Prediction of freezing times of foods

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1 Objective

To devise a simple method for the prediction of the freezing times of composite food products such as pizzas and meat pies. The prediction of the freezing time should be accurate to 10%.

2 Background

The freezing behaviour, more precisely the time dependence of the warmest point in the sample (known as the 'thermal centre') has to be determined by experiment for complex composite foods. This is because the heat-transfer coefficient is unknown and a full 3-D solution of the heat equation is difficult to obtain in industrial conditions. However, the 1-D or 2-D solutions of the heat equation give sufficiently accurate prediction of $T(t)$ if

(i) the effective thermal conductivity and enthalpy as functions of temperature are a realistic reflection of those of the real product, and

(ii) the correct value of the surface heat-tansfer coefficient is found by matching the predicted and experimental $T(t)$.

The 1-D and 2-D solutions are relatively easy to implement and run on personal computers. This is attractive for industrial use, for example by sales engineers, who would not be able to run large packages. Moreover, the engineers are interested only in the temperature at the thermal centre and do not require the full 3-D transient temperature field.

3 Overview

This problem split into two parts. One was the consideration of appropriate heat-transfer laws for given freezers and foods. The other was to determine suitable methods for analysing the cooling and solidification of composite foods. Whereas the heat-transfer coefficient has been used previously as a free parameter to obtain agreement with experiment for individual food types, the Study Group recommends that a single coefficient (or law) should be determined for a given freezer, taking into account air flow and other operating conditions. The focus of further investigation should then be on the appropriate modelling of the structure of different food types.

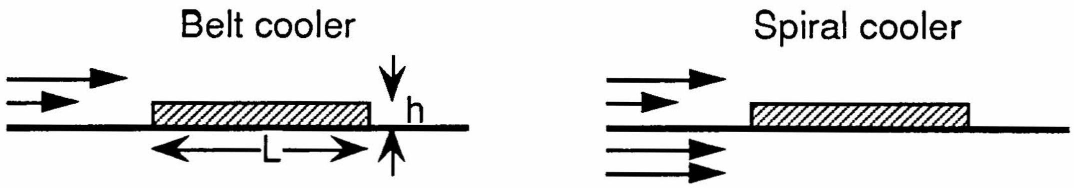


Figure 1: The geometries of typical industrial coolers

4 When is food one-dimensional?

There are principally two types of freezer: those in which cold air circulates only above the belt on which the food is placed; and those in which the cold air circulates above and below the belt (figure 1). This distinction must be borne in mind when considering the principal routes for heat transfer, which determine whether the food can be treated as essentially one-dimensional. Many foods can be treated with a 1-D model if one of the following conditions are satisfied.

(a) When there is flow above and below the plate, it is sufficent that the horizontal dimensions L of the food be much larger than its height h, so that $h/L \ll 1$.

(b) When there is flow just above the belt then heat can still be conducted along the belt and thence up through the food. However, since the belt is quite thin (of thickness d say) such transfer may not be very efficient. In this case, the food can only be treated as one-dimensional if

$$\frac{k_f}{k_s}\frac{L^2}{hd} \ll 1,$$

where k_f and k_s are respectively the thermal conductivities of the the food and the steel belt.

(c) For more equi-dimensional items of food, one-dimensional modelling may still be appropriate if the sides of the food are relatively well insulated; for example, an open flan with pastry sides, or a meat pie with dry crust but moist base, where heat transfer may be principally through the base.

5 The heat transfer coefficient

Based on an air flow U of about 5 ms^{-1} and food of horizontal dimensions L of about 10 cm, the Reynolds number $Re = UL/\nu$, where ν is the kinematic viscosity of the air, is about 5×10^4. For Reynolds numbers of up to about 5×10^5 the mean heat-transfer coefficient is well approximated by

$$h_T \sim 0.664\frac{k}{L}Re^{1/2}Pr^{1/3}$$

where k is the thermal conductivity of the air, $Pr = \nu/\kappa$ and κ is the thermal diffusivity of the air. For the dimensions given above, this formula yields a value of about 25, which is in good agreement with the empirically determined coefficients.

