JITTER IN ELECTRO-MECHANICAL DEVICES

1. Introduction

In various ways, the end product of technology is the construction of devices to perform tasks of varying degrees of sophistication. Such devices range from the domestic (such as household appliances including cooking aids, washing machines, radios and televisions) to the industrial (such as mechanical machinery including cranes and tractors; fabrication aids including lathes and drills; and fabrication machinery such as spinning and weaving devices for cloth making), and on to the advanced technological (such as the bionic ear, radar, robots and medical scanners). The majority of such devices contain moving parts often powered by electric motors. For obvious engineering reasons, such products are classified as *electro-mechanical devices.*

Though electromechanical devices can be *designed* to perform specific tasks exactly, the actual products will only perform the tasks with error, because manufacture can only reproduce designs approximately. As this error is usually (and can even be designed to be) of lower amplitude and of higher frequency than the task being accomplished, it is often referred to as *jitter.*

For many devices, such as household appliances like mixers, cleaners and washing machines, simple engineering design criteria can ensure that the jitter is virtually non-existent to the user and casual observer, except for some background noise and minor vibration at high operational speeds. Here, jitter itself is not a potential source for degrading the operational performance of the product. For more advanced technological devices such as robots and medical scanners, the situation changes. Interaction between the individual (local) jitters, produced by the various components of a complex system, such as a robot arm, can produce an accumulated (global) jitter which degrades performance and is difficult to model and predict. Now, it is not simply a matter of designing (with respect to given engineering criteria) a device to perform exactly some specific task; but of modelling the tolerances in the manufacturing process so that the jitter in the actual product can be predicted and thereby controlled.

In an industrial mathematics context, we have a situation where mathematical expertise is sought because the standard (engineering) approaches have failed. It is natural to first seek to control the degradation in the performance of a product within the framework in which it is designed. It is only necessary to move into a more sophisticated framework when it is clear that the original one is deficient in some fundamental way.

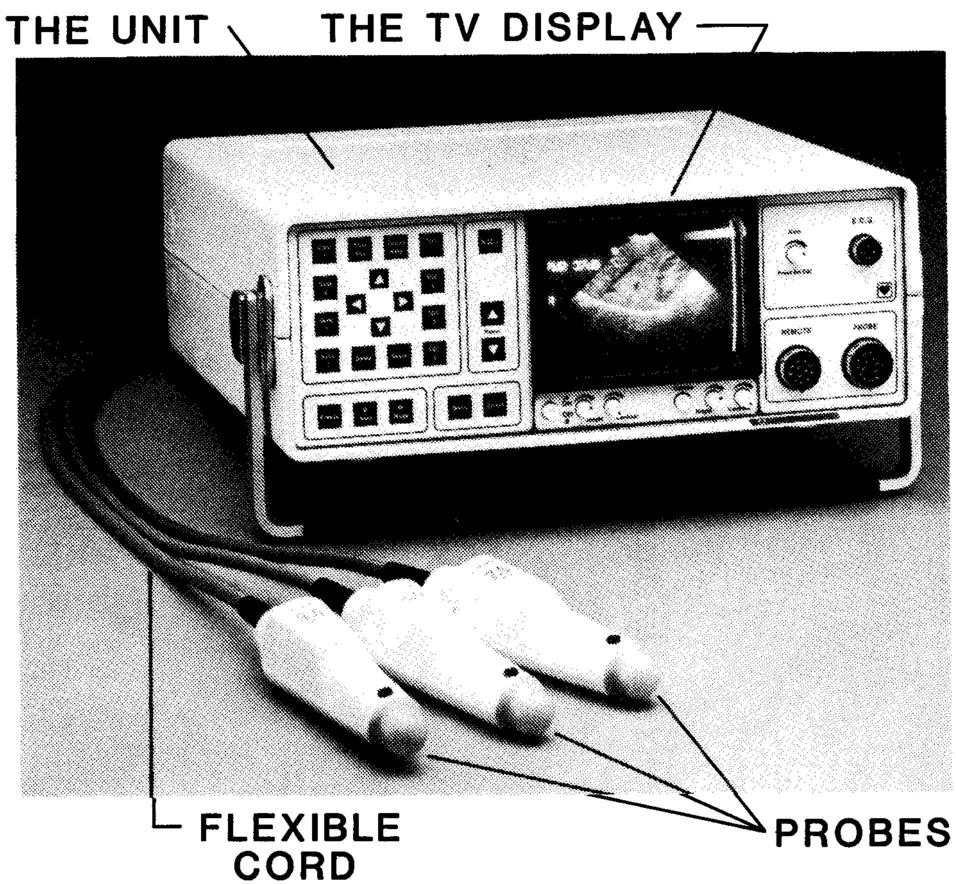

Figure 1: The Ausonics scanner with probes and unit.

This characterises the situation facing Ausonics. They manufacture ultrasonic body scanners (Figure 1). Each one consists of a light and robust hand-held *probe* (which can be placed on any part of the body) connected by a long flexible lead to a compact and semi-portable *unit* containing all the electronics (including power sources, controls and small TV-monitor) which do not need to be located in the probe. The probe itself is the electromechanical device. Because the underlying technology is ultrasonic radar imaging (echography), the scanner is able to operate in real-time to give local cross-sectional pictures (on the TV-monitor) of the body directly below and in the plane scanned by the probe (Figure 2). Their utility as a real-time diagnostic medical aid is therefore clear. In fact, Ausonics already have an established international market.

