PIPING IN NEWSPRINT ROLLS

1. Introduction

This problem was presented to the 1988 Mathematics-in-Industry Study Group (MISG) by Australian Newsprint Mills Ltd. The problem is concerned with the formation of 'piping' or 'moisture welts' on rolls of newsprint; The goal was to develop a mathematical model which would explain why and how these defects occurred, and what might be done to alleviate them.

Moisture welts are circumferential ripples on the surface of newsprint rolls. The ripples may occur individually or together, in which case a typical minimum separation distance is 0.1m. The height of the ripples ranges from 0.5mm to 2mm. They leave permanent creases in the paper, and in severe cases, might cause problems when the roll of paper is printed. The widely-accepted view is that piping is due to moisture, hence the name *moisture welts*. The question which arises, however, is: are the welts caused by uniform hygroexpansion of the outer sheets of a newsprint roll, or are they caused by non-uniform expansion of moisture in selected bands across the width of the roll? The latter view has some support from an industry technical information sheet (TAPPI TIS 016–31). No evidence was presented by the ANM representative at the MISG, however, to suggest that preferential moisture absorption in selected bands was the main cause of moisture welts; indeed, the industry practice is to produce newsprint which is as uniform as possible across the width of the sheet. Our view, therefore, was that the mathematicians at the MISG should attempt to understand piping with the aid of mathematical models based on the simplest mechanism – sorption of moisture by the outer sheets of a newsprint roll leading to uniform in-plane hygroexpansion and the subsequent formation of buckles to relieve the stress so induced.

The results of some industry practices and several crude experiments deserve to be recorded here. Moisture welts are a manageable phenomenon because manufacturers enclose the newsprint rolls in waterproof coverings immediately after production. The printers then ensure that the rolls are printed as soon as possible after the waterproof coverings are removed. It has been observed that the welts can first develop about 10 minutes after the coverings are removed, and that they gradually harden over a period of some hours. If the outer 0.02m of such a flawed roll is removed, the underlying paper appears to be flat at first, but then develops welts which first appear after about 10 minutes and again harden over a period of hours. Interestingly, the new welts often form with the same pattern and spacing as on the original roll. Those observations can all be explained using the ideas presented in the subsequent sections.

The work is based around two main factors – diffusion of water vapour through the outer sheets of a newsprint roll, and the subsequent buckling of the sheets to relieve the stresses caused by in-plane hygroexpansion of the paper. A very simple calculation to predict slippage of sheets across each other is also presented.

The main conclusions of our work can be simply highlighted: water vapour diffusing through the outer layers of a newsprint roll causes hygroexpansion and buckling of the outer sheet. Subsequently, the interior sheets will buckle in the same place because it will be the easiest place to relieve the stresses. The moisture welts grow by slippage of the layers of paper in an axial direction (towards the first welt) so as to relieve stresses unrelieved by the first buckling event. The model does not assume inhomogeities across the width of the newsprint roll.

The major recommendation for newsprint manufacturers is to monitor the moisture content of newsprint rolls which have developed piping: dry rolls should be more susceptible to this form of damage.

2. Radial diffusion of water vapour

The transport of water through porous media has been very extensively studied, particularly with application to water transport through soils (see *e.g.* Philip, 1969). In the case of water transport through newsprint, there is an extensive literature (see *e.g.* Hoyland & Field (1976–77) for a comprehensive treatment and Eklund & Salminen (1987) for recent developments). There are several key points to be made about the present application before embarking on mathematical model-building:

- The newsprint rolls are typically of 1m diameter and about 0.5 to 2m in length, whereas the available evidence suggests that moisture welts only penetrate to a depth of several centimetres. At these scales, the curvature of the rolls and non-axisymmetric effects can be neglected, and the only relevant co-ordinate is x (measured radially inwards from the surface).
- Axial transport of water is neglected. This assumption requires that axial diffusivities not be too much larger than radial diffusivities.
- The radial diffusion of water vapour is caused by changes in the relative humidity of the ambient air. For welts to form, the paper will be drier than the equilibrium moisture content corresponding to the ambient relative humidity. Moreover, the changes in water content are quite slight: for example Salmén, Fellers & Htun (1985) give the experimental moisture sorption isotherm at

23°C which is reproduced in Figure 1. This shows that for relatively large changes in the relative humidity, there are relatively small changes in the moisture content and that, moreover, the moisture sorption isotherm is effectively linear for relative humidities in the range 25% to 75%.