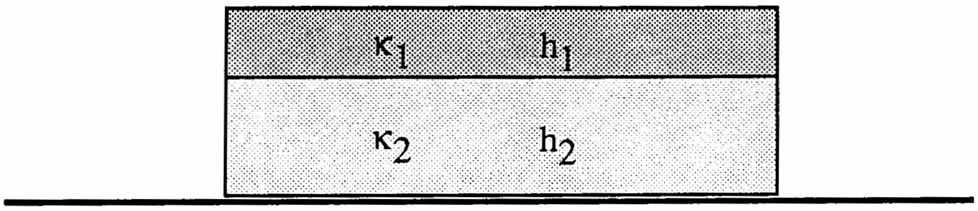


Figure 2: Parameters for horizontally layered food

This formula could be used in most situations but any empirical modification of it should be based more on the characteristics of the cooler (e.g. baffles included to make the air flow more turbulent) than on the characteristics of the food.

Another important parameter determining appropriate ways in which to model the cooling process is the Biot number

$$Bi = \frac{h_T L}{k_f}$$

which determines the relative importance of heat transfer from the air to the heat transfer within the food itself.

If the Biot number is small, $Bi \ll 1$, then a lumped-system approach can be used in which the rate of change of the total heat content of the food is set equal to the heat transfer at its surface. Such a zero-dimensional model requires only the solution of an ordinary-differential equation.

If the Biot number is close to unity or large $Bi \sim 1$ or $Bi \gg 1$ then modelling using partial-differential equations is necessary. In the case that $Bi \gg 1$ the model can be simplified by assuming that the surface temperature of the food is constant and equal to the temperature of the air.

Typical values of the Biot number for the foods that we considered during the week were about 2.

6 The structure of food – layered systems.

When calculating the cooling history of a composite food, it is extremely important to take account of the different thermal properties of the components of the food and the amount of space they occupy. For example, in a simple, horizontally layered food (figure 2), the ratio of timescales τ_1/τ_2, where $\tau_i = h_i^2/\kappa_i$, determines which of the layers is most significant and needs to be modelled most accurately, and the thermal centre of the food will be inside the layer with the largest value of τ_i.

This having been said, it is fairly straightforward to write a numerical code in conservative form that can take account of the different layers within the food. During the week, a code was written for one-dimensional situations and tested against known experimental data.

7 The numerical code

A widely used method for the one-dimensional linear heat equation with constant coefficients

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

is the *Crank-Nicholson scheme*

$$\rho c_p \frac{T_i^{n+1} - T_i^n}{\Delta t} - k \frac{T_{i+1}^{n+1/2} - 2T_i^{n+1/2} + T_{i-1}^{n+1/2}}{\Delta x^2} = 0$$

with

$$T^{n+1/2} := (T^{n+1} + T^n)/2,$$

which is stable for all time steps Δt and is second-order accurate. Although it is an implicit scheme it generates a tridiagonal matrix and thus the difference equations may be solved efficiently using the *Thomas algorithm.*

The Crank-Nicholson method may be adapted to allow ρ, c_p and k to be functions of x and T so that simulation of composite foods is possible. It is crucial that the new method may be written in *conservation form*

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \frac{F_{i+1/2} - F_{i-1/2}}{\Delta x} = 0$$

where $U_i = \rho(x_i, T_i)c(x_i, T_i)T_i$ is a numerical formulation of the conserved quantity, in this case enthalpy, and $F_{i+1/2}$ is the numerical heat flux from the cell centred on x_i to that centred on x_{i+1}. As well as conserving energy conservation form ensures that the correct boundary conditions will be applied automatically between layers where k, ρ and c_p have discontinuities.

