MODEL CONSTRUCTION FOR DECISION SUPPORT SYSTEMS FOR BEEF STATIONS

1. Introduction

Because the success of a Decision Support System (DSS) is measured in terms of the accuracy, reliability and relevance of the information (outputs) it generates about various scenarios (inputs), and thereby of the quality and perceptiveness of the decisions which that information invokes, the accuracy, reliability and relevance of the outputs become the criteria for focussing the construction of the quantitative model which describes the underlying process. Thus, when compared with the utilization of a DSS, the initial step of model formulation plays a hidden, but fundamental, role. In this paper, the focus is on *the Modelling* used by Monypenny and his colleagues (see References) to construct their Decision Support System for Beef Stations (DSSBS).

However, before turning to an examination of the Modelling, it is first necessary to clarify the relationship between Decision Support and the Modelling on which it is based. The role of Decision Support is to supply the relevant information required for decision making within a specific context such as the management of a beef cattle station. The role of the Modelling is to first identify the relevant factors and to then quantify their interrelationship as inputs and outputs, as well as the defining relations which connect them. For example, in the present situation, the Decision Support becomes: given the current conditions on a beef cattle station and inductive/deductive models of how the relevant factors interrelate, *the aim* is to make a decision about the future management of the station on the basis of a careful assessment of the profitability of possible management scenarios (options).

It therefore becomes clear that, though they depend on each other, the Modelling and the Decision Support are separate activities. One cannot have Decision Support without first constructing a suitable model. Once an appropriate model is available, then the Decision Support is an on-going activity. However, the Modelling must be performed within the framework of the particular application for which Decision Support is being designed. This is reflected in the fact that the outputs of the model are the inputs to the Decision Support. Thus, in the present context, profitability of the various management options in terms of the Net Cash Flow (NCF) is both the output of the model and the input to the Decision Support.

It follows that the success of a Decision Support activity depends crucially on the *tacit assumptions* that

- the underlying model is realistic and relevant to the problem context
- the outputs depend continuously (and stably) on the input parameters defining the profitability

The first assumption reflects the need, already flagged above, to do the Modelling within the framework of the particular application for which Decision Support is being designed. This requirement is driven by the fact that, the more faithfully the model represents the situation under consideration, the more likely the outputs will reflect the outcome of that situation. The second assumption highlights the fact that the model, in order to be useful for Decision Support, must be such that small changes in the inputs produce only small changes in the outputs. The study of this aspect goes under a variety of names including trend surface analysis, stability analysis, *etc.*

Usually, the validity of such assumptions is taken for granted. They were the prime consideration for the Study Group's deliberations. The opportunity was also taken to explore and examine related issues including general rationales for Decision Support systems, and the relationship between revenue and productivity.

In fact, in examining the validity and consequences of the above assumptions, as they related to the DSSBS problem, the Study Group focussed attention on the following questions:

(i) Does the model proposed by Monypenny and his colleagues represent a realistic assessment of the process being analysed?

(ii) Does a rationale exist for the methodology used by Monypenny and his colleagues to construct their DSSBS?

(iii) In what new ways can the model and its output be analysed and utilized?

(iv) How can the sensitivity of the model be studied and, if possible, quantified?

In Section 3, we explain and justify the modelling used by Monypenny and his colleagues to construct their DSSBS. This represents the Study Group's response to question (i). However, before we do this, it is necessary to develop a modelling framework into which it can be embedded naturally. This is done in Section 2, and answers question (ii). A basis for a sensitivity analysis of the outputs is the topic of Section 4. This lays a foundation for the future analysis of question (iv). Question (iii) is addressed in Section 5 where aspects connected with maximizing profitability (revenue) and productivity are pursued.

Concluding remarks are contained in Section 6, which contains some comments about modelling options for extending the utility and relevance of DSSBS.

