of stay distribution

The above approximation of the variance of the bed occupancy (5) can be generalized to allow for an *arbitrary* probability distribution of the length of stay. Note that in the following equations we are working with continuous time. A discrete time approach can also be developed.

If, as previously, we assume a Poisson arrival process of patients with mean μ_E, and write the length of stay as a cumulative distribution function F with mean μ_S, then by reasoning similar to the above, we obtain

$$\mathrm{Var}[B_{n+t} \mid B_n = b_n] \approx \mu_E \mu_S \left(1 - \left\{\frac{1}{\mu_S}\int_t^\infty (1 - F(x))dx\right\}^2\right) . \qquad (6)$$

The quantity on the right is zero at $t = 0$, and tends to $\mu_E \mu_S$ as $t \to \infty$, as expected. Furthermore, (6) seems to show that the variance is remarkably *insensitive* to the actual form of F.

For example, taking $\mu_S = 5$ and the four cases

C = constant distribution (length 5)
U = uniform distribution (on (0,10))
Exp = exponential distribution ($F'(x) = (1/5)e^{-x/5}$)
G_2 = gamma distribution of index 2 ($F'(x) = (2/5)^2 x e^{-2x/5}$)

and considering the quantity

$$q(t) = 1 - \left\{\frac{1}{\mu_S}\int_t^\infty (1 - F(x))dx\right\}^2$$

that appears in (6) (the proportion of ultimate variance for a t days ahead prediction), we obtain the following values for $q(t)$.

Distribution	Length of stay (t)	
	$t = 1$	$t = 5$
C	0.36	1.00
U	0.34	0.94
Exp	0.33	0.86
G_2	0.35	0.93

Table 2: Values of $q(t)$.

We conclude that the prediction of bed occupancy looks, in principle, like a fairly well organized and stable mathematical/statistical problem.

3. An optimal schedule for operations

3.1 A simple model for scheduling operations

Scheduling operations involves both planning and operational factors. Here we look at an initial model for planning a schedule of operations over a period such as a week. We assume that the hospital has been presented with the surgeons' lists and wishes to investigate how to best arrange the proposed operations, bearing in mind that emergency operations must be allowed for.

In practice, the operation and recovery times are stochastic, they are not known exactly until the operation is completed and until the patient is discharged. However, when planning for the coming week, the hospital staff need to estimate the time that a particular surgeon will require for a given operation and the number of days for which a recovering patient will need a bed.

With respect to the *planned* operations, we assume that all the variables are deterministic in the sense that we have a specified duration and recovery time for each nominated operation. These times correspond to the average values as estimated by the hospital staff. Essentially, we consider a single, averaged, scenario for the planned operations. At this stage we make no attempt to optimise over a number of different planned operation scenarios.

We allow for *emergency* operations by including a number of randomly generated operations on each day. Each emergency operation has an operating and recovery time generated from appropriate random distributions. We do not consider different emergency scenarios.

The general optimisation strategy is to generate the random emergency operations and treat them as 'fixed' operations on each day. We then reschedule the planned operations in an attempt to obtain an improved utilisation of the hospital's resources and facilities.

3.2 Parameters and variables for the model

In this section we define a series of parameters and variables which we use to model the movement of patients presenting for surgery, through the hospital. We use these terms to develop a set of equations for different performance measures related to the staff resources and facilities available in the hospital. These equations form the basis for a heuristic scheduling algorithm based on a simulated annealing approach.

In a later section we look at the results obtained from the algorithm.

Parameters

i — patient index, $i = 1, \ldots, P_2$
The patients for planned operations are represented by the indices $i = 1, \ldots, P_1$. We can reschedule these operations.
Emergency operations are represented by $i = P_1 + 1, \ldots, P_2$. We generate these operations and their characteristics using random distributions drawn from existing hospital data. Once assigned, the emergency operations cannot be rescheduled.

j — operating theatre index, $j = 1, \ldots, M$

n — a day in the planning horizon, $n = 1, \ldots, N$

B — the number of beds available in the ward

C_n — the existing bed occupancy from previous operations

T_i — operating theatre for patient i

S_i — scheduled day of operation for patient i

D_i — duration (in hours) of operation on patient i

R_i — days of recovery for patient i

Variables

x_i — the actual (possibly rescheduled if $i \leq P_1$) operation day for patient i

b_n — the number of beds occupied on day n

e_n — the number of beds in excess of B on day n

o_{jn} — the hours of overtime needed in theatre j on day n

With these variables and parameters, we can calculate the following terms related to the utilisation of facilities in the hospital ward.