The problem for Ausonics is that, though they make many probes with virtually no jitter, they also produce in the process a large number of probes with unacceptable jitter. For the user, this jitter manifests itself as high frequency small

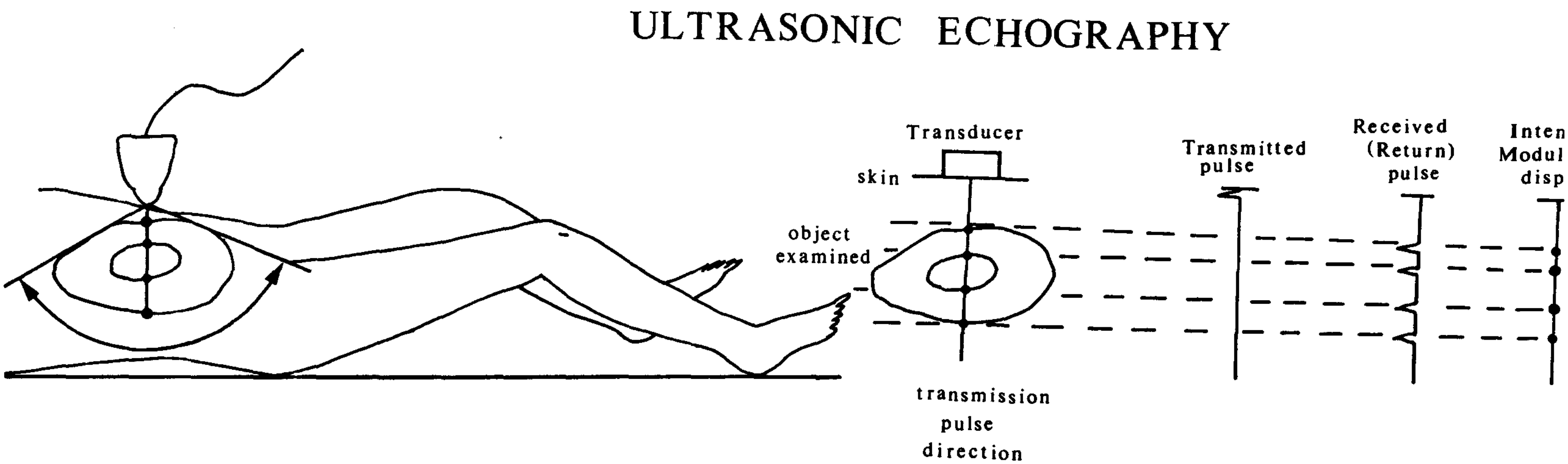

Figure 2: Diagrammatics of probe imaging.

amplitude movement of the image displayed on the TV-monitor which should not be there. The seriousness of the problem is reflected in the fact that the cost of manufacturing of a probe (because of its moving parts) is a major component in the price of a scanner, and, on occasions, up to one third of the probes are rejected because of unacceptable jitter.

For this problem, the *goal* of the Study Group became *the modelling and analysis of the electro-mechanical mechanism inside the probe.* Even though the electronic processing in the unit, of the signal transmitted from the probe, also produces jitter (in the image displayed on the TV-monitor), it was assessed to be of secondary importance at this stage, and was therefore not studied.

2. What is jitter?

The jitter in the Ausonics scanner manifests itself as a high frequency small amplitude *movement of* the image displayed on the TV-monitor, which should not be there. It is different from the low frequency *movement within* the image recording the dynamics of the object being probed, and must therefore be characterised accordingly. Thus, *jitter* is *error*, which is more complex than is characterisable using the usual statistical models. Compared with observational errors, which are modelled assuming the positions of the image pixels remain fixed in time, jitter must be characterised, modelled and analysed in terms of the changing positions of the image pixels as a function of time.

A corresponding situation arises when we watch a movie. Normally, we only see the movement which the image itself represents. Ignoring flickering in the light source, jitter occurs when the position of the image on the screen changes from frame to frame due to a malfunctioning in the projection equipment (which includes the film). This also highlights the fact that the *movement of* the image is not the origin of the jitter (in the scanner or movie), but its manifestation.

Just as for the movie, where one eliminates the jitter by correcting the malfunction in the projection equipment, the jitter in the scanner can only be eliminated by correctly identifying and analysing the source in the scanner of the movement in the positions of the image pixels. This is the basis for the *goal* articulated in the Introduction.

Independent justification for identifying and analysing the source in the scanner of the movement arises because jitter in the image cannot be removed by standard filtering techniques. In fact, a moment's reflection indicates that