2. Modelling rationales

In order to structure the subsequent discussion, it is first necessary to make some remarks about the nature of mathematical modelling. In one way or another, most texts on modelling draw a distinction between purely *inductive* (black box) models, where general inferences are derived from particular circumstances without knowledge of the underlying process, and fully *deductive* (white box) models, where particular conclusions are derived logically from general assumptions about the underlying process. Some even discuss modelling in terms of a spectrum of activities (gray boxes) ranging from the purely inductive (black; soft) to the completely deductive (white; hard). This is usually done in terms of the frameworks or contexts in which the models are being derived (*e.g.* social; ecological; physiological; hydrological; industrial; scientific). It reflects the extent to which deductive results can be utilized, and inductive steps are essential for generating structure. This aspect has been discussed at some length by Karplus (1977), and applied to the analysis of Geosphere–Biosphere models by Tucker (1988).

The point which concerns us here is the actual construction of a model (independent of the problem context). For purely inductive modelling, it is simply a matter of identifying the measured input and outputs along with some suitable framework (*e.g.* parameterized least squares; convolution modelling; Kalman filtering; stationary stochastic processes) in which to model their interrelationship. Except possibly when choosing the framework, nothing concrete or deductive is exploited from the problem context. For deductive models, the process is much more complex and involved. Because the measured inputs and outputs are often determined by considerations which do not relate to the underlying process, any modelling must accommodate this dichotomy in some appropriate manner. Therefore, it is not simply a matter of deciding to relate the inputs and outputs deductively on the basis of laws and structure from the problem context. There is an initial need to analyse how the inputs and outputs relate to and can be modelled in terms of the underlying process, and then to seek appropriate laws and structure from the problem context for achieving this.

There are various ways in which models can be and are constructed. The technique which is of relevance to the present deliberations is the use of a *link concept*, which acts as a stepping stone between the inputs and the outputs. The use of such a concept has a number of advantages:

i. **A link concept decouples the inputs from the outputs.** In many problems, the form of the available inputs and outputs is determined by considerations dictated by the problem context rather than the process which relates them. The introduction of a link concept therefore circumvents this difficulty by allowing the inputs to be decoupled from the outputs. In addition, its introduction represents an opportunity to re–emphasize the underlying process connecting the inputs and outputs. Thus, the introduction of a link concept can be used to both re-emphasize the underlying process and to suitably decouple (de-emphasize) the inputs from the outputs.

ii. **A link concept can enhance the relevance of the modelling.** Through a judicious choice of the link concept, the underlying nature of the problem can often be exposed. For example, the link concept can be the phenomenon one would like to measure about the process but cannot. The output is then seen as the only available information about the phenomenon. In addition, the identification of such a concept often assists in focussing on how the inputs should or could be modelled deductively.

iii. **The use of a link concepts adds flexibility to the modelling.** The model now becomes a multi–phase process each component of which can be modelled either inductively or deductively. In fact, the link concept can be used to partition the deductive aspects of the problem from the inductive.

From the point of view of the present DSSBS considerations, it is (ii) which is the crucial aspect. As explained in some detail in the next section, the *weight gain* of the cattle has been used as the link concept for the construction of the DSSBS and thereby gives it its utility. In addition, it yields a natural partitioning of the deductive aspects from the inductive.

3. The decision support system for beef stations

The actual decision support system (DSS) constructed by Monypenny and his colleagues (Gillard and Monypenny (1988,1990); Gillard *et al.* (1989); Monypenny and McIvor (1989)) is more complex than that described here. However, since the aim is to examine questions connected with the modelling, it is only necessary (and in fact sensible and desirable) to consider a model which suitably encapsulates the essential quantitative features of the DSS under consideration. In any analysis of modelling, such action is crucial to minimizing the effort devoted to peripheral considerations.