- The effect of hygroexpansion was also examined by Salmén, Fellers & Htun (see later in this report). We shall assume that the water transport is not affected by the hygroexpansion of the newsprint in the relative humidity range 25% to 75%.

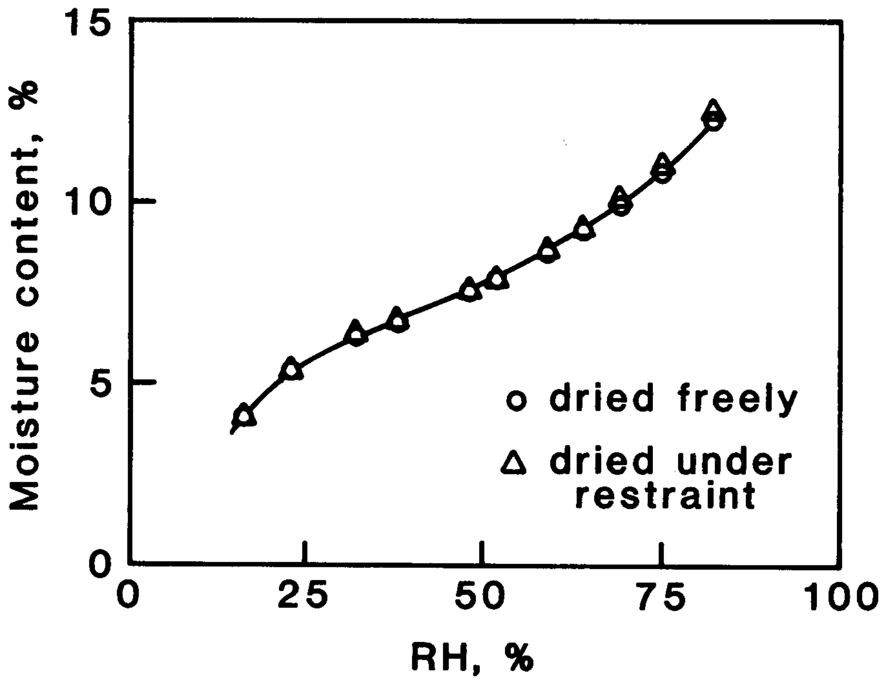

Figure 1: The moisture sorption isotherm at 23°C for sheets dried either freely or under restraint (from Salmén *et al.*, 1985).

Our simple transport model is now developed. Suppose that the newsprint roll is in a state of thermodynamic equilibrium with a gravimetric water content θ_{m} = *mass of water/total mass* corresponding to an ambient relative humidity of h_{vap}. The moisture sorption isotherm shown in Figure 1 relates the equilibrium values of θ_{m} and h_{vap}. We wish to consider the effects of a sudden increase in the ambient relative humidity. Locally, there will be gaseous diffusion of the excess water vapour molecules through the interstices between fibres and, instantaneously, condensation of water at the surface of the fibres in the paper. That is, at the low water contents that are found in newsprint rolls, it is assumed that transport occurs predominantly in the vapour phase (see Philip & de Vries (1957) or Philip (1957) for a more detailed treatment of this point). The moisture sorption isotherm gives the local moisture fraction as an almost linear function of the local relative humidity in the range 25% to 75%.

Locally, therefore, the rate of change of total water (liquid and gas) is balanced by divergence of the flux of water vapour. If Fickian diffusion is assumed to give the local flux, we have

$$\frac{\partial}{\partial t}(h_{\mathrm{liq}} + h_{\mathrm{vap}}) = D\frac{\partial^2 h_{\mathrm{vap}}}{\partial x^2} \tag{1}$$

where h_{liq} is the amount of water present as liquid (and expressed in equivalent relative humidity units), h_{vap} is the relative humidity of the water vapour, and D is the molecular diffusion coefficient for water vapour in air suitably modified by a factor to account for porosity and tortuosity.