- standard techniques for filtering images are based on the assumption that

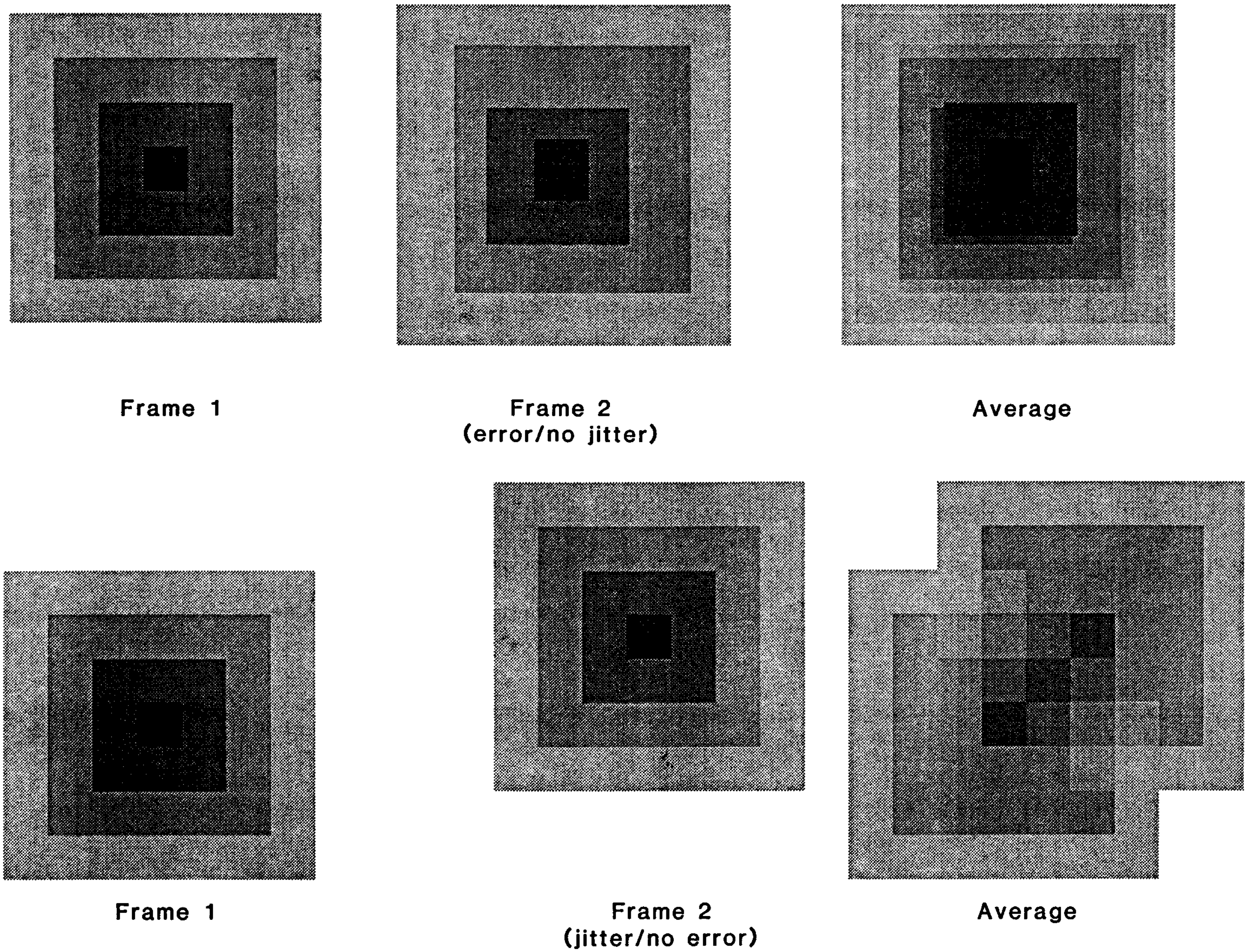

Figure 3: Effect of smoothing (averaging) on successive jittered and non-jittered frames

the only movement is low frequency dynamics within the image;

- filtering could only be sensibly applied if the nature of the movement could be explicitly characterised so that the positioning errors could be corrected;
- if applied, standard filtering (smoothing) procedures would only destroy resolution and confuse available information. This is illustrated in Figure 3.

Caveat. The movie analogy only goes so far, since the movement of the image pixels in the scanner will often be much more complex than the fairly systematic movement seen in movie jitter.#

3. Electro-mechanical operation of the probe

The first step is to identify the source of the jitter as manifest in the erratic movement of the image. Before this can be done, it is necessary to explain the essential character of the operation of the probe. As shown in Figure 4, it consists electromechanically of an electric motor which rotates a drive arm of radius R about a cylindrical drive shaft axis with the drive arm connected by a universal-joint to a transducer, which is tilted back and forth about an axis through its centre and perpendicular to the rotational axis of the drive arm. The universal-joint is designed so that the center of the transducer is a distance H above the plane of the drive arm and on the rotational axis of the drive arm.

The geometry of the drive arm and universal-joint is such that, as the drive arm rotates through 360^o, the normal to the face of the transducer (which is the direction in which the ultrasonic pulses are transmitted and received) scans from -45^o to 45^o (the forward scan) and back again (the backward scan). An optical encoder at the base of the motor controls the transmission and reception of the ultrasonic pulses. Thus, in many ways, the probe operates like a limited angle radar scanning from -45^o to 45^o and back again about the central axis of the probe (*i.e.* the rotational axis of the drive arm) and in the plane containing the central axis and the perpendicular to the axis of the transducer. It is for this reason that the probing process is often referred to as *ultrasonic echography.*

Thus, the mathematical modelling of the electromechanical mechanism of the probe reduces to determining the relationship between the position of the drive arm and the direction of the normal to the transducer's face. Though this can be done in various ways, the aim is to do it in such a manner which is ideal for the subsequent analysis of the problem.

Remark 1. Ausonics had in fact derived a number of such models for a variety of situations, but used a framework more cumbersome than that presented below.#