Topographically, a beef-cattle station (BS) consists of a large piece of land (2000 or more hectares) which has been partitioned into areas of (i) native pasture (in the hilly and/or poor soil regions); (ii) cleared native pasture (in undulating and/or moderate good soil regions); (iii) oversown legumes (in the flatter regions of good soil adjacent to (semi-) permanent water). This partitioning defines the different pasture types p. Fuller details can be found in Monypenny(1990).

As a commercial enterprise, the BS grazes different cattle types q on the various pasture types p, usually in a fairly well-defined stratified manner, in the sense that different cattle types are grazed on different pastures so that the model for each type decouples. For management purposes, the cattle types q are partitioned into (a) breeders and (b) non-breeders, with both groups sub-partitioned on the basis of age. The role of the breeders is to maintain the stock levels so that the need for restocking (purchase of breeders or non-breeders) is minimized. The breeders therefore tend to play only a passive role in revenue generation.

During the year, older non-breeders are sold to generate the revenue R, and hence, the net cash flow (NCF) for the BS. The *aim* then of the DSSBS is to determine possible stocking (management) strategies which will tend to ensure that NCF is maximized, within the framework of expected pasture growth, which is defined as the number of green weeks g_t which summer rains in year t guarantee for that year.

However, we are confronted by a situation where we have inadequate data for an inductive model, and the problem is too complex to derive a fully deductive model. Hence, the need arises for an alternative approach such as the use of a link concept.

There are various ways in which these factors could be interrelated. A purely inductive model could be derived. But, the construction of such a model assumes the availability of appropriate data which highlights the underlying pattern which needs to be modelled. However, this is not the situation in the present circumstances. A fully deductive model is clearly out of consideration given the nature of the problem. Monypenny and colleagues constructed a model based on a *link concept*. For this concept, they chose *the annual weight gain* $w = w(g_t)$ of the cattle as a function of green weeks g_t. The number g_t of green weeks is determined at the end of summer on the basis of the nature and extent of that summer's rainfall. It is an assessment of the growth potential the pasture has in the coming autumn and winter seasons. This turns out to be a highly perceptive choice for the following reasons:

(a) It is realistic and appropriate for the matter under consideration, since the whole process is concerned with the weight gain of the animals. It gives a natural intuitive appeal to the model, since weight gain is a concept with which the station manager works regularly. It is also used as an indicator of other factors including the overall health of the animals.

(b) It allows a natural decoupling of the inputs from the outputs in that (i) a deductive type model can be constructed relating weight gain to revenue; and (ii) an inductive/deductive model can be constructed relating p, q, and g_t to $w(g_t)$.

The additional facts, definitions and constraints which fully characterise the problem quantitatively are:

1. The area of a pasture class p is denoted by $A(p)$.
2. The weight gain $w(g_t)$ stratifies with respect to the pasture class p and the cattle type q. However, in the discussion below, this aspect is suppressed, since it adds nothing to the modelling considerations.
3. In year t, the amount of feed available is determined by the number of green weeks g_t which in turn is determined by the extent of the summer rains at the beginning of that year.
4. In year t, the amount of feed available determines the carrying capacity of the BS, and hence, whether there is too few or too many stock for the coming season. A decision can then be made about stocking (*i.e.* whether cattle should be bought or sold) before the season commences.

Note: For simplicity, we have invoked the Markovian assumption that the circumstances in year t only depend on those in year $t-1$. In fact, as we shall see below in equation (1), the connection is implicit because the cattle feed on pasture which has grown as the result of earlier rains. Clearly, the interrelationship is more complex in reality.

One is now in a position to utilize the link concept in the manner outlined above in (b)(i) and (b)(ii).