It is necessary to relate the local gravimetric water content θ_{m} to h_{liq} measured in units of equivalent relative humidity. Now the density of newsprint is about 0.6gm cm^{-3}, and so the mass of water per unit total volume is about $0.6\theta_{\mathrm{m}}$ gm cm^{-3}. Therefore, the number N of water molecules per cm^3 (present as a liquid) is about $0.6\theta_{\mathrm{m}}/m_w$ where $m_w = 18/(6 \times 10^{23})$ is the mass in gms of one water molecule. Hence N is approximately $2\theta_{\mathrm{m}} \times 10^{22}$ molecules/cm^3.

This number of water molecules/cm^3 is now converted to an equivalent relative humidity. The gas pressure due to N molecules is

$$p = NkT$$

where k is Boltzmann's constant ($1.38 \times 10^{-16}\mathrm{ergs}K^{-1}$) and T = 300 K say. Thus

$$\begin{aligned} p &= 2\theta_{\mathrm{m}} \times 10^{22} \times 1.38 \times 10^{-16} \times 300 \\ &= 8.3\theta_{\mathrm{m}} \times 10^{8} \text{ dynes/cm}^2. \end{aligned}$$

The equivalent relative humidity of the water present as a liquid is therefore $8.3\theta_{\mathrm{m}} \times 10^8/p_{\mathrm{sat}}$ where p_{sat} is the saturation vapour pressure of $H_20(= 3.16\times10^4\mathrm{dynes/cm}^2$ at 300 K). Hence we have (in c.g.s. units)

$$h_{\mathrm{liq}} = 8.3\theta_{\mathrm{m}} \times 10^8/3.16 \times 10^4$$

or

$$h_{\mathrm{liq}} = 2.6\theta_{\mathrm{m}} \times 10^4 \tag{2}$$

We now return to equation (1) and write the term $\frac{\partial}{\partial t}(h_{\mathrm{liq}})$ as

$$\begin{aligned}\frac{\partial}{\partial t}(h_{\mathrm{liq}}) &= \frac{dh_{\mathrm{liq}}}{dh_{\mathrm{vap}}}\frac{\partial h_{\mathrm{vap}}}{\partial t} \\ &= \frac{dh_{\mathrm{liq}}}{d\theta_{\mathrm{m}}}\frac{d\theta_{\mathrm{m}}}{dh_{\mathrm{vap}}}\frac{\partial h_{\mathrm{vap}}}{\partial t}\end{aligned}$$

In this expression, equation (2) gives

$$\frac{dh_{\mathrm{liq}}}{d\theta_{\mathrm{m}}} = 2.6 \times 10^4$$

and a linear approximation to the moisture sorption isotherm in the central range (25% to 75%) gives

$$\frac{d\theta_{\mathrm{m}}}{dh_{\mathrm{vap}}} = 0.05/0.50$$

Therefore we see that

$$\frac{\partial}{\partial t}(h_{\mathrm{liq}}) = 2.6 \times 10^3 \frac{\partial h_{\mathrm{vap}}}{\partial t} \tag{3}$$

so that the term $\frac{\partial}{\partial t}(h_{\mathrm{liq}})$ in equation (1) completely dominates the term $\frac{\partial}{\partial t}(h_{\mathrm{vap}})$. Equation (1) now becomes (in c.g.s. units)

$$(2.6 \times 10^3 + 1)\frac{\partial h_{\mathrm{vap}}}{\partial t} = D\frac{\partial^2 h_{\mathrm{vap}}}{\partial x^2}$$

or

$$\frac{\partial h_{\mathrm{vap}}}{\partial t} = D^* \frac{\partial^2 h_{\mathrm{vap}}}{\partial x^2} \tag{4}$$

where $D^* \simeq D/(2.6 \times 10^3)$ and D is the molecular diffusion coefficient for water vapour in air suitably modified by a factor to account for porosity and tortuosity. Numerical values (in c.g.s. units) are given below for D^*.

