Temperature variations and control of a Calciner

Elkem

1 Background

The electrodes used in arc-furnaces are made by heating anthracite to about 2500°C in a 'calciner', thereby removing volatiles and producing some graphite. The calciner is a stationary cylinder 8m high and 2m in diameter, see figure 1. Raw anthracite is fed continuously from bins around the top electrode and removed by two rotating scrapers at the bottom, after residing in the calciner for about 15 hours. The anthracite is heated ohmically by passing an electric current of 14kA between the electrodes, which are separated by about 3m. The anthracite conducts electricity poorly below 1500°C.

Although the particulate flow of the anthracite is stable and near to a uniform plug flow, the temperature distribution seems not to be axisymmetric within the calciner. Moreover if the particulate flow is disturbed (temporarily reduced or enhanced), the cold side becomes significantly colder for about 10 hours. The Study Group was set the problem of explaining the temperature variations cause by changes in the flow rate, and to consider the effect of different standard control strategies.

In the following, the governing partial differential equations are introduced in section 2. Then section 3 develops a slender body theory for the heated plume downstream of the top electrode, while section 4 looks at the thermal boundary layer on the top electrode which pre-heats the anthracite. Simplified lumped-parameter models governed by ordinary differential equations are considered in sections 5 & 6, and finally in section 7 the control of one of these through flow rate and current is studied.

2 Governing equations

The quasi-steady conduction of electric current is governed by

$$\nabla \cdot (\sigma \nabla \phi) = 0,$$

where σ is the electrical conductivity and ϕ is the electric potential. We will approximate the electrical conductivity by zero below a critical temperature $T_C = 1500\,^\circ\mathrm{K}$ and the constant $\sigma = 2500\,\mathrm{mho\,m^{-1}}$ above T_C.

The heat equation including advection, diffusion and ohmic heating is

$$\rho c T_t + \rho c U T_z = \rho c \kappa \nabla^2 T + \sigma |\nabla \phi|^2.$$