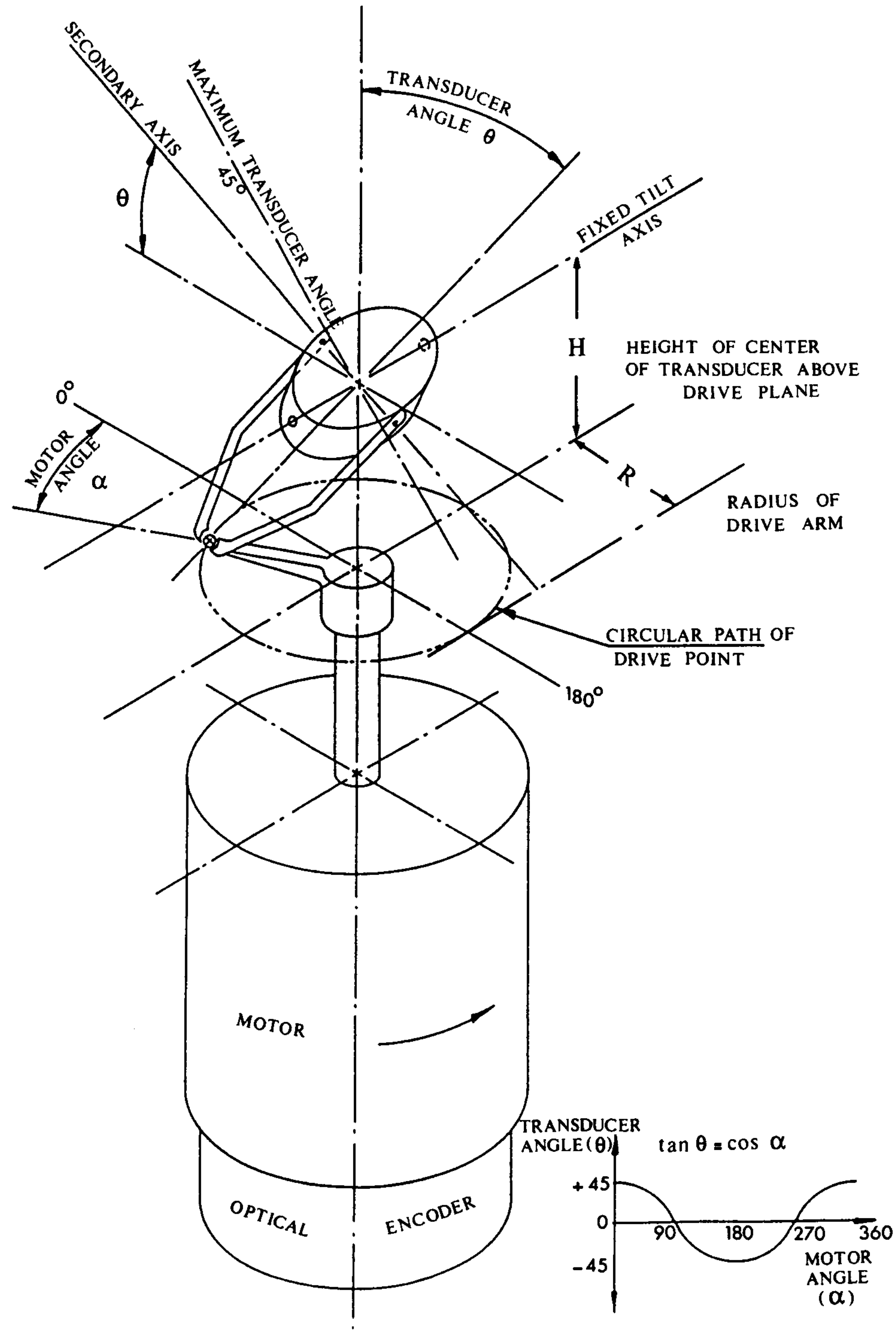

Figure 4: Diagrammatics of probe construction.

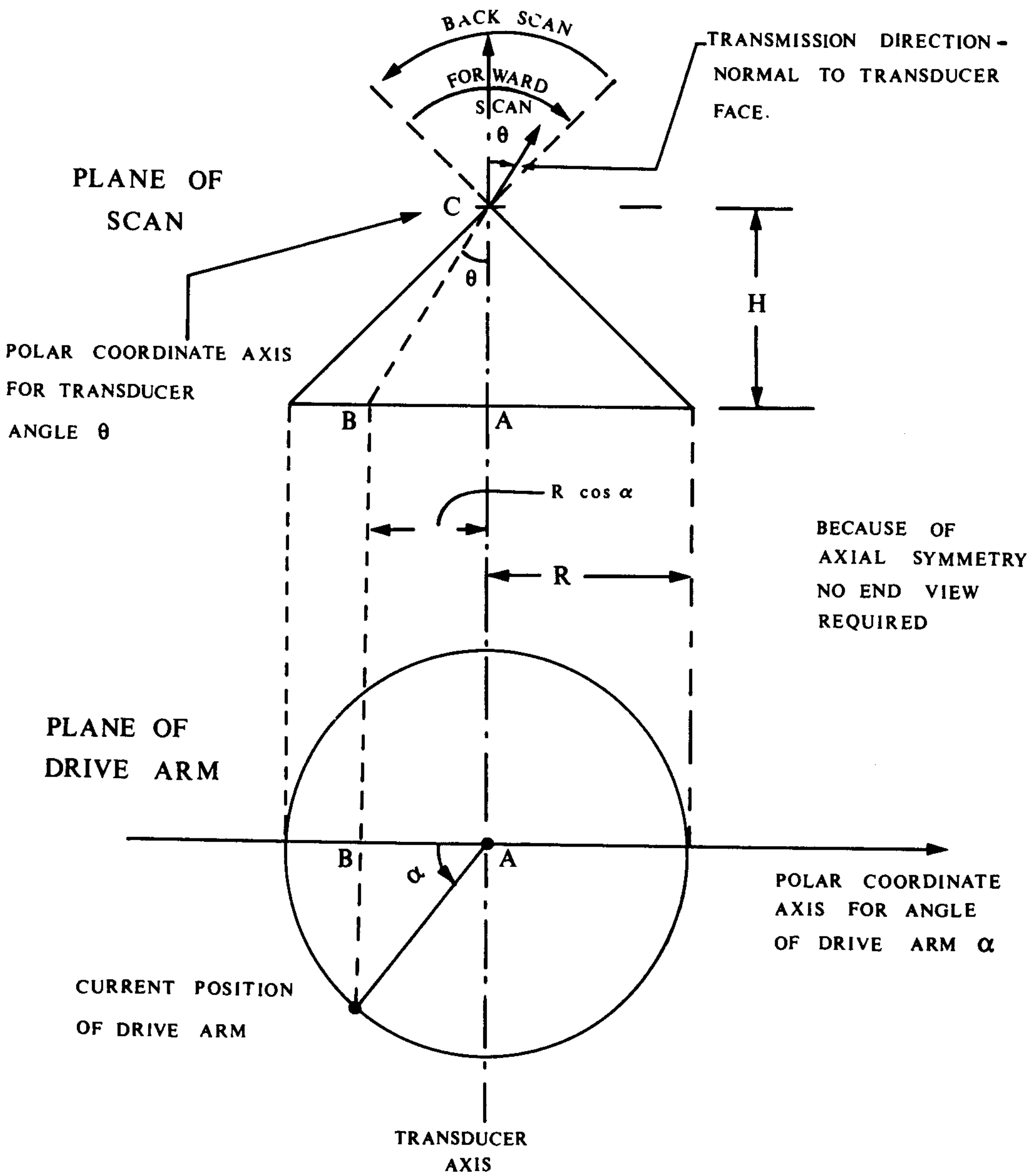

Figure 5: Basic derivation of *tan* θ-model.