Thus, we can now pose and solve a problem for transport of water vapour into a newsprint roll. If the roll is initially in thermodynamic equilibrium and the initial relative humidity h_0 is suddenly changed to a value h_1, the local relative humidity at a depth x into the roll is given by solving equation (4) subject to the conditions

$$h_{\mathrm{vap}}(x,0) = h_0, \quad h_{\mathrm{vap}}(0,t) = h_1.$$

The solution is

$$h_{\mathrm{vap}} = h_0 + [h_1 - h_0]\mathrm{erfc}\{x/2\sqrt{D^* t}\} \tag{5}$$

where

$$\mathrm{erfc}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du.$$

Now D^* takes the approximate numerical value $0.25/2.6\times10^{-3} \simeq 10^{-4}$ cm^2/sec if tortuosity and porosity are neglected. If porosity and tortuosity reduce the diffusion coefficient by an additional factor of 10, then $D^* = 10^{-5}$ cm^2/sec approximately. In general, it takes a time of order $x^2/4D^*$ sec for excess humidity effects to be transported through a distance x cm. Thus, using $D^* \simeq 10^{-5}$ cm^2/sec, we estimate that it requires about 7 hours for water vapour to be transported through 1 cm of newsprint, and about 1.5 sec for transport through 1 sheet of newsprint (about 80 μ).

We conclude this section with several observations. Firstly, the above estimates for D^* and the time taken for water transport through 1 cm fall within reasonable ranges. Also, it deserves to be emphasized that the transport theory has eventuated in a linear model. This has occurred for two reasons. Firstly, the moisture sorption isotherm (Figure 1) has been linearly approximated in the region of interest. Secondly, it has been assumed that, at low water contents, all significant transport takes place in the vapour phase and consequently the variable used in the model is the local equivalent relative humidity and not the total water content.

3. In-plane buckling

The presence of an excess moisture fraction in a newsprint roll causes hygroexpansion, both in-plane and out-of-plane. Figure 2 is reproduced from Salmén, Fellers & Htun (1985) and shows the in-plane hygroexpansion $\Delta\ell/\ell$ as a function of moisture content θ_{m} for sheets dried either freely or under restraint. We believe that axial in-plane hygroexpansion causes the formation of moisture welts, whilst out-of-plane hygroexpansion and circumferential in-plane hygroexpansion merely cause an increase in the radius of the newsprint roll.

The results of Salmén, Fellers & Htun permit an estimation of the axial strain $\Delta\ell/\ell$. To fix ideas, suppose that a newsprint roll is in thermodynamic equilibrium with a moisture content corresponding to 50% relative humidity. If the ambient relative humidity is increased to 75%, the equilibrium moisture fraction will increase from 8% to 11% (Figure 1) and there will be a corresponding in-plane strain $\epsilon = \Delta\ell/\ell$ of 0.0015 (Figure 2, line B).

This strain ϵ causes a stress which will be relieved by the buckling of the outermost sheet of the newsprint roll. Once the outermost sheet has buckled, each interior sheet will tend to buckle immediately below the buckle on the adjoining