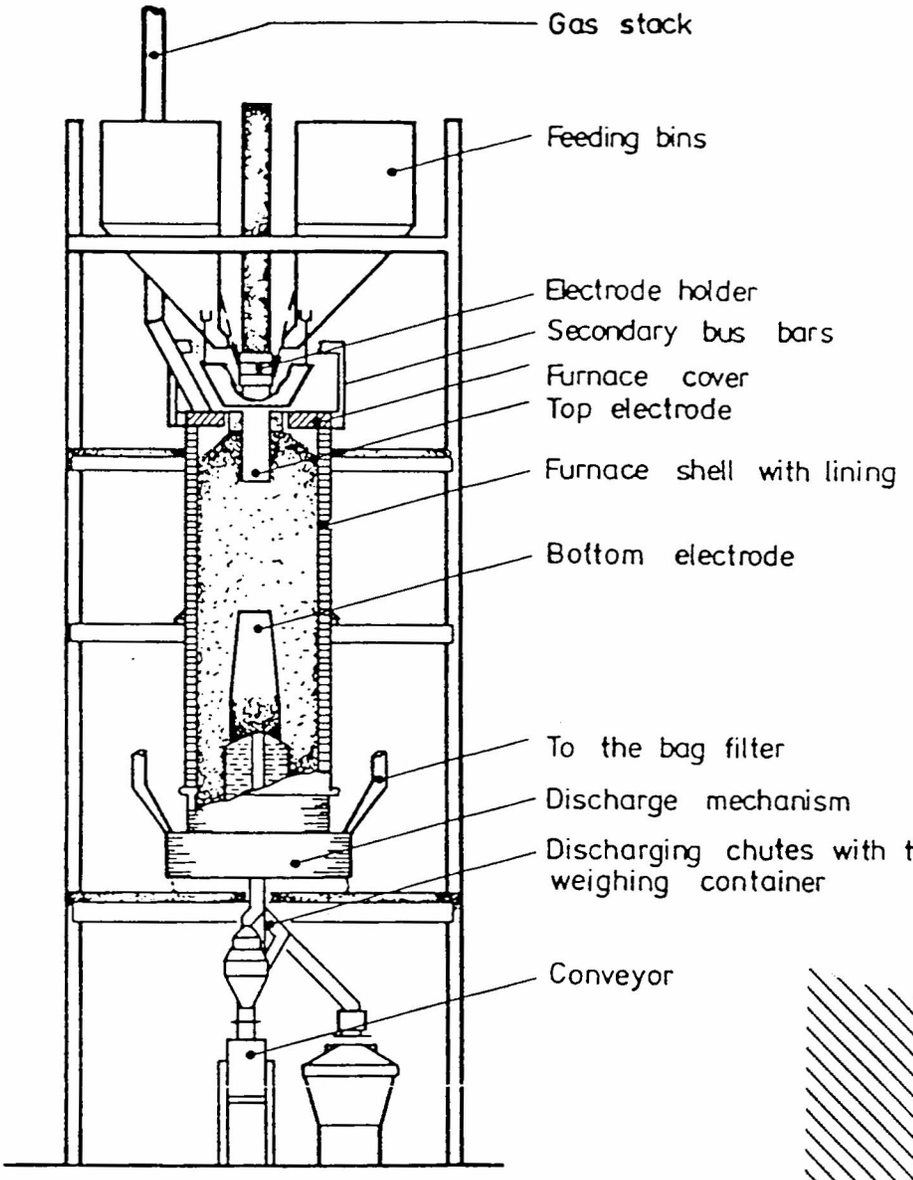

Figure 1 Elkem Calciner

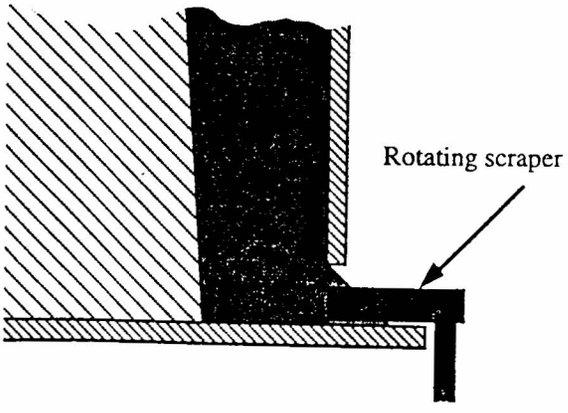

Figure 2 Discharge mechanism

Typical values for the anthracite in the calciner are :- density $\rho = 900\,\mathrm{kg\,m^{-3}}$, specific heat $c = 1700\,\mathrm{J\,kg^{-1}\,{}^\circ K^{-1}}$, temperature $T = 1500\,{}^\circ\mathrm{K}$, time $t = 6\times10^4\,\mathrm{s}$, advection velocity $U = 1.4\times10^{-4}\,\mathrm{m\,s^{-1}}$, (axial) length $L = 5\,\mathrm{m}$, heat conductivity $\kappa = 10^{-5}\,\mathrm{m^2\,s^{-1}}$, electrical conductivity $\sigma = 2500\,\mathrm{mho\,m^{-1}}$, and electrical potential $\phi = 70\,\mathrm{V}$. From these values we can form two important non-dimensional groups, the Péclet number $Pe = UL/\kappa = 70$ and the ohmic heating number $\alpha = \sigma\phi^2/\rho c\kappa T = 500$. Nondimensionalising the variables with the above scales, the heat equation takes the form

$$0.65T_t + PeT_z = \nabla^2 T + \alpha\sigma(T)|\nabla\phi|^2.$$

where now $\sigma = 0$ for $T < 1$ and $\sigma = 1$ for $T > 1$.