Bed occupancy

Patient i will occupy a bed between days x_i and $(x_i + R_i)$, and if we define

$$U_{in} = \begin{cases} 1 & \text{for} \quad x_i \leq n \leq (x_i + R_i), \\ 0 & \text{otherwise}, \end{cases} \qquad \begin{matrix} i = 1, \ldots, P_2 \\ n = 1, \ldots, N \end{matrix} \tag{7}$$

then the number of beds occupied on day n is

$$b_n = \sum_{i=i}^{P_2} U_{in} + C_n, \tag{8}$$

and the mean and variance of the bed occupancy are

$$\mu_b = \frac{\sum_{n=1}^{N} b_n}{N} \quad \text{and} \quad \sigma_b^2 = \frac{\sum_{n=1}^{N} b_n^2}{N} - \mu_b^2. \tag{9}$$

The excess bed requirement on each day is

$$e_n = \begin{cases} b_n - B & \text{for} \quad b_n > B, \\ 0 & \text{otherwise}. \end{cases} \tag{10}$$

Deviation from scheduled day of operation

The planned operation on patient $i \leq P_1$ involves rescheduling by $(x_i - S_i)$ days.

Theatre utilisation

The number of hours of operations carried out in theatre j on day n is given by

$$h_{jn} = \sum_{i=1}^{P_2} \{D_i : T_i = j\,,\, x_i = n\}\,. \tag{11}$$

If we set 8 hours as the normal working hours each day in an operating theatre, then the overtime worked in theatre j on day n is given by

$$o_{jn} = \begin{cases} h_{jn} - 8 & \text{for} \quad h_{jn} > 8\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{12}$$

3.3 An objective function for the model

There are a number of factors that need to be balanced in order to obtain a good operating schedule. The particular factors and their relative weights will depend largely on the systems used in a particular ward. In general however, we are looking for an optimal balance of some or all of:

- high bed occupancy with hopefully a low variation in this occupancy,
- a low need for overtime during the planning horizon,
- minimum deviation from scheduled operation times.

Considering each of these items in turn, we could develop an objective function, *to be minimised*, consisting of the following terms. In the discussions of these terms, the weights $W_v, W_m, W_e, W_o, W_r \geq 0$ indicate the relative importance we give to each of: the variation in bed occupancy, the level of bed utilisation, the use of excess beds, payment of overtime and rescheduling planned operations.

Bed occupancy factors

The factors relating to bed occupancy can be expressed as

$$W_v \sigma_b^2 - W_m \mu_b + W_e \sum_{n=1}^{N} e_n\,. \tag{13}$$

The third term in (13) indicates that we are penalising the use of excess beds rather than enforcing a hard constraint such as

$$b_n \leq B \qquad n = 1, 2, \ldots, N\,.$$

This allows for a situation in which a hospital may choose to pay for the transfer of recovering patients to beds in another hospital. It also simplifies the development of a simulated annealing algorithm for the optimisation problem. By incorporating the excess bed factor as a penalty term in the objective we remove the need to maintain feasibility (at least for this factor) as candidate solutions are generated. If necessary, we can effectively prevent excess bed use by setting a very high penalty for $e_n > 0$.

Overtime

The contribution to the objective from overtime worked during the planning period is given by

$$W_o \sum_{n=1}^{N} \sum_{j=1}^{M} o_{jn} \,. \tag{14}$$

If necessary, we could allow for an overtime budget of say $\$Y$ and use a term of the form

$$W'_o \{ \bar{p} \sum_{n=1}^{N} \sum_{j=1}^{M} o_{jn} - Y \} \,,$$

where $\bar{p}$ is some mean overtime payment rate. We could also formulate the overtime factor in terms of a hard constraint.