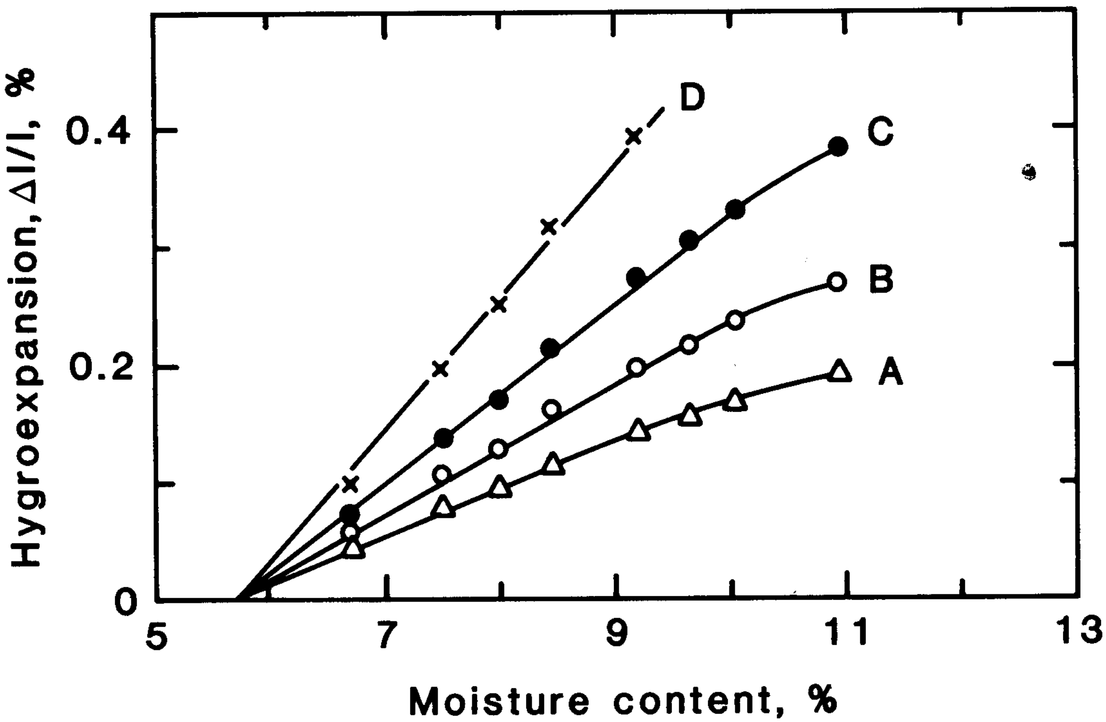

Figure 2: In-plane hygroexpansion (relative length increase $\Delta\ell/\ell$) *versus* moisture content for isotropic sheets dried either freely or under restraint down to different RH levels: A 25% RH, B 50% RH, C 90% RH, D dried freely (from Salmén *et al.*, 1985).

sheet. An important consideration is to determine the length of a single sheet of newsprint which can resist the compressive stresses caused by the in-plane hygroexpansion, and this question is resolved by using well- known buckling criteria (see *e.g.* Segel, 1977, p.214).

A rod of length L which is compressed longitudinally by a load P will buckle when P reaches the critical value

$$P_c = \pi^2 EI/L^2$$

where E is Young's modulus for the material and $I = d^3/12$ is the second moment of area of the cross-section of the rod of thickness d. This result assumes that the ends of the rod are held at zero displacement without curvature. For linear elasticity theory, $P = \epsilon Ed$, so that the critical length of a single sheet of newsprint over which buckles will form is

$$L_c = \pi d/2\sqrt{3\epsilon}. \tag{6}$$

To obtain an estimate of L_c, we use the numerical values $d = 8 \times 10^{-3}$ cm and $\epsilon = 0.0015$. This gives

$$L_c = 0.2\text{cm approximately}. \tag{7}$$

An interpretation of the consequences of this estimate (7) is given in the next section. Note that once buckling begins, the compressive stress in the buckled region will decrease whilst the end-points of the region remain fixed. Thus, in fact, the buckling length must be larger than the critical length L_c.