(b)(i) The "input–weight gain" model

The model used to relate weight gain $w(g_t)$ to the various inputs has both a deductive and an inductive aspect. The deductive aspect defines, for a given cattle type q, the number N_t of the cattle alive at the end of year t given there were N_{t-1} cattle alive at the end of year $t-1$. In fact, we have

$$N_t = N_{t-1}[1 + b(w(g_t)][1 - d(w(g_t)][1 - c(w(g_t)][1 + r(w(g_t)] \qquad (1)$$

where the terms modifying N_{t-1} correspond respectively to the effect of *birth*, *death*, *culling* of non-breeders for market, and the purchase of breeders and/or non-breeders for *restocking.*

The inductive aspect defines, for a given cattle type q, the various functions $b(\cdot)$, $d(\cdot)$, $c(\cdot)$, and $r(\cdot)$, entering into the above deductive formula in terms of the weight gain $w(g_t)$. For example, on the basis of available data, the following models have been constructed for $b(\cdot)$ and $d(\cdot)$:

$$b(w(g_t)) = \begin{cases} 0 & \text{non} - \text{breeders} \\ \bar{a}/(w(g_t) + \bar{b}) & \text{breeders} \end{cases} \tag{2}$$

where $\bar{a}$, $\bar{b} \equiv$ consts,

$$d(w(g_t)) = \begin{cases} 1/(\bar{c} + \bar{e}(w(g_t) + \bar{f})) & \text{non} - \text{breeders} \\ 1/(\bar{g} + \bar{h}(w(g_t) + \bar{k})) & \text{breeders} \end{cases} \tag{3}$$

where $\bar{c}$, $\bar{e}$, $\bar{f}$, $\bar{g}$, $\bar{h}$, $\bar{k} \equiv$ consts, and where weight gain as a function of the number of green weeks g_t is assessed inductively to be

$$w(g_t) = a - \frac{N_t}{A}(\frac{e}{g_t} - f), \quad a, e, f \equiv \text{consts} \tag{4}$$

An equation for the culling proportion $c(w(g_t))$ is based on whether or not the number of available breeders on the property exceed the maximum number required for the coming year. Culling only takes place when the number of available exceeds the number required. Because it lay outside the needs of the Study Group's deliberation, formulas for the restocking where not pursued, which reduces to assuming that $r(w(g_t)) \equiv 0$.

(b)(ii) The "weight gain–output" model

This reflects one of the advantages of having made a judicious choice for the link concept. The model relating the weight gain to NCF takes a simple (pseudo-deductive) form

$$(NCF)_t = R_t - C \tag{5}$$

where C denotes the running "costs" of the cattle property, which can be assumed to be fixed (*i.e.* C does not depend strongly on N_t); and R_t denotes the "revenue in year t" which is determined by

$$R_t = H_t W_t \$_t \tag{6}$$

where $\$_t$ denotes the "price per kilo in year t" (which often has a dependence on the weight of the non-breeders sold); W_t denotes the "weight per non-breeder sold in year t", which is determined by

$$W_t = W_{t-1} + w(g_t) \tag{7}$$

and H_t denotes the "number of head sold in year t", which is determined by

$$H_t = N_t(1 - d(w(g_t))) - \hat{H}_t + \bar{H}_t \tag{8}$$

where $\hat{H}_t$ and $\bar{H}_t$ denote respectively the number of non-breeders sold and bought throughout year t.

The above modelling illustrates the clear advantages in the use of an appropriate link concept. One has in (1) a precise formula for predicting the number N_t of cattle in year t from that available in year $t-1$ (assuming the individual terms $b(\cdot)$, $d(\cdot)$ *etc.* are known exactly). One has relegated to (2), (3) and (4) the inductive aspect of the modelling so that its role in the overall process is clearly differentiated. The perceptiveness of the choice of weight gain $w(g_t)$ as the link concept is reflected in

i. the fact that $w(g_t)$ is a concept regularly used on the BS for management purposes as well as the key factor in determining the market price
ii. the economic and biological relevance of (5)-(8) in modelling the revenue aspect of the process

An interesting aspect to the Study Group's deliberations arose out of the observation that, because difference equations can be reinterpreted as differential equations, it must be possible to give a differential equation interpretation to equation (1). However, though interesting in its own right, it does not relate directly to the process under investigation because (1) is modelling a clearly discrete process (integer number of cattle with time steps of 1 year). Nevertheless, it did yield qualitative insight into the underlying process.