A possible choice for $F_{i+1/2}$ is

$$-k\frac{\partial T}{\partial x}\bigg|_{i+1/2} \simeq F_{i+1/2} = -\frac{(k_{i+1} + k_i)}{2}\frac{(T_{i+1} - T_i)}{\Delta x}$$

For second-order accuracy F should be evaluated at the mid-step time by averaging F^n and F^{n+1}. Unfortunately this will lead to non-linear difference equations due to the dependence of k on the unknown T at the new timestep. An alternative is to perform a half timestep $\Delta t/2$ using k, c_p, ρ evaluated at $t = n\Delta t$ and use the result to approximate $k^{n+1/2}$. This will maintain second-order accuracy at the expense of calculating twice as many timesteps. The full scheme used to produce the results in this report is

$$\rho_i^n c_i^n \frac{T_i^* - T_i^n}{\Delta t/2} - \frac{k_{i+1/2}^n T_{i+1}^{n+1/4} - (k_{i-1/2}^n + k_{i+1/2}^n)T_i^{n+1/4} + k_{i-1/2}^n T_{i-1}^{n+1/4}}{\Delta x^2} = 0$$

with

$$T^{n+1/4} := (T^* + T^n)/2$$

and

$$k_{i+1/2} := (k_i + k_{i+1})/2$$

to calculate T^*, an approximation to T at $t = (n + 1/2)\Delta t$. This is followed by

$$\rho_i^* c_i^* \frac{T_i^{n+1} - T_i^n}{\Delta t} - \frac{k_{i+1/2}^* T_{i+1}^{n+1/2} - (k_{i-1/2}^* + k_{i+1/2}^*)T_i^{n+1/2} + k_{i-1/2}^* T_{i-1}^{n+1/2}}{\Delta x^2} = 0.$$

8 Phase Changes

For one-dimensional equations *front-tracking* methods are very effective at calculating the position of a phase change. However, these do not easily translate to 2D and 3D problems and the composite nature of foods means that there is often no sharp freezing front to track. Ideally, the enthalpy method would be used for this problem to allow for the latent heat released on freezing to be distributed over a temperature range. However, for an implicit solver this would lead to non-linear difference equations requiring iteration. This may be avoided by including the latent heat in the specific heat capacity of the food (see figure 3b) and simply calculating the temperature using the scheme described above. In the ideal case of a sharp phase change this would require a delta function to be included in the specific heat while for a real food the latent heat will be distributed over a larger temperature range. Moreover, if the specific heat capacity is smoothed further (while maintaining its total area) the numerical method may produce more accurate long-time temperature distributions at the expense of a loss of accuracy in the position of the phase boundary.

9 Results

The first test case was made for a chicken burger. This is a simple case in which the food is macroscopically homogeneous. The specific heat and conductivity of a chicken burger are given in figure 3. These were used as input data for the numerical model. The heat-transfer coefficient used in the model was adjusted so that the time to cool to 0°C agreed with experimental observations (figure 4). This one-point correlation yielded the excellent agreement for the cooling history shown in figure 4. Note that the heat-transfer coefficient obtained in this way was 21, which is close to the value 25 obtained from the formula given earlier. We kept the heat-tansfer coefficient fixed at 21 for the remainder of the trials, on the assumption that this is a property of the cooler.

The same code was used to predict the cooling history of a pizza base at two different air temperatures (figure 5a, b). No parameters were adjusted from the previous case of the chicken burger except of course that the known conductivity and specific heat capacity of a pizza base were used. Again there is very good agreement, which lends support to the idea that it is not necessary to adjust the heat-transfer coefficient between foods.

The final trial was for a composite pizza. Unfortunately, we did not have available the correct thermal properties of the topping, so the agreement found is very poor. (figure 6). In particular, the composition of the topping (especially its water content) has a significant influence on both the latent heat content and the thermal conductivity of the food. In the calculations, the thermal properties of the topping were taken to be those of minced fish, whereas the actual topping is likely to have been based on a tomato puree with much higher water content.

10 Conclusions

Simple numerical modelling, using a code in conservative form is sufficient to determine the cooling history of many foods.

It is of paramount importance to model the composite structure of foods appropriately in order to gain accurate predictions of the cooling history.

A simple heat-transfer law for laminar flow is probably adequate for the sorts of applications in mind. The heat-transfer coefficient is more a property of the cooler than of the food to be cooled.

Participants

Gail Duursma, Warren Smith, John Taylor, Grae Worster.