In fact, the basic movement of the drive arm and universal-joint can be modelled schematically as in Figure 5. It is clear from this diagram how to derive the required relationship between the direction θ of the normal to the tranducer and the angular position α of the drive arm. Since, for the triangle ABC, we know that $AC = H$, we find that $AB = R\cos\alpha$, and we see that the angle $BCA = \theta$, it follows that we can always calculate *tan* θ in terms of the other parameters defining the position of the drive arm and the geometry of the universal-joint; namely,

$$\tan\theta = R\ \cos\alpha/H.$$

With $R = H$, we obtain the exact situation for Ausonics' ideal probe which scans back and forth between -45^o and 45^o; namely

$$tan\ \theta = cos\ \alpha. \tag{1}$$

Because this type of schematic representation and mathematical model is central to any comprehensive analysis of the movement of the drive arm and universal-joint, whether ideal or actual, it will be referred to as the *tan* θ*-model.*

At any instant of time, the image displayed on the TV-monitor is simply an echograph (radar image) produced by the object being scanned. The echograph is built up from the amplitudes and return times of the echoes received from each of the transmitted pulses; and therefore displays the strength as a function of depth of the ultrasonic reflectivity of the object. Because of technical engineering constraints related to the speed at which the transducer can be tilted back and forth and the need to have a clear image on the TV-monitor, successive frames of the image correspond to the echographs obtained from successive forward and backward scans of the transducer. This successive use of forward and backward scans is believed to be *the primary source of the electromechanical jitter* in a probe. If the electromechanical mechanism of the probe could be constructed perfectly, then the directions of the individual transmitted pulses on the forward and backward scans would match perfectly. However, because an actual probe is only an approximate realization of its design, this matching is in error with obvious consequences for the position of the echograph pixels in successive frames of the image.

It is this aspect of electromechanical jitter which was analysed in some detail by the Study Group. For obvious reasons, it will be referred to as *scan alignment jitter.*

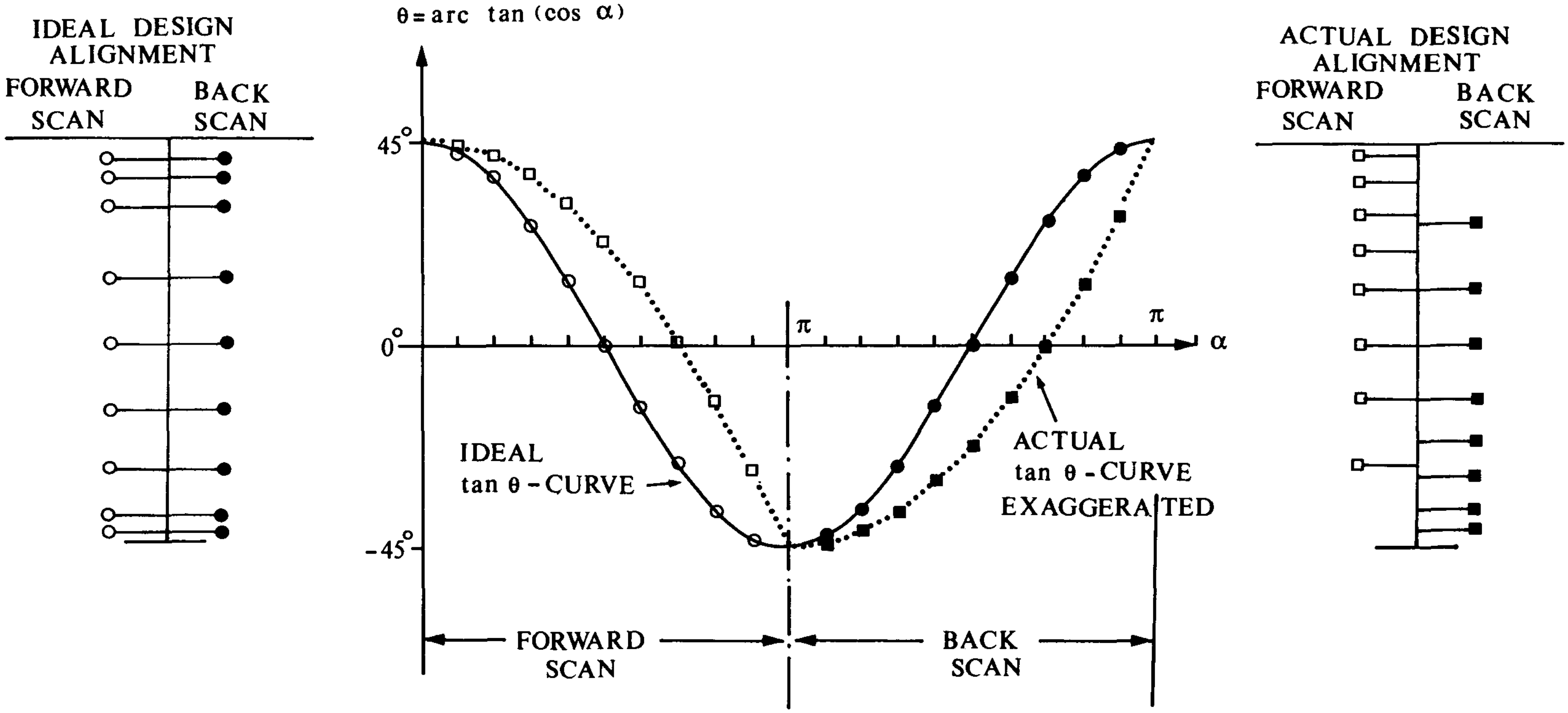

Figure 6: Diagrammatics of scan misalignment.

