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UMTS Network Planning – The Impact of User Mobility

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Abstract

The impact of user mobility on network planning is investigated. For a system of two base stations the stationary distribution of a Markov chain, including mobility, is computed.

7.1   Introduction

For the 3rd- generation mobile communication system UMTS (Universal Mobile Telecommunication System) a network has to be planned. Therefore locations of the base stations have to be chosen. One question to answer here is, what should be the base station density. This is an optimization problem with a trade off between on the one hand the investment costs, and on the other hand the quality of service, here expressed by (i) the *probability of blocking*, i.e. the probability that a new request for a call has to be blocked because there are no channels available and (ii) the *handover blocking probability*, i.e. the probability that for an existing call, moving from one base station to another, a so called hand over, there is no channel available at the new base station.

We have to deal with two effects. There are fresh calls arriving in a cell. This is typically a Poisson process. Secondly, we assume mobility in the system. While making a call, the user might move from one cell to another. We reserve capacity in the base stations to serve hand over calls. From a quality of service point of view it is much worse to drop an existing call when the user moves form one cell to another, than to block a new call.

The main problem to be addressed here is to quantify the effects of user mobility on the density of the base stations.
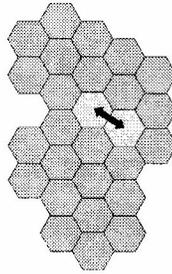
Figure 7.1: Hexagonal covering of the area. In this paper we focus on the two-base-station system indicated by the light-grey cells

7.2   A simple model

We assume, in the planning stage, that the area we are interested in is covered by hexagons. In each hexagon there is a base station that can accept a number of calls. We drastically simplify the analysis in this paper by concentrating on a *two-base-station system*. In addition we assume these cells to be equal. See Figure 7.1. The state of the system at a particular time instant is characterized by the two-tuple (n_1, n_2), where n_i is the number of calls at cell i. We assume that for both cells the capacity is N. The state space of the system is depicted in Figure 7.2.

Our approach is to calculate the stationary distribution $p(n_1, n_2)$. In the stationary distribution the transition rates coming from a state balance with the transition rates entering into that state. Therefore the distribution doesn't change in time. The stationary distribution tells us the average fraction of time the system is in a certain state.

Once the distribution is known the probability of blocking, P_{block}, can be obtained by identifying the states that don't allow an extra fresh call and adding up their probabilities. In Figure 7.2 these states can be found on the lower and right boundaries.

The probability of dropping a call P_{drop} is the fraction of the calls that, when moving from one cell to another, is dropped. We take into account that different states have different weight by the number of calls in that state. This probability follows from the transition rates due to mobility and is given explicitly in Formula (7.4).

If we make no distinction in the treatment of fresh calls and hand over calls, we obtain a Markov process for which we can easily solve the stationary distribution.

We assume that the arrivals of fresh calls in the cells is a Poisson process. Let λ be the arrival rate of fresh calls and let $\frac{1}{\mu}$ be the mean of the exponentially distributed call-length. The transition rate due to terminated calls is proportional to the number of existing calls in the cell. Therefore it is expressed as μn_i. Without mobility we have the following transitions around the state with $(n_1, n_2) = (i, j)$: To obtain the
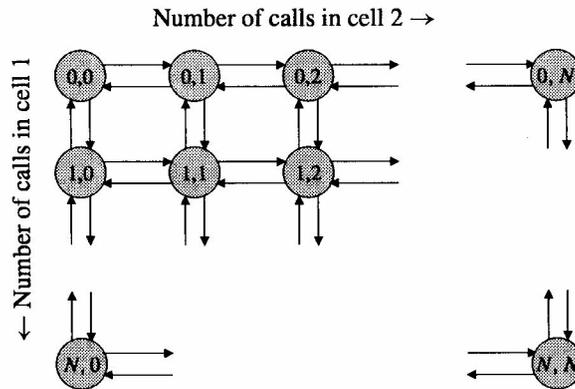
Number of calls in cell 2 →

Figure 7.2: State space for the two-base-station system. N denotes the maximum number of calls accepted in a cell

stationary distribution we need to solve the balance equations:

$$
\begin{aligned}
\lambda(p(i-1, j) + p(i, j-1)) &+ \mu((i+1)p(i+1, j) + (j+1)p(i, j+1)) \\
&= (2\lambda + \mu(i+j))p(i, j).
\end{aligned}
\tag{7.1}
$$