3 Slender heated plume

Downstream of the top electrode, there is a plume in the moving anthracite which is above the critical temperature to conduct electricity and so is heated ohmically. The plume spreads as heat diffuses radially. Because the Péclet number is large, the plume is slender, with radial extent $(z/Pe)^{1/2}$. It should be noted that the calciner is designed so that this plume reaches the outer boundary at the exit, i.e. all the anthracite is treated, just.

The Study Group constructed a similarity solution for this plume, but in two-dimensional rather than axisymmetric geometry. Let x be the cross-stream coordinate. This is rescaled by $x = Pe^{-1/2}X$. Let $X = h(z)$ be the edge of the plume defined by where $T = 1$, so that inside the plume $\sigma = 1$ and outside $\sigma = 0$. For long slender plumes, the quasi-steady conduction of heat becomes at leading order

$$h\phi_z = I,$$

where I is the total current flowing. At leading order the temperature equation becomes

$$T_z = T_{XX} + \frac{7I^2}{h^2}.$$

inside the plume $X < h(z)$ and without the final term outside. (Here $7 = \alpha/Pe$.)

A similarity solution is sought in the form

$$T(X,z) = \theta(\xi) \quad \text{and} \quad h = Hz^{1/2} \quad \text{with} \quad \xi = X/z^{1/2}.$$

The function θ then obeys

$$\theta'' + \frac{1}{2}\xi\theta' + \frac{7I^2}{H^2}$$

inside $\xi < H$ and without the final term outside. Boundary conditions to be applied are $\theta'(0) = 0$ and $\theta \to 0$ as $\xi \to \infty$. The constant H is determined by the condition $\theta(H) = 1$. It was left as a simple numerical task to find $\theta(\xi)$ and H. A series solution is easy to obtain inside

$$\theta(\xi) = \theta(0) + \frac{7I^2}{H^2}\left(-\frac{1}{2}\xi^2 + \frac{1}{24}\xi^4 - \frac{1}{360}\xi^6 + \frac{1}{6720}\xi^8\right),$$

while outside $\theta \propto \operatorname{erfc}\xi$. [Note: a similarity solution can be found for the axisymmetric plume, which has the same $z^{1/2}$ sideways spread but has the temperature decaying downstream like z^{-1}.]

4 Thermal boundary layer on top electrode

Now the electrodes are long and are themselves heated ohmically. Along the length of the electrode, heat will diffuse radially out forming a thermal boundary layer in the anthracite moving along the electrode. It is essential that the anthracite is pre-heated (in sufficient quantity) to the critical temperature T_C before the end of the electrode in order that thermal plume can conduct electricity.

Let the electrode carry current I, be of radius a and have a length ℓ. Let the thermal boundary layer have a thickness δ at the end of the electrode: $\delta = (\kappa\ell/U)^{1/2}$. The time for anthracite to move along the electrode is ℓ/U, so that the total ohmic heating in the electrode during that time is $(I/a^2)^2/\sigma_E \times a^2\ell \times \ell/U$ (dropping factors like π), where σ_E is the electrical conductivity of the electrode. Increasing the temperature of the anthracite to T_C from effectively 0 within the boundary layer requires heat $\rho c T_C \times \delta\ell a$. Equating these two, we find that electrode must be at least as long as

$$\ell = \left(\frac{\rho c T_C \sigma_E a^3}{I^2}\right)^2 \kappa U.$$

We use the following values of the relevant parameters: density $\rho = 850\,\mathrm{kg\,m^{-3}}$, specific heat $c = 1700\,\mathrm{J\,kg^{-1}\,{}^\circ K^{-1}}$, critical temperature $T_C = 1500\,{}^\circ\mathrm{K}$, conductivity of the electrode $\sigma_E = 10^5\,\mathrm{mho\,m^{-1}}$, radius of electrode $a = 0.25\,\mathrm{m}$, total current $I = 14\,\mathrm{kA}$, advection velocity $U = 1.4 \times 10^{-4}\,\mathrm{m\,s^{-1}}$ and heat conductivity $\kappa = 10^{-5}\,\mathrm{m^2\,s^{-1}}$. These produce the estimate for the length of the electrode necessary in order to reach the critical temperature as $\ell = 0.5\,\mathrm{m}$, which agrees with the Elkem design. At this length, the thermal boundary layer has a thickness of 0.17 m, which is similar to the radius of the electrode in the Elkem design, and hence allows an easy transition to the thermal plume.