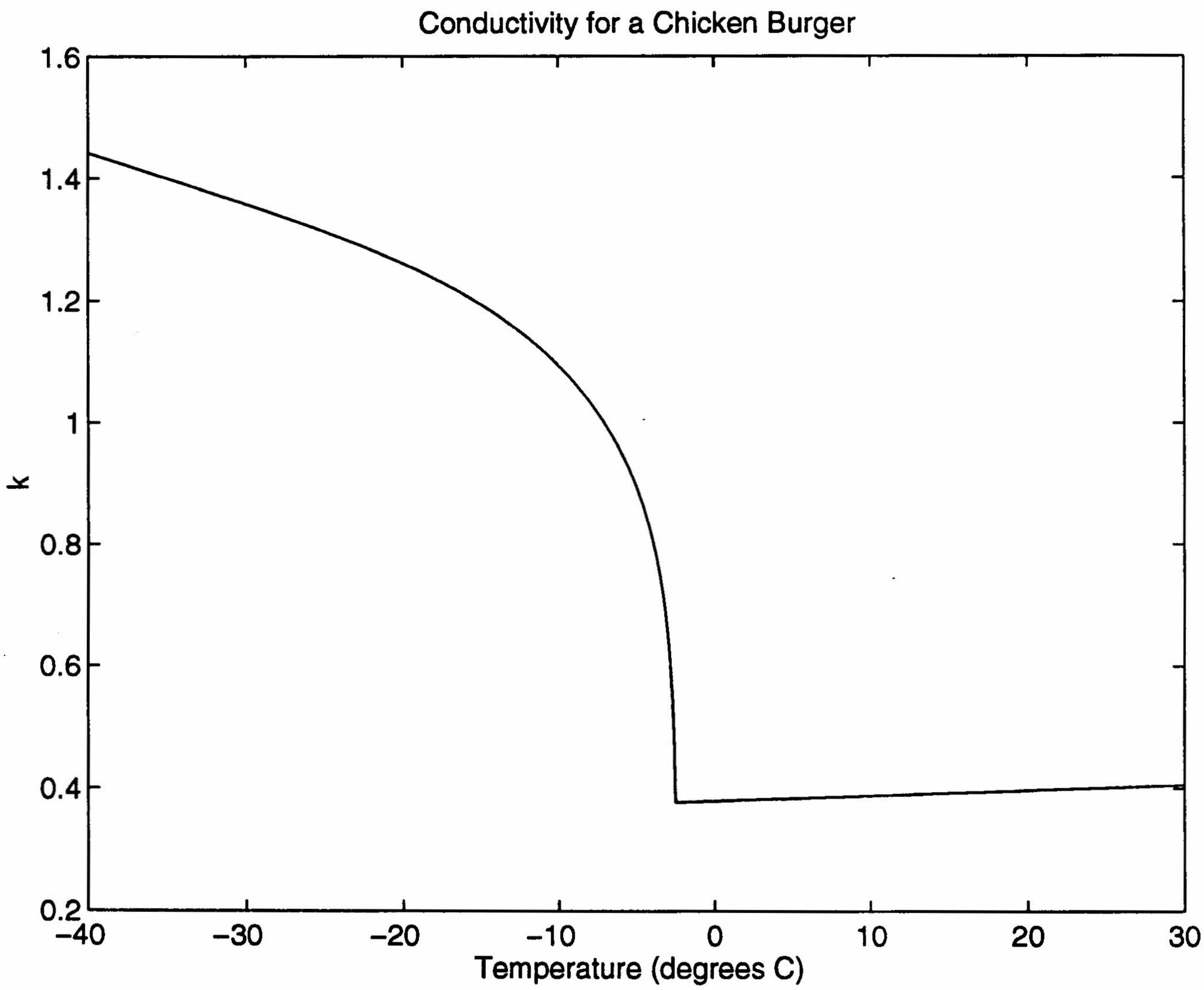