Rescheduling operations

We use the expression $e^{|x_i - S_i|}$ to penalise a change in the operation for patient i from day S_i to day x_i. The exponential function is used here to try and discourage shifting operations by too many days. The particular changes allowed to S_i will depend on hospital policies. In particular, they will need to reflect the arrangements that the surgeons have with the hospital. The contribution to the objective from rescheduling operations is

$$W_r \sum_{i=1}^{P_1} e^{|x_i - S_i|} \,. \tag{15}$$

The objective function

Combining (13)–(15) gives the following objective:

Minimise:

$$W_v \sigma_b^2 - W_m \mu_b + W_e \sum_{n=1}^{N} e_n + W_o \sum_{n=1}^{N} \sum_{j=1}^{M} o_{jn} + W_r \sum_{i=1}^{P_1} e^{|x_i - S_i|} \,. \tag{16}$$

3.4 Using simulated annealing to minimise the objective

We have written the operation scheduling problem as an objective function with a series of weighted penalties. The terms for excess bed use and overtime could be treated as hard constraints by using suitably large weights in the objective function.

As formulated, we have a mixed Non-Linear Integer Program (NLIP) problem to solve. Given the complexity of the objective, a heuristic approach, such as simulated annealing (SA), is probably the most appropriate method to apply to the problem.

We developed a computer program to solve the mixed NLIP minimisation problem given by (16) and describe some of the factors involved in implementing the algorithm below. We also discuss some of the results obtained from the program.

3.5 Developing the simulated annealing program

There are two major aspects associated with developing the SA code for our problem.

1. The first is reasonably straightforward and involves writing a set of housekeeping subroutines to calculate each of the terms in the objective for a given candidate solution of x_i values.

2. The second is a little more complicated. It involves setting up suitable transitions for moving from one candidate solution to another. Essentially we want to explore the near neighbourhood of the current solution. To restrict major changes to a solution we employed the following two transition operations:

    - a pairwise interchange of the operating days for two patients, that is, swap the values of x_i and x_k for randomly chosen i and k,

    - a shift in the day of operation for a patient, that is, $x_i \leftarrow x_i \pm d$ for a randomly chosen i, with d a random number of days.

    In practice, these transitions will be restricted to a set of allowed moves that reflect the arrangements that surgeons have with a given hospital. Patients will need to be rescheduled to other days (and possibly time periods) on which their surgeon has time booked in the operating theatres.

At the time of writing, we did not have the data needed to set up a rule base which ensured that patients are rescheduled to other days on which their surgeon operates. In order to test the program, we allowed operations to be rescheduled to any other day in the planning period. Essentially this gives us a relaxed problem that should give 'better' solutions with respect to factors, such as bed utilisation and overtime, which are of interest from the viewpoint of hospital administrators.

3.6 Results from the optimisation program

In the absence of data from hospital records, we used a small test problem in which 40 patients are scheduled for planned operations in 4 theatres over a one week (5 day) period, 45 beds are available in the ward for recovering patients.

The emergency operations for each day are generated from a Poisson distribution with a mean of 4 operations per day. For the test case, a total of 21 operations were generated. The operation durations and recovery periods are taken from exponential distributions with means of 0.6 hours and 5 days respectively. The emergency operations are spread evenly among the operating theatres. We obtained the 'known' bed occupancy for the current week, resulting from operations carried out during the previous week, from a prior run of the program with a similar set of operation data.

Table 3 gives a summary of the some of the performance measures obtained from the program before the optimisation process starts. These values were calculated from the initial list of planned operations, together with the expected emergency operations.

Operations on each day		6	4	15	22	14
Overtime				1.6	6.4	1.4
Beds occupied on each day		36	38	38	51	58
Bed occupancy:	Average	44.20				
	Variance	95.20				

Table 3: Results for initial list of operations.

From Table 3 we note that there are a large number of operations on the last 3 days and that we need more beds than are available (45) on the last 2 days.

Running the program with weights[4]: $W_v = 0.3$, $W_m = 1.0$, $W_e = 2.0$, $W_o = 1.0$ and $W_r = 0.0$, gave the following results.