A solution is given by

$$
p(i, j) = c\frac{\left(\frac{\lambda}{\mu}\right)^{i+j}}{i!\,j!},
\tag{7.2}
$$

where c is the normalizing constant. From this distribution we can calculate the probability of blocking by

$$
P_{\text{block}} = \sum_{j=0}^{N} p(N, j) = \frac{\left(\frac{\lambda}{\mu}\right)^{N}}{N!\sum_{j=0}^{N}\frac{1}{j!}\left(\frac{\lambda}{\mu}\right)^{j}}.
\tag{7.3}
$$

7.2.1 Including mobility

Next we allow a call to move from one cell to the other. We assume for this process an exponential distribution with mean $\frac{1}{\nu}$, i.e. the average time that a call resorts under the same antenna is $\frac{1}{\nu}$. Apart from the transitions in Figure 7.3 there are a number of extra possible transitions. In the interior of the state space there are two diagonal 'arrows' leaving each state, corresponding to calls moving from one cell to another. Their transitions rates are proportional to the number of calls in the cell that the caller is leaving. Likewise there are two additional transitions towards each state. Note that these four transitions balance each other if the distribution (7.2) is used.
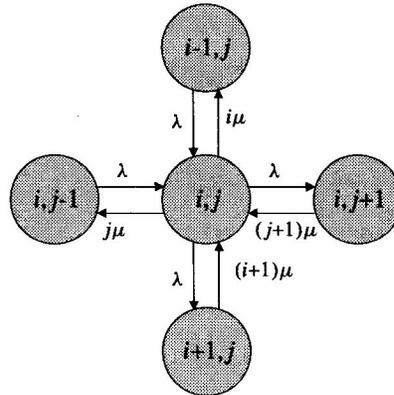
Figure 7.3: Transitions without mobility around the state (i, j)

On the boundary of the state space things are more complicated, as a hand over call is not accepted by the new base station. Suppose a call moves from cell 1 into cell 2, which has got no capacity available. This corresponds to the arrow in Figure 7.4 going to the upper-right. However since the call is not accepted in cell 2, the number of calls in this cell is not raised. The call disappears from cell 1, so instead of pointing to the state $(i - 1, j + 1)$, the arrow points to the state $(i - 1, j)$. Similarly, there is another transition from the state $(i + 1, j)$ to the state (i, j). This leads to the transitions as depicted in Figure 7.4.
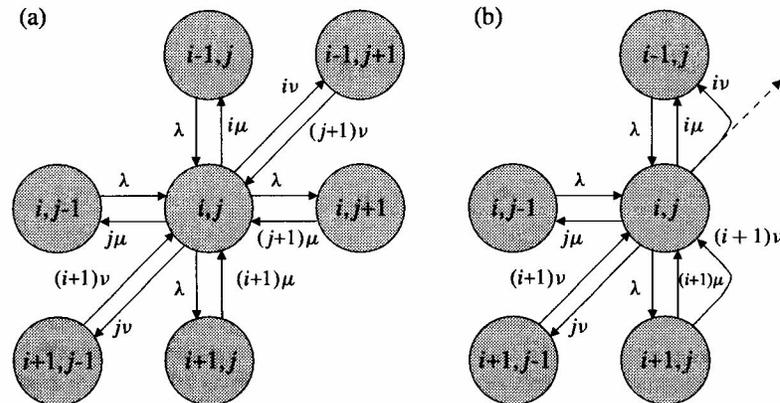

Figure 7.4: Transitions and transition rates with mobility, (a) for a state in the interior of the state space and (b) for a state on the boundary of the state space

The balance equations for states on the boundary are different from those for states in the interior of the state space. Therefore the stationary distribution is altered. Qualitatively the number of calls present in the cells is slightly lowered, but is not possible to solve the new balance equations analytically. As a first approximation we use distribution (7.2). This can be justified if the probabilities for the boundary states are small, i.e. if $\frac{\lambda}{\mu} \ll N$. We obtain for the probability of dropping

$$P_{\text{drop}} = \frac{1}{\bar{n}} \sum_{i=0}^{N} i p(i, N), \tag{7.4}$$

where $\bar{n}$ is the average number of calls in a cell

$$\bar{n} = \frac{1}{2} \sum_{i=0}^{N} \sum_{j=0}^{N} (i + j) p(i, j). \tag{7.5}$$

To check whether this gives the correct numbers we have written a computer program to compute numerically the stationary distribution. In the next section the results of these computations are compared to the results obtained for the second scenario.

