Risk/Reward

UBS

UBS want to consider ways in which a client can raise money by using various kinds of debt (fixed rate, floating rate, etc). Debt of type i is issued in a proportion α_i, where

$$\sum_{i=1}^{N} \alpha_i = 1.$$

The client may also independently use options to hedge revenues from a natural resource (eg oil), with a hedge ratio Δ ($0 \leq \Delta \leq 1$). UBS have a (confidential) model which takes a strategy $(\alpha_1, \ldots, \alpha_N; \Delta)$ and assigns to it a point in a risk-reward diagram; see figure 1. (Unlike in classical portfolio theory, the UBS model does not require the efficient frontier to be convex.)

The curve joining A and B in this diagram is the "efficient frontier" on which the portfolio should ideally be situated. A client might at present be at P, ie exposed to excessive risk, for a given return and want to move to Q.

There were two aspects to the problem:

(i) find the strategies corresponding to Q and the most robust of these (in terms of sensitivity to small changes, either in the model or in the strategy);

(ii) for the benefit of clients, interpret the pictures graphically (more below).

For point (i), two possibilities were suggested. The first is to search directly for the require minimum risk point; simulated annealing seems a good algorithm given that the map from strategies to the x-y plane is only known numerically. The second is, as at present, to cover the strategy space reasonably uniformly with randomly chosen points and to plot the images in the x-y plane. After this, it is not difficult to locate the strategies leading to point Q.

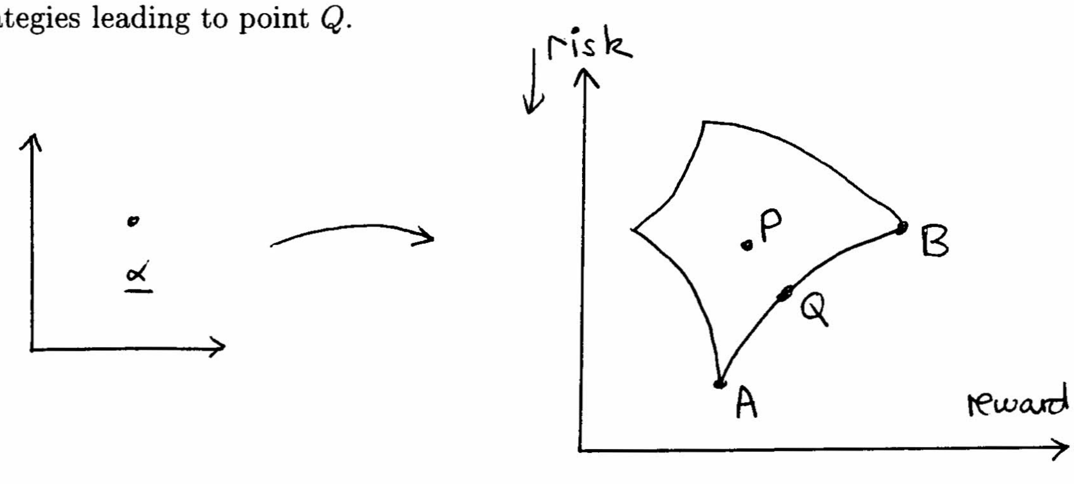


Figure 1:

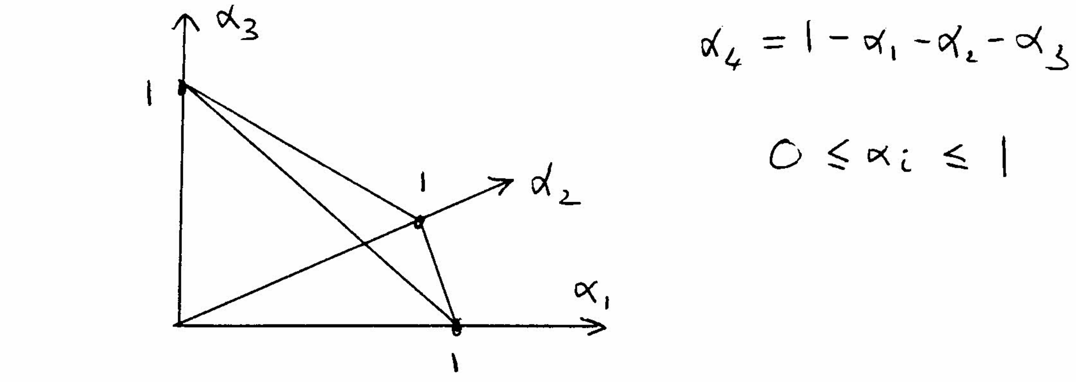


Figure 2:

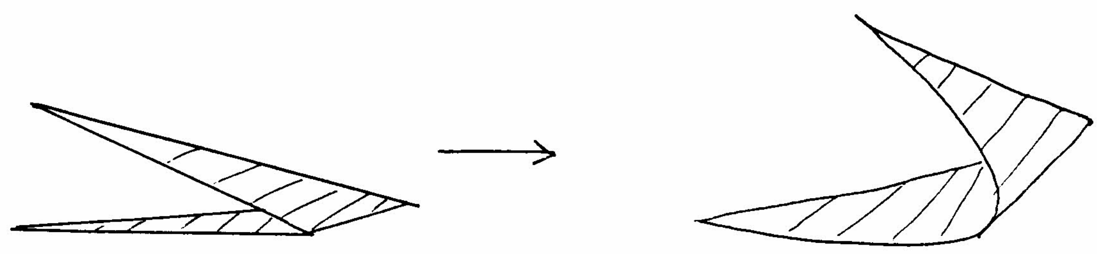


Figure 3:

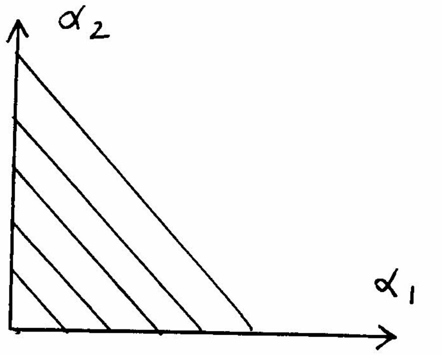
In both cases, robustness can simply be tested by looking at the effect of changing the strategy parameters by small amounts (either systematically, one at a time, to evaluate partial derivatives of y (which is minimised at Q for given x), or randomly).

Concerning graphics, at present the risk-reward diagram is computed for four types of debt (hence three independent types) and one hedge ratio. The debts lie in a tetrahedron and the hedge ratio is independent, see figure 2. For graphical purposes, a two-dimensional slice of the tetrahedron is taken and its image in the x-y plane is shown. Often, the map from the slice (parametrised by (γ_1, γ_2): (α, β) in the problem statement) is many-to-one. For visualization purposes it would be helpful to "uncouple" this multiple-valued map in some way by expanding into a third dimension, see figure 3. It was suggested that a good way to do this would be via an "equiareal" map from the (γ_1, γ_2) plane to $\Re^3$ via $(\gamma_1, \gamma_2) \mapsto (x(\gamma_1, \gamma_2), y(\gamma_1, \gamma_2), z(x, y))$ so that elements of area in the (γ_1, γ_2) plane give the same area on the image surface. This entails solving an equation rather similar to the equation of geometric optics and the following difficulties become apparent:

(i) the equation is nonlinear and in general its solutions have singularities (caustics, etc). It is not at all clear (although an interesting question) whether there is any smooth solution at all;

(ii) the boundary conditions to be applied are far from obvious;

(iii) even if there is a smooth solution, it may still self-intersect.



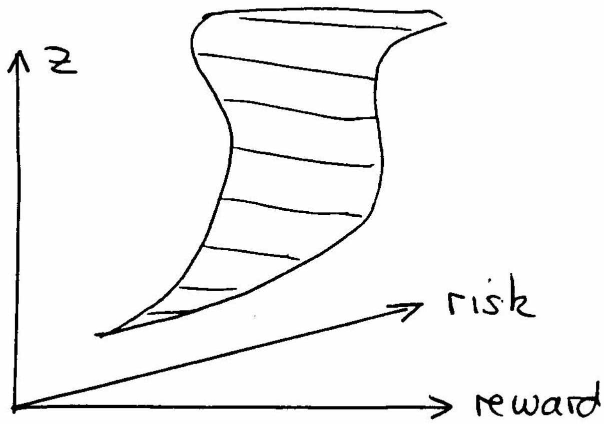


Figure 4:

There is a large literature on representation of surfaces. One simple alternative suggestion follows. Suppose, for simplicity, we have only three types of debt: α_1, α_2 and $\alpha_3 = 1 - \alpha_1 - \alpha_2$ (rather than a 2-dimensional subset of the 3-dimensional space above). Given α_1, α_2 we can calculate $(x(\alpha_1, \alpha_2), y(\alpha_1, \alpha_2))$ in the risk-reward diagram is multi-valued, as noted above. Open the multi-valued map up by looking at $(x(\alpha_1, \alpha_2), y(\alpha_1, \alpha_2), \alpha_1 + \alpha_2)$ This has the advantage that level curves of the surface are lines α_3 =constant; see figure 4. The map is not equiareal and it, too, may self-intersect; but it is easy to compute.

Participants

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