Operations on each day	12	5	15	16	13
Overtime	1.8		0.4	2.3	0.1
Beds occupied on each day	42	45	45	45	45
Bed occupancy: Average	44.40				
Variance	1.800				

Table 4: Results for optimised operating schedule, no rescheduling penalty.

The results in Table 4 were obtained with $W_r = 0.0$, that is, there is no penalty if operations are rescheduled. The total overtime has been reduced from 9.4 hours (see Table 3) to 4.6 hours. Bed occupancy never exceeds the maximum number of beds available and is more evenly spread across the week. These results have been obtained by rescheduling operations as shown in Table 5.

Patients 1–10			2		1			−2	−2	1
Patients 11–20	−1	−3	−2					−1	2	
Patients 21–30		−1	−2	−3		−1	−1	−2	−3	1
Patients 31–40	−1	−1	2	−3		−2		−2	2	1

Table 5: Rescheduling of operation days, no rescheduling penalty.

If we use a value $W_r = 0.3$ to restrict the extent to which operations are rescheduled then we obtain the results shown in Tables 6 and 7. The effect is to reduce overtime from 4.6 hours to 4.3, there is a slight decrease in the average bed occupancy[5] and an increase in its variance.

Operations on each day	10	7	15	14	15
Overtime			2.2	0.4	1.7
Beds occupied on each day	40	45	45	45	45
Bed occupancy: Average	44.00				
Variance	5.000				

Table 6: Results for optimised operating schedule, $W_r = 0.3$.

[4]Note that these weights do not in themselves give a direct idea of the relative contribution of each term in the objective function, we also need to look at the relative magnitudes of the terms μ_b, σ_b^2, e_n and so on.

[5]It is often difficult to predict how changing the weights in an objective function such as that given by (16) will affect the solution.

For the case with $W_r = 0.0$, 26 out of 40 planned operations are rescheduled. The average change to the schedule, for those patients affected, is 1.7 days. With $W_r = 0.3$, the number of rescheduled operations has decreased to 15 with an average shift of 1.3 days.

Patients 1–10		−1	−1					−2	−1	
Patients 11–20	−1	−1	−2		1					
Patients 21–30	−1		−2					−2	−1	
Patients 31–40	−1	−1	−1							

Table 7: Rescheduling of operation days, $W_r = 0.3$.

4. Conclusions

The models described here are simplified representations of patient behaviour and hospital processes, and the systems used to organise surgical procedures in a hospital. Nevertheless, we feel that these provide a reasonable starting point for developing an optimisation procedure for planning theatre schedules.

There is a wealth of data already available on hospital patients, such as information on arrivals, surgery times, and recovery times. This can provide the basis for the application of often straightforward statistical techniques not only to estimate, for instance, bed occupancy the day after tomorrow, but to give measures of accuracy of such estimates. These predictions can be implemented in the short term (in a computer system), and used in a systematic way to assist human schedulers. An eventual goal may be to use such predictive technology to produce partial inputs for an automatic but interactive scheduling program based on optimization techniques (§3).

Individual hospitals have their own systems for allocating surgeons to theatres and rostering the support staff for operations. As well, different performance measures may be used in different hospitals. With regard to optimizing theatre schedules, any further work probably needs to concentrate on developing the model to meet the needs of a particular hospital. The most practical initial steps would be to:

- Adapt the different terms in the objective to reflect the performance measures used at the hospital.
- Incorporate the rules needed to ensure that the transitions used to generate solutions in the SA reflect the arrangements that the surgeons have with the hospital. Essentially, the association of patients with surgeon theatre times constrains the possible values for each x_i.

- With the search space constraints for the hospital implemented, optimise the various SA control parameters.

Acknowledgments

The moderators Danny Ralph and David Sier would like to thank the industry representatives Paul Fahey and Steve Gillett for their participation in the MISG 1994 and insight into the health problem. We also thank other participants in this problem, especially those whose contributions directly or indirectly appeared in this report: John Bithell, Carol Callaghan, David Cromwell, Mark Westcott, Stuart Wilson, Jim Youngman.

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