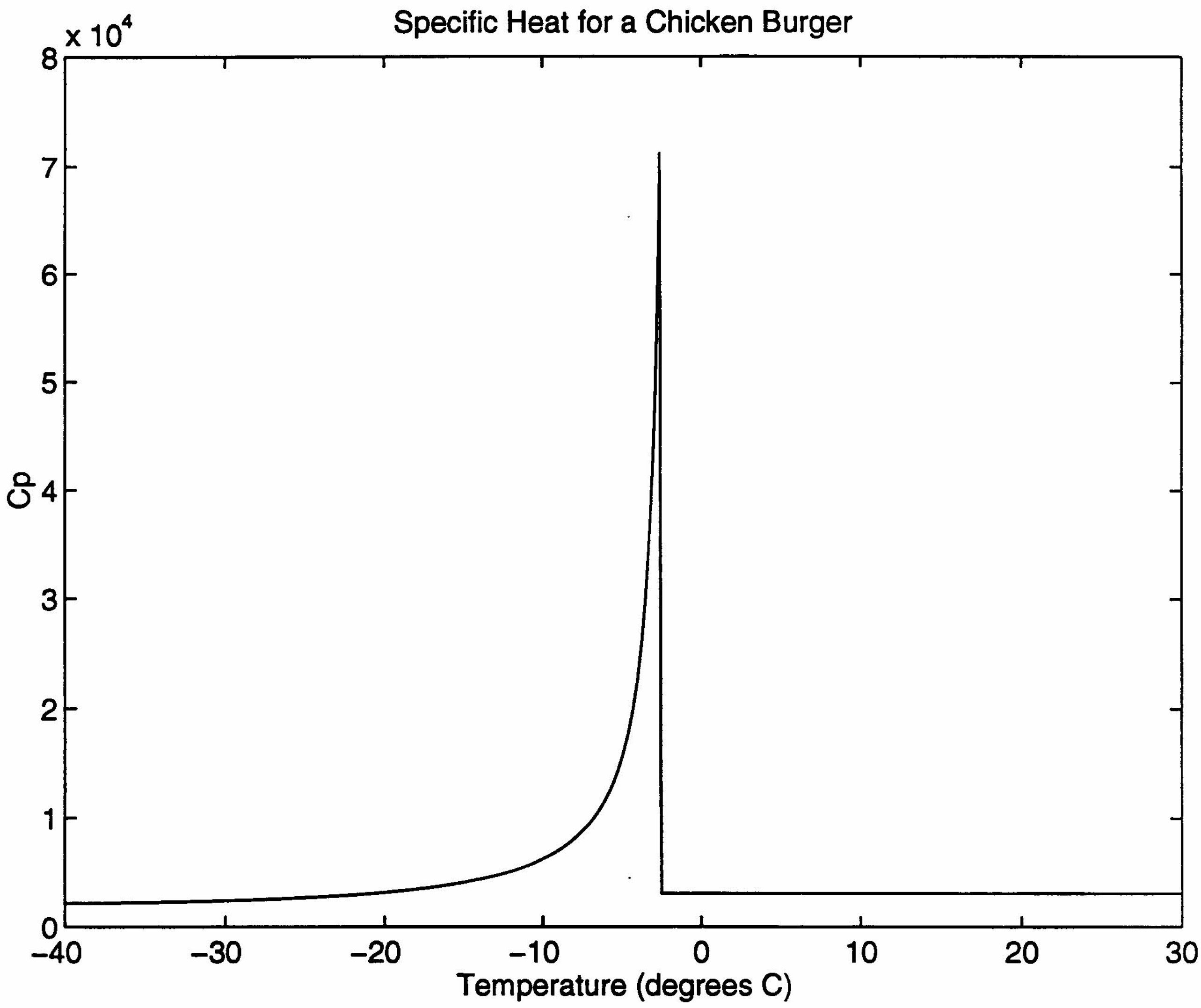