4. The *tan* θ-model in the analysis of scan alignment jitter

Mathematically, the study of scan alignment jitter reduces to quantifying the misalignment between forward and back scans in terms of the errors which arise in constructing an actual probe (when compared with its ideal design). The crucial role of the *tan* θ-model in quantifying such misalignment is illustrated in Figure 6. In order to interpret it, we first note that the time between successive transmissions of the ultrasonic pulses is held constant. This is achieved by coupling the optical encoder to the rotation of the drive shaft since it moves with constant angular velocity. Thus, for the interpretation of Figure 6, we assume that the transmissions (and hence the recording of the return pulses) occur at equal steps along the α-axis.

For the ideal design, we see immediately from Figure 6 that misalignment is zero because of the symmetry about $\alpha = 180^o$ of the ideal *tan* θ-curve (which is the graphical realization of the algebraic formula (1)). The severity of the misalignment which can occur is also illustrated for an actual *tan* θ-curve which has been distorted from the ideal (including symmetry about $\alpha = 180^o$) by the errors in the construction of the actual probe it represents.

The goal set by Ausonics was not to explicitly quantify the nature of the misalignment so that it can be partially corrected at some subsequent stage (*e.g.* electronically in the unit); but to identify the construction errors which contribute the most to the misalignment so that appropriate quality control can be introduced at the production stage. Thus, from the point of view of the present investigation, the examination of scan alignment jitter reduced to deriving and analysing *tan* θ-models for various configurations of actual probes (in how they differ from the ideal design).

The construction of such *tan* θ-models is always possible, because, as the top half of Figure 5 illustrates, a pulse transmission direction will always be relatable to a triangle ABC which characterises the current position of the drive arm (as a function of the drive angle α and the errors in the construction) to the effective height of the origin C of the transmission directions above the plane of rotation of the drive arm.

5. Derivation and analysis of *tan* θ-models for actual probes

In this Section, we identify and discuss some of the possible *design errors* (*i.e.* the ways in which an actual probe can differ from its ideal design) as well as list for each the relevant *tan* θ-model. They are easily derived by generalising the argument built around Figure 5 which was used above to derive the *tan* θ-model

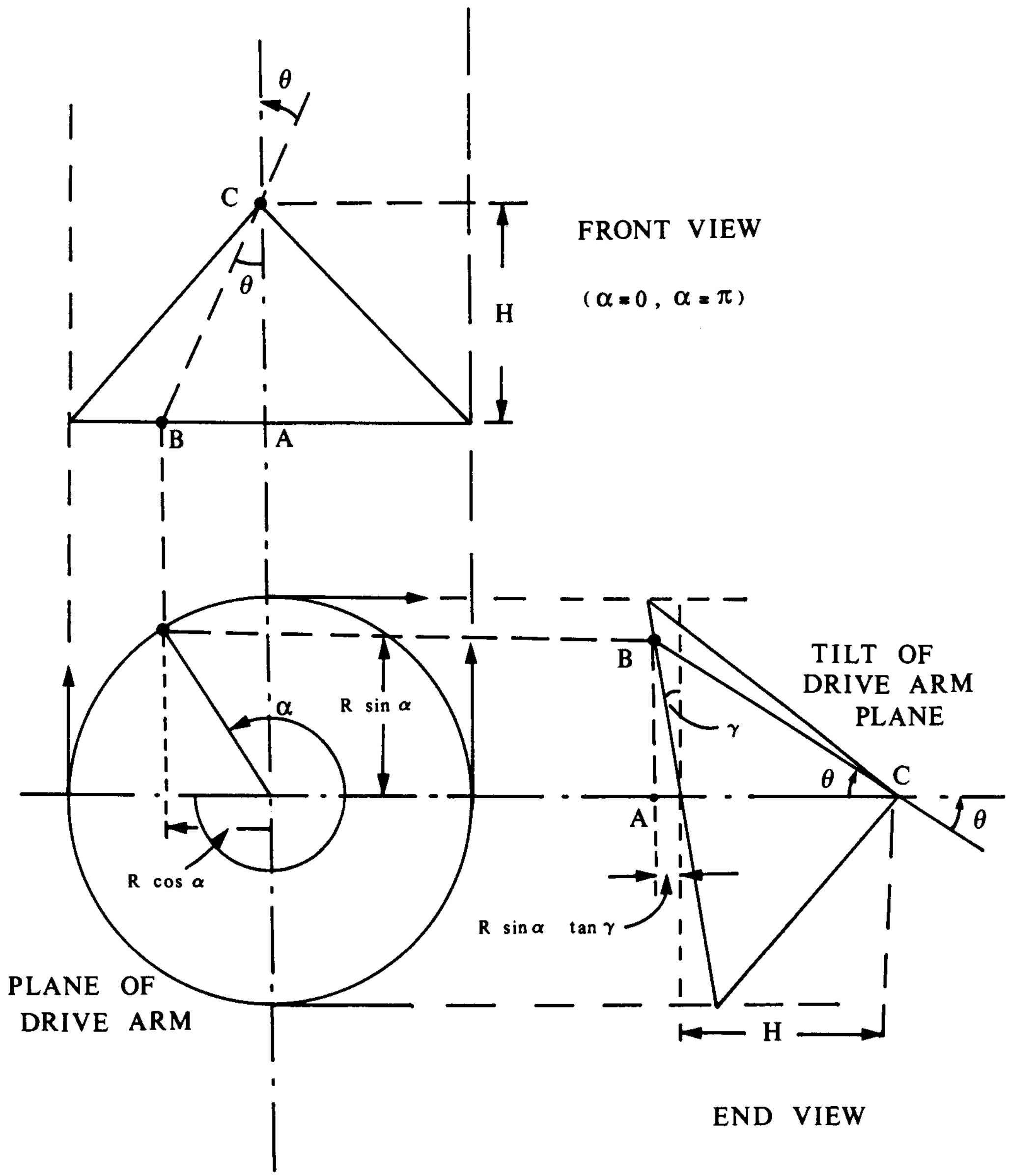

Figure 7: *tan* θ-model when transducer axis is misaligned.

for the ideal design. Finally, for what is surmised to be a highly relevant *tan θ*-model, we make a detailed error analysis and give our interpretation of the results as they apply to Ausonics' situation.