We note that the required length of the electrode ℓ is proportional to the velocity of the anthracite. Hence if the feed is increased too much, there will be insufficient pre-heating of the anthracite to the critical conduction temperature T_C in the thermal boundary layer.

5 Lumped models

In order to understand the temporal development, and possible instability, of non-axi-symmetric differences between one side of the calciner and the other, the Study Group investigated a simple lumped-parameter model. The full temperature field was replaced by four temperatures: T_1 and T_2 the temperatures in the thermal plume on the left and right sides respectively, and T_3 and T_4 the temperatures in the thermal boundary layers again on the left and right sides respectively. These temperatures were taken as being typical of their respective region and to be only a function of time. The temperature of the electrode was taken to be T.

The heat balance for the left plume region of length L and cross-sectional area A is

$$\rho c A L \dot{T}_1 + \rho c U A (T_1 - T_3) = k_1(T_2 - T_1) + k_2(T_3 - T_1) + k_3(T - T_1) + \Omega_1,$$

with coefficients k_i for the heat transfer between different regions, which could be estimated from the studies above, and with ohmic heating Ω_1. The equation for the right

plume regions is similar with T_2 replacing T_1 and T_4 replacing T_3. The heat balance for the left thermal boundary layer is

$$\rho c A \ell \dot{T}_3 + \rho c U A T_3 = -k_2(T_3 - T_1) + k_4(T - T_3),$$

and a similar equation for the right thermal boundary layer with T_2 replacing T_1 and T_4 replacing T_3.

There was much discussion about the form to be adopted for the ohmic heating. In the model eventually adopted, most of the voltage drop V was taken to occur in the thermal plume, and so

$$\Omega_1 = \frac{A}{L}\sigma(T_1)V^2,$$

and a similar expression in the right thermal plume with T_2 replacing T_1. The alternative model which was eventually rejected had most of the heating occurring near to the electrode

$$\Omega_1 = \gamma(T_1, T)V^2,$$

with the function γ still to be determined somehow.

More sophisticated models with the cross-sectional area A varying were not pursued.

6 Reduced model

In order to understand the possibility of non-axisymmetric states, the above lumped-parameter model was further simplified by deleting the thermal boundary layers, leaving the two-variable model

$$\begin{aligned} \dot{T}_1 &= -UT_1 + \lambda(T_2 - T_1) + \gamma\sigma(T_1) \\ \dot{T}_2 &= -UT_2 + \lambda(T_1 - T_2) + \gamma\sigma(T_2) \quad . \end{aligned}$$

Recall that $\sigma(T) = 0$ for $T < T_C$ and $\sigma(T) = 1$ for $T > T_C$. It is important to realise now that σ rises very sharply from 0 to 1 in a neighbourhood of T_C, and so we may consider it to take on arbitrary values between 0 and 1 for $T = T_C$.

If $U > \gamma/T_C$, the only solution is the symmetric solution $T_1 = T_2 = 0$, i.e. too cold for any ohmic heating.

If $U < \gamma/T_C$, there are three symmetric solutions

$$T_1 = T_2 = 0, \quad T_C \quad \text{and} \quad \gamma/U,$$

corresponding to no heating ($\sigma = 0$), partial heating ($\sigma = UT_C/\gamma$) and full heating ($\sigma = 1$), respectively.

