PROBLEM 4

EXPLOSIVELY INDUCED STRESS WAVE FROM A FINITE CYLINDRICAL SOURCE

1. INTRODUCTION

Blasting is a fundamental technique in mining operations, and there is a long history of attempts to analyse the sequence of events in the vicinity of an explosion in rock. At present factors such as amount of explosive, placement and depth of holes, type of detonator and speed of explosive, are decided on experience, and could probably be improved, with great savings to the industry, if there were a more analytic basis available.

The problem of "subsonic" radiation of elastic waves is well within the domain of classical applied mathematics, and several papers have been published. A little less conventional is the analysis of radiation from a detonation wave travelling faster than the elastic waves, but there is at least one adequate paper in this area (Paplinski, 1977), and the acoustic analogy works quite well.

All this analysis fails if the medium begins to fail as the elastic waves pass through, affecting their propagation. A major problem is to decide whether the propagation of cracking, probably by hot gases, is in some region faster than the elastic waves; or whether the elastic waves, probably modified by reflection, precede and cause the cracking.

2. CLASSICAL ANALYSIS

A well-known early paper in the field is that of Heelan (1953). Like several that followed, it considers a pressure instantaneously applied to a portion of the wall of an infinite cylinder in an infinite medium. Advanced and elegant mathematic methods produce asymptotic wave forms of a relatively easily calculated shape. Unfortunately, as asserted by Jordan (1962) and demonstrated more convincingly by Abo-Zeno (1977), the results are wrong. Jordan does convincingly show, by trying to do the analysis correctly, that no easy answers are to be obtained that way.

The wave propagation problem treated by those three authors is now described. Two elastic potentials ϕ and ψ are defined which determine the radial and axial displacements u_r and u_z by the equations

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial\psi}{\partial z},$$

$$u_z = \frac{\partial\phi}{\partial z} + \frac{1}{r}\frac{\partial(r\psi)}{\partial r} .$$

The potentials satisfy the wave equations

$$\nabla^2\phi = \frac{1}{c_L^2}\frac{\partial^2\phi}{\partial t^2} ,$$

$$\nabla^2\psi = \frac{1}{c_T^2}\frac{\partial^2\psi}{\partial t^2} ,$$

where $c_L^2 = (\lambda+2\mu)/\rho$ and $c_T^2 = \mu/\rho$, in which λ,μ are the Lamé constants.

The boundary condition is an impulsive rise in the radial stress

$$\tau_{rr} = 2\mu\frac{\partial u_r}{\partial r} + \lambda\Delta^2\phi.$$

Heelan allows non-radial symmetry. An important difficulty with these methods is the reliance on the maintenance of an infinite cylindrical boundary. It is possible to impose various pressure conditions there, but both the finiteness of the cylinder and the modification by the explosion of parts of the wall are difficult to include.

Jordan solves the p.d.e. by taking Fourier transforms with respect to t and z, leaving a Bessel equation in r. The result gives transformed stresses as linear functions of $H_0(\zeta R)$, $H_1(\zeta R)$, $H_0(\xi R)$, $H_1(\xi R)$ where R is the radial co-ordinate, ξ and ζ are given by

$$\xi^2 = \sigma^2 + (s/C_T)^2,$$
$$\zeta^2 = \sigma^2 + (s/C_L)^2,$$

s and σ are the transform variables, and the denominator includes a term

$$\Delta = \frac{2\zeta}{a}\frac{s^2}{C_T^2} H_1(a\zeta)/H_1(a\xi) + (\sigma^2+\xi^2)H_1(a\xi)/H_0(a\zeta)$$
$$- 4\sigma^2 \xi \zeta H_1(a\zeta)H_0(a\xi)$$

in which a is the radius of the hole. Jordan then replaces the Bessel functions by large argument asymptotic expansions to show the emergence of a dilation wave front and a shear wave front travelling with different speeds (previously shown by Selberg 1952).

Abo-Zena (1977) covers much the same ground to produce the response to a δ-function in the radial stress

$$\tau_{rr} = F(t,z)\ \delta(t-t_0)\ \delta(z-z_0).$$

The displacements are found to be

$$u_r = \frac{a^2}{4RC_L}\left[1 - \frac{2\mu}{\lambda+\mu}\cos^2\zeta\right] F(t_0,z_0)\ \delta\left[t - \frac{R}{C_L} - t_0 + \frac{z_0}{C_L}\cos\zeta\right],$$
$$u_z = \frac{a^2}{2RC_T}\cos\zeta\ \sin\zeta\ F(t_0,z_0)\ \delta'\left[t - \frac{R}{C_T} - t_0 + \frac{z_0}{C_T}\cos\zeta\right].$$

3. SHOCK WAVE SOLUTIONS

Shocks were briefly treated in Jordan's paper, and more extensively by Paplinski and Wlodarczyk (1977). We decided treatment of this case was essential because commonly quoted detonation speeds of 4-10 km/sec. exceeded common

P-wave velocities of 2-3.5 km/sec. The details of Paplinski's analysis are reminiscent of that of Jordan, but are somewhat simpler. The detonation wave is given uniform velocity V, leading to a linear function for the radial displacement velocity

$$V_r(\xi,\eta) = \frac{\beta}{M}\left[\frac{1}{\sqrt{\xi}}\,\delta(\eta-\xi_\beta) + H(\eta-\xi_\beta)\int_0^\infty f_v(\xi,\rho)e^{-\rho\eta}\,d\rho\right]$$

where

$$f_v(\xi,\rho) = \frac{K_1(\rho\beta\xi)\ I_0(\rho\beta) + K_0(\rho\beta)\ I_1(\rho\beta\xi)}{K_0^2(\rho\beta) + \pi^2\ I_0^2(\rho\beta)}$$

and ξ, η and ξ_B are defined by

$$\xi = r/r_0,\quad \eta = (z+c_0t)/v_0,\quad \xi_\beta = \beta(\xi-1).$$

4. SOLUTION BY SUPERPOSITION OF SPHERICAL SOLUTIONS

In a 1969 paper, Favreau developed a solution with spherical symmetry which used the gas equations of state to provide a boundary condition allowing for the effect of expansion of the cavity on the gas pressure. The result gives travelling wave solutions which can be determined from a second order ordinary linear differential equation with constant coefficients. This would also have been true had no allowance been made for gas expansion (Selberg 1953), the only difference being in the complexity of the coefficients.

Although the mathematics is rendered only slightly more complicated, there is also little improvement in physical realism, because rock is so much stiffer a material than gas. A 1% increase in cavity radius diminishes the gas pressure by something like 3%, but the change in rock stress is enormous. The notes of Bland (1980) reinforce this point.

One can try to incorporate the same effect in a blast moving up a cylindrical borehole by superimposing spherical explosions translated in time and space. But there are several drawbacks. Favreau's analysis based on gas confined in a sphere is now applied to gas not so confined. Energy moving up the borehole in the gas phase and as surface Rayleigh waves is not allowed for. And in common with most solutions, no account is taken of the crumbling region behind the detonation front.

5. CONCLUSIONS

The method of superposition of spherical solutions of the type due to Favreau should not be used because the assumption of radial gas expansion is not justified, and Favreau's refinement of Selberg's notation is neither significant nor applicable in this case. The most attractive replacement is a conical wave solution, probably a shock wave. This idea is also suggested by Bland, and some of the theory in Section 3 should be useful.

REFERENCES

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