DESIGN ERRORS

When constructing an actual probe, some possible design errors which can occur are:

1. *Slap (slack, slop) in drive arm and transducer mountings*

   Because the probe is a dynamic (not a static) mechanism, and because the joining of components is not completely rigid, the tilting of the transducer may not be directly coupled to the movement of the drive arm. The appropriate *tan θ*-model is

   $$\tan(\bar{\theta} + \delta_{\bar{\theta}}) = \cos(\alpha + \delta_{\alpha}) \tag{2}$$

   where $\bar{\theta}$ denotes the transducer angle relevant to this situation, $\delta_{\bar{\theta}}$ the tilt-slap in the transducer mounting, and δ_{α} the rotational-slap in the drive arm mounting. This *tan θ*-model can also be used to account for the situation where the α-axis is not perpendicular to the axis of tilt of the transducer.

   Remark 2. We express all *tan θ*-models in terms of the same drive arm angle α, since it is the position of the drive arm which determines the tilt of the transducer. This easily allows for the incorporation of errors in the optical encoder, since these errors will only affect the points along the α-axis at which the ultrasonic pulses are transmitted.#

2. *Misalignment of transducer axis*

   Through errors in the construction of the universal-joint, the axis of the transducer may not be parallel to the drive arm plane. By considering the equivalent situation where the drive arm plane is tilted relative to the transducer axis, the corresponding *tan θ*-model is found to be (Figure 7)

   $$\tan\hat{\theta} = R\cos\alpha \;/\; (H + R\sin\alpha\tan\gamma) \tag{3}$$

   where $\hat{\theta}$ denotes the transducer angle relevant to this situation, and γ the angle the transducer axis makes with the drive arm plane. Without loss of generality and in order to simplify the form of the right hand side of (3), γ was chosen as shown in Figure 7.

   Remark 3. Though Ausonics had previously obtained a version of this formula, it was more complex than (3), even though it assumed that $R = H$. The simplicity of (3) results from the way we have defined γ, and the explicit implementation of the *tan θ*-modelling formalised above.#

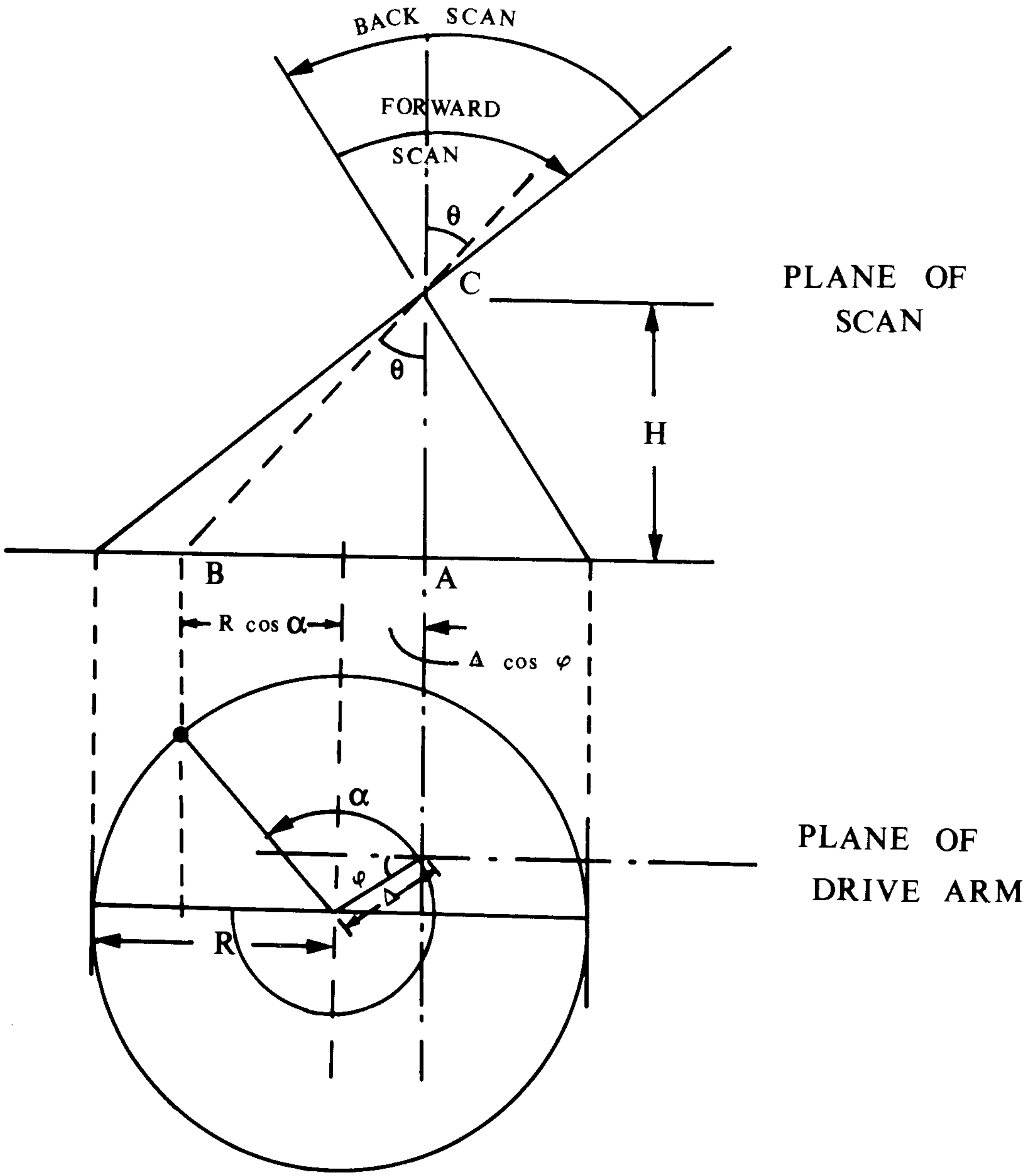

Figure 8: *tan θ*-model when transducer centre is mis-positioned.