If $\gamma(\lambda + U)/T_C(2\lambda + U) > U > \gamma\lambda/T_C(2\lambda + U)$, there are also two asymmetric solutions with full heating in one side $\sigma(T_1) = 1$ and no heating in the other $\sigma(T_2) = 0$

$$T_1 = \frac{\gamma(\lambda + U)}{U(2\lambda + U)}, \quad T_2 = \frac{\gamma\lambda}{U(2\lambda + U)},$$

and a similar solution with T_1 and T_2 interchanged.

If $\gamma/T_C > U > \gamma\lambda/T_C(2\lambda+U)$, there are two asymmetric solutions with full heating on one side $\sigma(T_1) = 1$ and partial heating on the other $T_2 = T_C$

$$T_1 = \frac{\gamma + \lambda T_C}{\lambda + U}, \quad T_2 = T_C \quad \text{with} \quad \sigma_C = \frac{U(2\lambda+U)T_C - \lambda\gamma}{(\lambda+U)\gamma},$$

and a similar solution with T_1 and T_2 interchanged.

Finally if $U < \gamma(\lambda+U)/T_C(2\lambda+U)$, there are two asymmetric solutions with partial heating on one side $T_1 = T_C$ and no heating on the other $\sigma(T_2) = 0$

$$T_1 = T_C \quad \text{with} \quad \sigma_C = \frac{U(2\lambda+U)T_C}{(\lambda+U)\gamma}, \quad T_2 = \frac{\lambda}{\lambda+U}T_C$$

and a similar solution with T_1 and T_2 interchanged.

A simple phase-plane analysis find that the solutions with some partial heating are all unstable (saddle points or unstable nodes). At high flow rates $U > \gamma/T_C$ the only stable solution is the cold symmetric state $T_1 = T_2 = 0$, i.e. the anthracite fails to reach the critical temperature before it passes out of the calciner and so in this simplified model there is no heating anywhere. At lower flow rates $U < \gamma/T_C$, in addition to this cold stable symmetric state there is the symmetric state $T_1 = T_2 = \gamma/U$, in which ohmic heating can exceed the critical temperature before the material exits the calciner. In the range $\gamma(\lambda+U)/T_C(2\lambda+U) > U > \gamma\lambda/T_C(2\lambda+U)$ there are also two stable asymmetric states, one side with ohmic heating and one without. In these asymmetric states, the ohmic heating on the one side is sufficient for the anthracite to reach the critical temperature on that side, but via heat transfer between the two sides is insufficient for the other side. Clearly one way to eliminate the asymmetric state would be to decrease the flow rate until $U < \gamma\lambda/T_C(2\lambda+U)$.

Future work should incorporate the electrode into the two-variable theory, and to return to the four-variable theory of section 5 which could be solved numerically.

7 Control

The Study Group considered the control of the two-variable model of section 6 through monitoring the total current $I = A(\sigma(T_1)+\sigma(T_2))V/L$ and in response adjusting the flow U of the anthracite. It is to be expected that some control mechanisms $U = f(I)$ might be unstable: the time-delay resulting from the non-zero residence time (not in the model of section 6) may require responding to the time-integral of the current rather than the instantaneous value in order to avoid control instabilities.

The simplest control strategy is 'bang-bang', in which *either* the flow runs at its normal constant value, $U = \mathcal{U}$ say, if both sides of the thermal plume are being ohmically heated, $I > 2A\sigma(T_C+)V/L$, *or* the flow is shut down totally, $U = 0$, if both sides are not above the critical temperature for ohmic heating.