4. The sensitivity analysis of outputs

A major goal behind most modelling is to gain a more accurate understanding of the interdependence of the outputs (which characterize the performance of the process) on the inputs (which charactize how the process is driven). This is usually approached in some form of sensitivity analysis. Like the modelling itself, such an analysis can be accomplished in a variety of ways.

One possibility is to simply apply the standard formula for the differentiation of a function of two or more variables to any component of the model as well as the full model. For example, consider equation (4), which defines weight gain as a function of green weeks. It is basically a function of two parameters $g(= g_t)$ and $\alpha(= N_t/A)$ which can be written in the form

$$w(g, \alpha) = \bar{a} - \alpha(\frac{\bar{b}}{g} - \bar{c}) \tag{9}$$

It follows that

$$\begin{aligned} dw(g,\alpha) &= \frac{\partial w}{\partial \alpha}\delta\alpha + \frac{\partial w}{\partial g}\delta g \\ &= -(\frac{\bar{b}}{g} - \bar{c})\delta\alpha + \frac{\alpha b}{g^2}\delta g \end{aligned} \tag{10}$$

which allows one to conclude that

- When the number of green weeks is held constant, the weight gain correlates negatively with cattle numbers.
- When the number of cattle is held constant, the weight gain correlates positively with the number of green weeks.
- Ignoring drought conditions (which is a pathological situation for the model), weight gain is more sensitive to changes in the number of green weeks than to changes in the number of cattle.

An alternative way for studying sensitivity is to perform a more comprehensive stability analysis of the type discussed by Rheinboldt (1986) for the analysis of parameterized non–linear equations. In fact, as indicated by Rheinboldt: "While the local stability is important in practice, it is equally important to understand those variations of the parameters which produce a change of behaviour." This is achieved by simply reinterpreting in some appropriate way the model as parameterized non–linear equations. For example, one could examine the actual determination of the parameters in the weight gain equation (4). If the y_j, $j = 1, 2, \ldots, n$, denote the observations of the weight gain (as a function of the green weeks j), which are used to determine the parameters in (4), then the least squares determination of these parameters reduces to solving

$$\min_{a,e,f} F(a,e,f;\{y_j\})$$

where

$$F(a,e,f;\{y_j\}) = \sum_{k=1}^{n}(w(a,e,f;j) - y_j)^2$$

with $w(a,e,f;j)$ denoting the right hand side of (4) with $g_t = j$.

Further details are outside the scope of this report. The aim was to indicate ways in which the sensitivity analysis could be performed. A general theory for and practical applications of that theory to parameterized non–linear equations can be found in Rheinboldt (1986).

5. Optimization of revenue and productivity

A model, once constructed for the implementation of a DSS, can then be utilized to examine related matters connected with the underlying process. For the DSSBS model, one can examine, among other things, the interrelationship between revenue and productivity. It is not simply a matter of maximizing revenue independent of the consequences. For example, one could aim to maximize revenue by maximizing productivity in terms of cattle numbers; but this is not necessarily the *best* way to proceed, as it can involve environmental implications such as overgrazing (as we shall show below).

In fact, one obtains different conclusions depending on how one defines the revenue $R(t)$ and the productivity $P(t)$ as a function of the time t. We illustrate by examining the following cases:

1. **Revenue and productivity defined as linear functions of the total weight of the non–breeders sold.** This corresponds to the situation examined by Monypenny and colleagues under the assumption that the price per kilo as a function of time remains fixed, and is modelled by

$$\begin{aligned} R(t) &= k_1 H(t)W(t) + k_2, \quad k_1 \neq 0 \\ P(t) &= k_3 H(t)W(t) + k_4, \quad k_3 \neq 0 \end{aligned}$$

   where k_1, k_2, k_3 and k_4 denote constants. Clearly, the revenue and productivity maximize simultaneously at a time when

$$\dot{H}(t)W(t) + H(t)\dot{W}(t) = 0$$

   where "$\cdot$" denotes differentation with respect to t.