3. *Positioning of centre of transducer above drive arm plane*

   Because of the complex structure of the probe from a manufacturing point of view, it will be difficult to ensure that the centre of the transducer is
   (a) exactly a distance R above the drive arm plane; and
   (b) on the drive arm axis.

   The corresponding *tan* θ-models are

$$\tan \tilde{\theta} \;=\; R \cos\alpha / H, \tag{4}$$

   where $\tilde{\theta}$ denotes the transducer angle relevant to the situation identified in (a); and

$$\tan \tilde{\tilde{\theta}} \;=\; (R \cos\alpha + \Delta \cos\phi)/H \tag{5}$$

   where $\tilde{\tilde{\theta}}$ denotes the transducer angle relevant to the situation identified in (b), and Δ and ϕ define the offset between the centre of the transducer and the axis of the drive arm as shown in Figure 8.

Remark 4. In all the formulas derived by Ausonics, it was assumed that $R = H$. The consequences of this will be pursued below. #

Thus, the general design error situation is characterised by a misaligned transducer axis and a poorly positioned transducer. It follows from (3) and (5) and their derivations that the general *tan* θ-model is given by

$$\tan \Theta \;=\; (R \cos\alpha + \Delta \cos\phi)/(H + R \sin\alpha \tan\gamma), \tag{6}$$

where Θ denotes the transducer angle relevant to this general situation.

As mentioned above, from Ausonics' point of view, the goal is to determine which design errors (in terms of α, R, H, δ, ϕ and γ) affect most the alignment between the transmission directions on the forward and back scans. In part, the problem thereby reduces, for a given *tan* θ-model, to analysing the effect on the corresponding transducer angle of perturbations in the design variables, which corresponds to a classical error analysis. Since a comprehensive account of such deliberations is beyond the scope of this Report, we briefly sketch such an analysis and discuss the resulting interpretations for a simple (but important) situation not previously considered by Ausonics; namely, the *tan* θ-model of equation (4) where H is assumed to depend on α. An examination of the construction of the probe indicates that the drive arm is a single solid component of fixed length R whereas the height H of the centre of the transducer above the drive arm plane depends on the construction and movement of the universal joint in which the transducer is mounted. It is therefore natural to assume that R is fixed (independent of α), and that H depends on α and, up to first order, equals R.

It therefore follows that, under these assumptions,

$$d\tilde{\theta} = -\frac{RH}{(H^2 + R^2\cos^2\alpha)}(\sin\alpha \; d\alpha + \cos\alpha\frac{dH}{H}),$$

or

$$d\tilde{\theta} \doteq -\frac{\sin\alpha}{1+\cos^2\alpha}\, d\alpha - \frac{\cos\alpha}{1+\cos^2\alpha}\,\frac{dH}{H}. \tag{7}$$

But, from (1), we have

$$d\theta = -\frac{\sin\alpha}{1+\cos^2\alpha}\, d\alpha. \tag{8}$$

Hence, (7) becomes

$$d\tilde{\theta} \doteq d\theta - \frac{\cos\alpha}{1+\cos^2\alpha}\,\frac{dH}{H} \tag{9}$$

Together, equations (7) and (9) give a detailed picture of the effect of H, when compared with the ideal situation characterised by (8). In fact, we can draw the following conclusions:

(a) For the ideal design, it follows from (8) that the transducer angle is most sensitive to perturbations in α when $\alpha \sim \pi/2$ and $3\pi/2$ and least sensitive when $\alpha \sim 0$ and π. From a design point of view, this has good and bad aspects. Since, when examining an image on the TV-monitor, the eye seeks information from the central (rather than peripheral) region of the picture, this is bad as the above interpretation implies that, for errors in the optical encoder, jitter will be worst in the central region of the image. It is good in the sense that encoder induced jitter will be least where slap in the drive arm and transducer mountings could have most effect.

(b) From (9) it follows that the effect of H is to introduce an additive error into the ideal situation.

(c) Equation (7) shows that the effect of H will be out of phase with encoder induced errors in the ideal situation. Thus, its jitter will be less crucial in the central region of the image.

(d) From a manufacturing point of view, the most disturbing aspect about the structure of the additive error due to H is that it is a *relative* error term, since in construction tolerances are specified as *absolute* errors (which we know do not necessarily control relative errors).

6. Conclusions

Acknowledging that other aspects would have to be examined in a more comprehensive investigation of the electronic and electro-mechanical jitter in Ausonics' scanner, the Study Group concentrated attention on what was believed to be a primary source of the jitter; namely, scan alignment jitter. As outlined in the Report above, the Study Group made considerable progress with the modelling, analysis and interpretation of this source, and thereby laid a basis for a more detailed investigation of scan alignment jitter. In particular, the Study Group

- identified the central importance of the *tan* θ-modelling strategy;
- formulated a quite simple procedure for determining *tan* θ-models and then derived them for a variety of design errors;
- gave an error analysis for a simple (but important) situation not previously considered by Ausonics; and, thereby,
- derived conclusions about the scan alignment jitter not previously appreciated by Ausonics.

Acknowledgement

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