Conductivity for a Chicken Burger
1.6
1.4
1.2
1
k
0.8
0.6
0.4
0.2
−40
−30
−20
−10
0
10
20
30
Temperature (degrees C)

FIGURE 3a



Specific Heat for a Chicken Burger
x 10^4
8
7
6
5
4
3
2
1
0
Cp
-40
-30
-20
-10
0
10
20
30
Temperature (degrees C)

FIGURE 3b

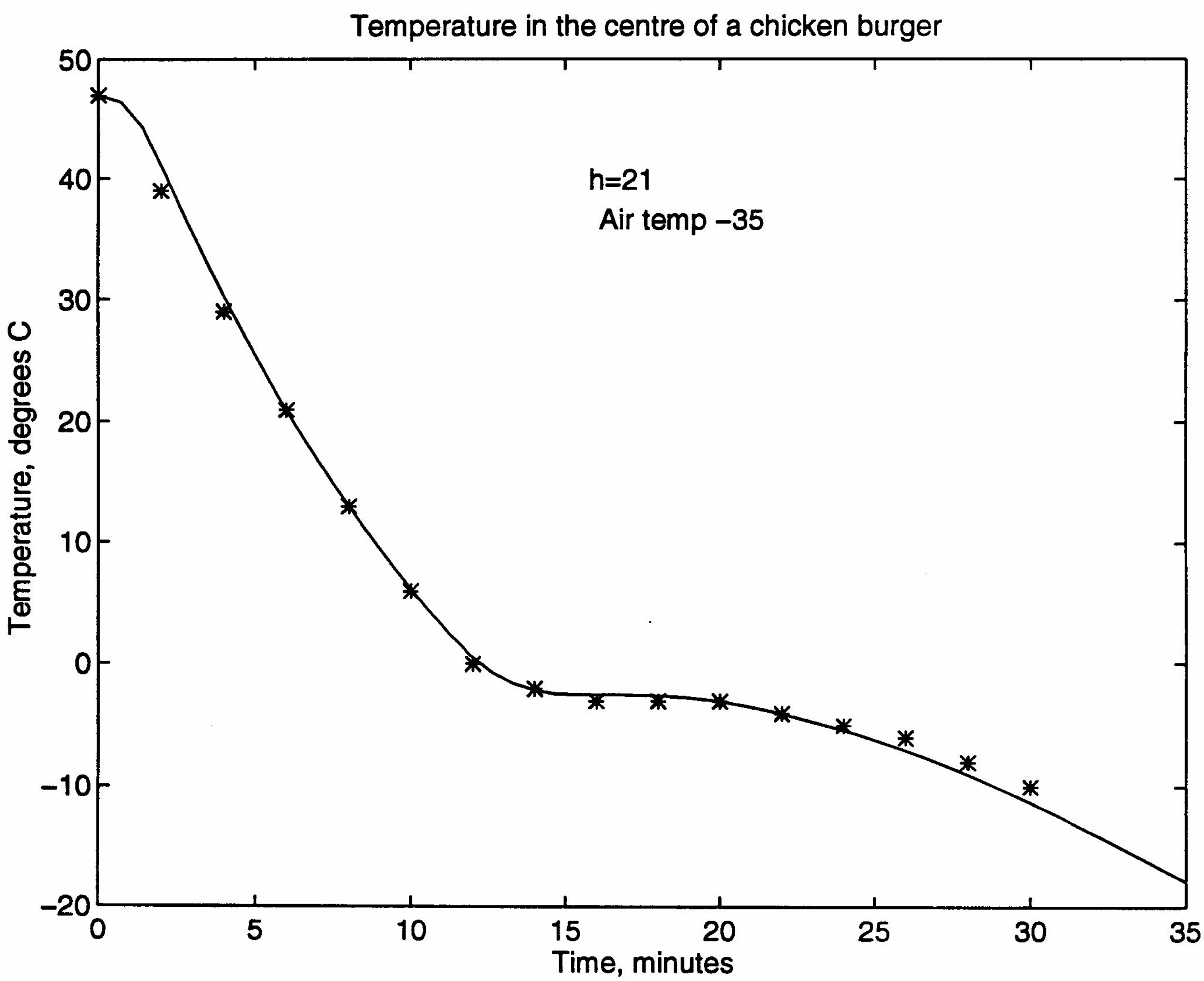

Temperature in the centre of a chicken burger
h=21
Air temp −35
Temperature, degrees C
Time, minutes
50
40
30
20
10
0
−10
−20
0
5
10
15
20
25
30
35