2. **Revenue a linear function of the total weight of the non–breeders sold and productivity simply the total number $H(t)$ of head sold.** This corresponds to the situation where cattle station owners try to maximize the number of head sold in the belief that it maximizes revenue, and is modelled by

$$\begin{aligned} R(t) &= k_1 H(t)W(t) + k_2, \quad k_1 \neq 0 \\ P(t) &= H(t) \end{aligned}$$

   where k_1 and k_2 denote constants. The revenue is maximized when

$$\dot{H}(t)W(t) + H(t)\dot{W}(t) = 0$$

   while the productivity is maximized when

$$\dot{H}(t) = 0$$

Hence, unless $W(t)$ is constant, simultaneous maximization is quite unlikely to occur. In addition, the fact that $W(t)$ must be constant, in order, to guarantee simultaneous maximization, is indicative of the suboptimality of the above strategy as it implies that the number of head $H(t)$ must be so large that the individual animals have no weight gain. This is clearly indicative of overgrazing. It therefore implies that production should be defined in terms of the total weight of non–breeders sold and not the number of non–breeders sold.

3. **The price per kilo, $\$(t)$, depends on the weight $W(t)$, and productivity is defined as the total weight of the non–breeders sold.** This corresponds to a marketing strategy where price is a funtion of quality (weight), and a non–overgrazing model for production has been used. It can be modelled as

$$\begin{aligned} R(t) &= H(t)W(t)\$(t) \\ P(t) &= H(t)W(t) \end{aligned}$$

Simultaneous maximization will not in general occur, because the derivatives of $R(t)$ and $P(t)$ will tend to differ. In addition, the interrelationship between $R(t)$ and $P(t)$ will depend heavily on the assumptions made about $H(t)$, $W(t)$ and $\$(t)$.

In particular, because of the importance of reducing overgrazing, it is important to identify economically sensible scenarios where revenue maximizes at lower stocking rates than production. For example,

   i. if price $\$(t)$ increases with $W(t)$, then one wants fewer head for sale in prime condition (rather than a larger number of head in poorer condition)
   ii. in order to discourage overgrazing, rewards should be given to cattle stations which produce prime stock

Clearly, a complete analysis of case 3 involves considerations outside the scope of this report since, as explained above, the maximization of $R(t)$ and $P(t)$ depend heavily on the assumptions made about $H(t)$, $W(t)$ and $\$(t)$.

6. Conclusions

Acknowledging that other aspects would have to be examined in a more comprehensive investigation of DSSBS, the Study Group concentrated attention on the particular questions listed in the Introduction. As outlined in the report above, the Study Group made considerable progress with all the matters raised in those questions. In particular:

- Clear confirmation for the model developed by Monypenny and his colleagues has been obtained. In fact, their work represents a good example of how
  - the modelling for a DSS should be done if one wants more sophistication and relevance than is achievable with a black–box model
  - the use of an appropriate link concept forces the modelling to be more relevant, and hence, to make the modelling more "informative"

In this report, no attention has been paid to the implementation of the model discussed here. Such an implementation has been developed by Monypenny and his colleagues using a spread sheet system on a PC (Monypenny(1990)). For its construction, it requires that the in–year activities such as birth, death, *etc.* be decoupled from the between–year activities such as policy for the number of breeders, culling *etc.* Thus, implementation involves questions and expertise outside the scope of the deliberations of this report. In addition, risk based on perturbations (uncertainty) associated with the number of green weeks is not considered, but the discussion above represents a first step. In fact, the above discussion of the sensitivity analysis of the outputs represents a possible starting point.

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