Under bang-bang control, if the initial state has ohmic heating on both sides, $T_1 > T_C$ and $T_2 > T_C$, then the system will evolve to the stable symmetric state with ohmic heating, $T_1 = T_2 = \gamma/\mathcal{U}$, so long as the flow is sufficiently slow for this state to exist, $\mathcal{U} < \gamma/T_C$. This follows immediate from the governing equations in this regime

$$\dot{T}_1 + \dot{T}_2 = -\mathcal{U}(T_1+T_2) + 2\gamma, \quad \text{and} \quad \dot{T}_1 - \dot{T}_2 = -(\mathcal{U}+2\lambda)(T_1-T_2).$$

If on the other hand the initial state has no ohmic heating on both sides, the system would evolve under bang-bang control to a symmetric state at the average of the initial temperatures, and no ohmic heating. Again this immediately follows from the governing equations in this regime

$$\dot{T}_1 + \dot{T}_2 = 0, \quad \text{and} \quad \dot{T}_1 - \dot{T}_2 = -2\lambda(T_1 - T_2).$$

The initial condition of one side ohmically heated and the other not, as would occur in the stable asymmetric states, is more complicated. If one side is significantly above the critical temperature and the other side is only just below, then the cold side will warm up to the critical temperature and the system will then evolve into the stable symmetric state with both sides heated ohmically. If one side is only just above the critical temperature and the other side is significantly below, then the hot side may cool down to the critical temperature, with the system then evolving into a symmetric state with no ohmic heating, or if the rate of heating is high enough the cold side may warm up to the critical temperature. To distinguish between these different cases, we consider the governing equations for the left side being heated $T_1 > T_C$ and the right side not $T_2 < T_C$

$$\dot{T}_1 + \dot{T}_2 = \gamma, \quad \text{and} \quad \dot{T}_1 - \dot{T}_2 = -2\lambda(T_1 - T_2) + \gamma.$$

i.e. with $S = T_1 + T_2$ and $D = T_1 - T_2$

$$\frac{dD}{dS} = -\frac{2\lambda}{\gamma}D + 1$$

with solution

$$D = \frac{\gamma}{2\lambda} + \left(D_0 - \frac{\gamma}{2\lambda}\right) e^{-2(S-S_0)\lambda/\gamma},$$

where D_0 and S_0 are the initial values.

We now need to write down the boundaries of this regime of one sided heated and the other not in terms of the new variables D and S. The constraint that $T_1 > T_C$ and $0 < T_2 < T_C$ gives $D > 0$ and $S > T_C$ at least. The right side will warm up to the critical temperature while the left side is still hot if according to the above solution $T_2 = T_C$ while $T_1 > T_C$, i.e. $D = S - 2T_C$ in $S > 2T_C$. The left side will cool down to the critical temperature while the right remains cold if $T_1 = T_C$ while $T_2 < T_C$, i.e. $D = 2T_C - S$ in $T_C < S < 2T_C$.

From the solution $D(S)$ it is clear that if the rate of heating is sufficiently large $\gamma > 2\lambda T_C$ all the trajectories will move away from the boundary $D = 2T_C - S$ in $T_C < S < 2T_C$ and so the system must go to the symmetric heated state.

If $\gamma < 2\lambda T_C$, we have to contemplate solution trajectories intersecting this boundary. The last trajectory to intersect will be tangent, and so has slope dD/dS equal to -1 at the point of intersection. We thus find the point of intersection as

$$D = \gamma/\lambda, \quad \text{and} \quad S = 2T_C - \gamma/\lambda.$$

We note that this intersection does not occur in the domain $S > T_C$ (i.e. $T_1 > T_C$) if the heating is too high $\gamma > \lambda T_C$. For the case of weak heating $\gamma < \lambda T_C$, we can follow the

trajectory back to $S = T_C$ and so deliminate all the initial conditions which would cause the whole system to cool down to below the critical temperature

$$2T_C - T_1 - T_2 < T_1 - T_2 < \frac{\gamma}{2\lambda}\left(1 + e^{-2+2(2T_C - T_1 - T_2)\lambda/\gamma}\right), \quad \text{and} \quad T_C < T_1 + T_2 < 2T_C - \frac{\gamma}{\lambda}.$$

We conclude that bang-bang control will drive the system to the stable symmetric solution with ohmic heating if initially at least one side is ohmically heated, and if under dangerous conditions the heating rate is sufficiently strong $\gamma > \lambda T_C$.

8 Contributors

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