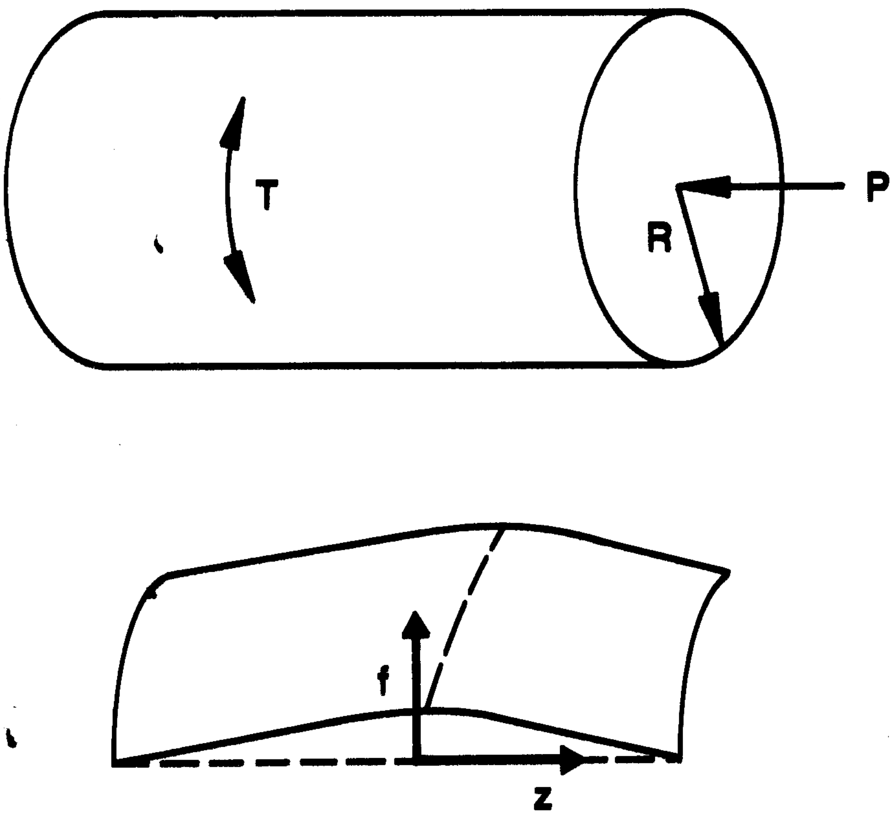

Figure 3: Illustrating the second buckling calculation: P is the axial load per unit of circumference, T is the tangential load per unit axial length.

The value for L_c is rather smaller than we expected, so a modified estimate was developed in which the buckles have to form against the restoring effect of radial stress. Consider the situation sketched in Figure 3. The equation for the displacement $f(z)$ outwards as a function of the axial co-ordinate z is

$$EI_2 f^{iv} + Pf'' = -\frac{T}{R} \tag{8}$$

where $I_2 = d^3/12$ is the second moment of area per unit circumferential length, P is the axial load per unit circumferential length and $-T/R$ is the effective radial stress due to the hoop tension T per unit axial length. Consider a length L of newsprint with zero at its centre, and enforce the fixed end conditions $f = f' = 0$ at $z = \pm L/2$. The first buckled solution of equation (8) is

$$f(z) = A\cos(\beta z) + B - \frac{Tz^2}{2RP}$$

where

$$\begin{aligned} A &= -\frac{TL}{2RP}/\beta\sin(\beta L/2), \\ B &= \frac{TL}{2RP}(\frac{L}{4} + \frac{\cot(\beta L/2)}{\beta}), \\ \beta &= \sqrt{P/EI_2}. \end{aligned}$$

Physical conditions require $f(z) \geq 0$ for $|z| \leq L/2$. Graphical reasoning shows that in the first buckled mode this will be satisfied provided $f''(L/2) \geq 0$ so that L must lie in the range

$$\frac{4\pi^2 EI_2}{P} < L^2 < \frac{4\xi^2 EI_2}{P} \tag{9}$$

where $\xi \simeq 4.493$ is the smallest positive number such that $\tan\xi = \xi$. The left hand member of the result (9) gives an estimate for the critical length L_c which is twice that of equation (6) (in which the effects of radial stress have been neglected). Note, however, that the end conditions are not the same for both calculations. The new result still gives a surprisingly small value for L_c. The same observation as before applies about the decrease in compressive stress and the end-points remaining fixed.

4. Sliding

The buckling calculations of the last section predicted that welts with a spacing of the order of 1 cm or less would form on the outermost sheet of a newsprint roll. This estimate for the welt spacing is rather small, and it is necessary to explain why the moisture welts generally tend to form rather further apart than 1 cm and with a substantial amplitude. The additional physical mechanism which we now invoke is sliding: suppose that one buckle has formed on the outermost sheet by the mechanisms described in the last section, and so we have the situation sketched in Figure 4. It is clear that the moisture welt provides no resistance to residual