7.3   Distinguishing between fresh and hand over calls

In the previous section we have not distinguished between new incoming and hand over calls. As remarked before it is much worse to drop an existing call than to reject a new call. At each base station we want to reserve some space for hand over calls. The simplest, and definitely not the best, strategy is to fix a certain capacity for hand over calls only. Therefore we have

$$N = N_f + N_h, \tag{7.6}$$

where the capacity of the base station, N, is divided into capacity for fresh calls, N_f, and capacity for hand over calls, N_h. At a certain time instant the state of our two-base-station system is characterized by the four tuple $(n_1^f, n_1^h, n_2^f, n_2^h)$, where n_i^f denotes the number of fresh calls in station number i and n_i^h denotes the number of hand over calls in station number i, $i = 1, 2$.

Like before we can write down the balance equations that yield the stationary distribution. From each state in the interior of the state space there are no less than ten possible transitions. Therefore we refrain from listing the balance equations. The solution to these balance equations do not take a nice form like (7.2).

However it is possible to obtain the probability of blocking explicitly. Thereto we look at the two dimensional subsystem (n_1^f, n_2^f) consisting only of the parts of each cell reserved for fresh calls. To this subsystem a hand over call is exactly the same as a terminated call. After all the subsystem doesn't see where the call is going,

it just sees it disappear. Therefore the subsystem has a state space as in Figure 7.2, with μ replaced by $\mu + \nu$ and N replaced by N_f. The stationary distribution in the subsystem is given by

$$\tilde{p}(i, j) = c \frac{\left(\frac{\lambda}{\mu+\nu}\right)^{i+j}}{i! \, j!}, \tag{7.7}$$

where $\tilde{p}(i, j)$ denotes the stationary distribution in the subsystem, irrespective of the distribution over the capacity for hand over calls.

$$\tilde{p}(i, j) = \sum_{k,m} p(i, k, j, m). \tag{7.8}$$

To compute the probability of dropping we have written a Mathematica program[1], which computes numerically the stationary distribution. This allows to play with the parameters to obtain reasonable figures for blocking and dropping rates.

As an example, the following table shows the results of a number of computations on a small cell. Each computation was performed using the same total capacity, i.e. $N = 10$ and using the same parameter values, i.e. $\mu = 0.1$, $\lambda = 0.4$, $\nu = 0.04$. In Figure 7.5 we show the probabilities in one cell for one of these computations.

Strategy	$P_{block}(\%)$	$P_{drop}(\%)$
According to (7.2)	0.531	0.531
Correction to (7.2) making no distinction	0.519	0.493
Using distinction, $N_f = 10$	0.057	100.
Using distinction, $N_f = 8$	0.634	20.641
Using distinction, $N_f = 5$	9.801	0.263
Using distinction, $N_f = 2$	51.414	0.000

In this example, we see that our second strategy can reduce the probability of dropping, but the probability of blocking is increased enormously. Therefore we conclude that for the model of two cells this strategy reduces the probability of dropping but in the same time increases the blocking probability. This is due to the fixed number of available channels in an antenna. In practice we have interaction between one cell and the neighbouring cells. Similar balance equations can be written for the system of seven cells or more. In order to keep the blocking probability less then a desired value using the proposed strategy, we have to increase the density of antennas; this is the impact of the user mobility on the density of base stations.

7.4 Conclusions

- We have concentrated on a two cell system. To extend this to a seven hexagon system would still be doable, but the extension to a larger system requires large scale computations.

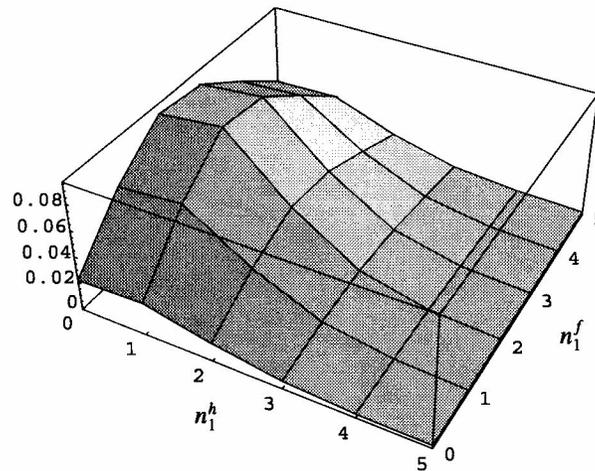[1]This program is available on request.

Figure 7.5: Probabilities in the first cell when $N_f = 5$, $N_h = 5$, $\lambda = 0.4$, $\mu = 0.1$, $\nu = 0.04$

- Setting up a small model with maple gives quickly some insight in the qualitative behavior.

- Additional modeling is required. We would like to investigate whether it is possible to treat a seven hexagon system as a single antenna with adjusted parameters. This would scale done the complexity of the computations.

Bibliography

[1] L. Kleinrock and R. Gail, *Queuing systems: problems and solutions*, New York, Wiley, 1996.