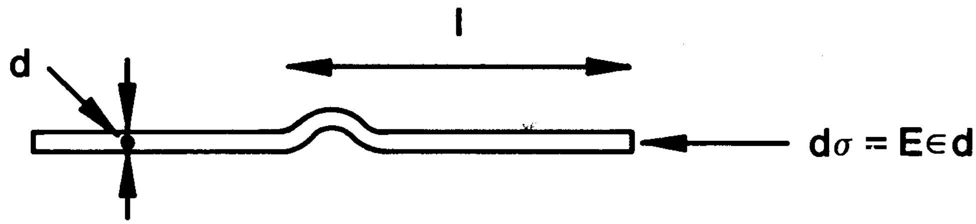

Figure 4: Illustrating the sliding calculation.

axial stresses, and a length of the outermost sheet will slide towards the buckle. An indication of the length which will slide is now presented.

Suppose that a length ℓ will slide under the effect of the axial force $\sigma d = \epsilon E d$ per unit circumferential length on a single sheet of newsprint. The frictional force per unit circumferential length which acts to oppose sliding is $\mu T\ell/R$ where μ is the limiting static friction coefficient and T/R is the radial force per unit area. Hence we have $\epsilon E d$ is approximately $\mu T\ell/R$, or

$$\ell = \epsilon E d R/\mu T \tag{10}$$

To get an idea of the length ℓ, we use the numerical values $d = 8 \times 10^{-3}$ cm, $E = 2 \times 10^5$ N/cm^2, T=600 N/m, ϵ=0.0015 and R=50 cm. We find that

$$\ell = 20/\mu \ \text{cm}, \tag{11}$$

or $\ell = 40$cm if μ=0.5 (a value chosen without the benefit of experimental data). Slippage of the outermost sheet towards a moisture welt would probably stop before the full length of $20/\mu$ cm because sliding friction would bring the sliding sheet to rest before all the compressive axial stress was relieved. In this sense, the estimate (11) can be seen as a modest over-estimate for the spacing between moisture welts.

In work performed after the MISG, Dr I.D. Howells has combined the calculation for buckling (when there is hoop tension) and for slippage with the requirement of fixed end-points under buckling. The calculation considers the prescribed integrated compressive strain over the combined length that undergoes buckling and slippage. The calculation results in an equation which sets a minimum value for ϵ and involves the buckling length L_c and the total slippage length 2ℓ. If ϵ increases from zero as the moisture content increases, buckling will start when the minimum value of ϵ is reached. It seems likely, moreover, that the size and spacing of the buckles will be set during the initial buckling. For the coefficient of friction $\mu = 0.75$, Dr Howells finds that the critical value of ϵ is $\epsilon = 0.0008$, and that $L_c = 0.9$ cm and the spacing $L_c + 2\ell = 11$ cm.

5. Conclusions and recommendations

The three elements - water transport, buckling and slippage - presented above give a rational explanation for moisture welts and account for the observational evidence on the depth and spacing of moisture welts. The situation in which a defective roll (*i.e.* one with moisture welts) forms welts with exactly the same axial distribution after the outer 1 or 2 cm is removed can also be explained as follows. Infinitesimal buckles are formed at substantial depths into the newsprint roll. Once these layers of the roll are exposed and absorb moisture and expand, sliding of the sheets towards the infinitesimal buckles leads to the same pattern as previously.

Our theory implies that dry newsprint rolls would be susceptible to the formation of moisture welts, and we suggest that a quality control program could be initiated to record the moisture content of newsprint rolls which form moisture welts. If the damage-prone rolls are generally found to have a low moisture content, then Australian Newsprint Mills should consider the likely benefits of increasing the moisture contents of the rolls.

Another minor suggestion follows from estimate (10) for ℓ. If the hoop tension of the outermost layers of newsprint could be reduced, then the welts would be likely to be more widely spaced (although with a greater amplitude). We point out this possibility, although we are unable to comment on its feasibility or desirability.

In conclusion, the moderator for this problem (Noel Barton) would like to thank the representatives from Australian Newsprint Mills Ltd (Dr J. Bonham and Mr A. Rosson) and to acknowledge the substantial contributions to this report by Prof. L. White and Drs. P. Broadbridge, I. Howells, K. Landman and D. Yuen.

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