FIGURE 4

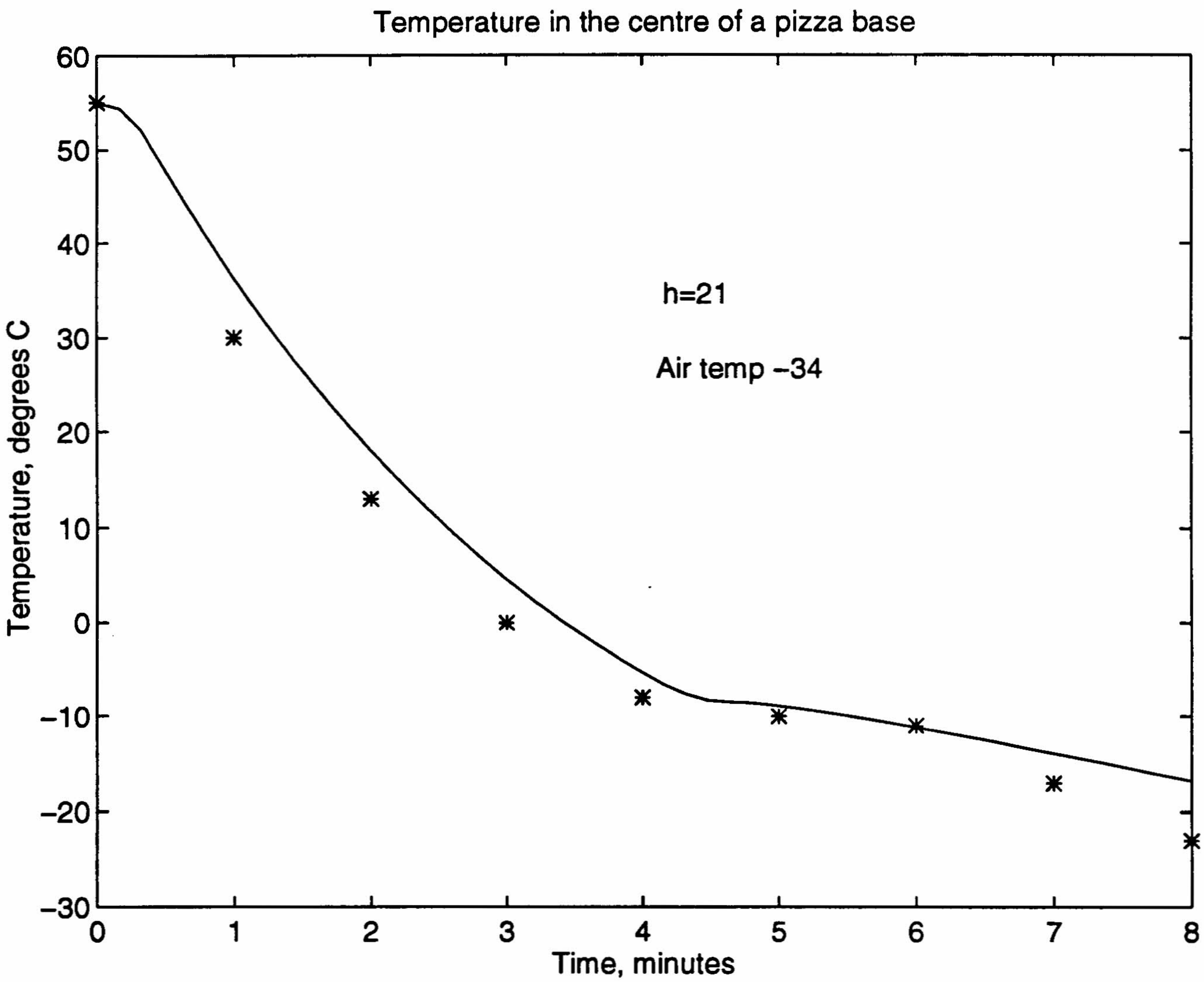

Temperature in the centre of a pizza base
h=21
Air temp −34
Temperature, degrees C
Time, minutes
60
50
40
30
20
10
0
−10
−20
−30
0
1
2
3
4
5
6
7
8

FIGURE 5a

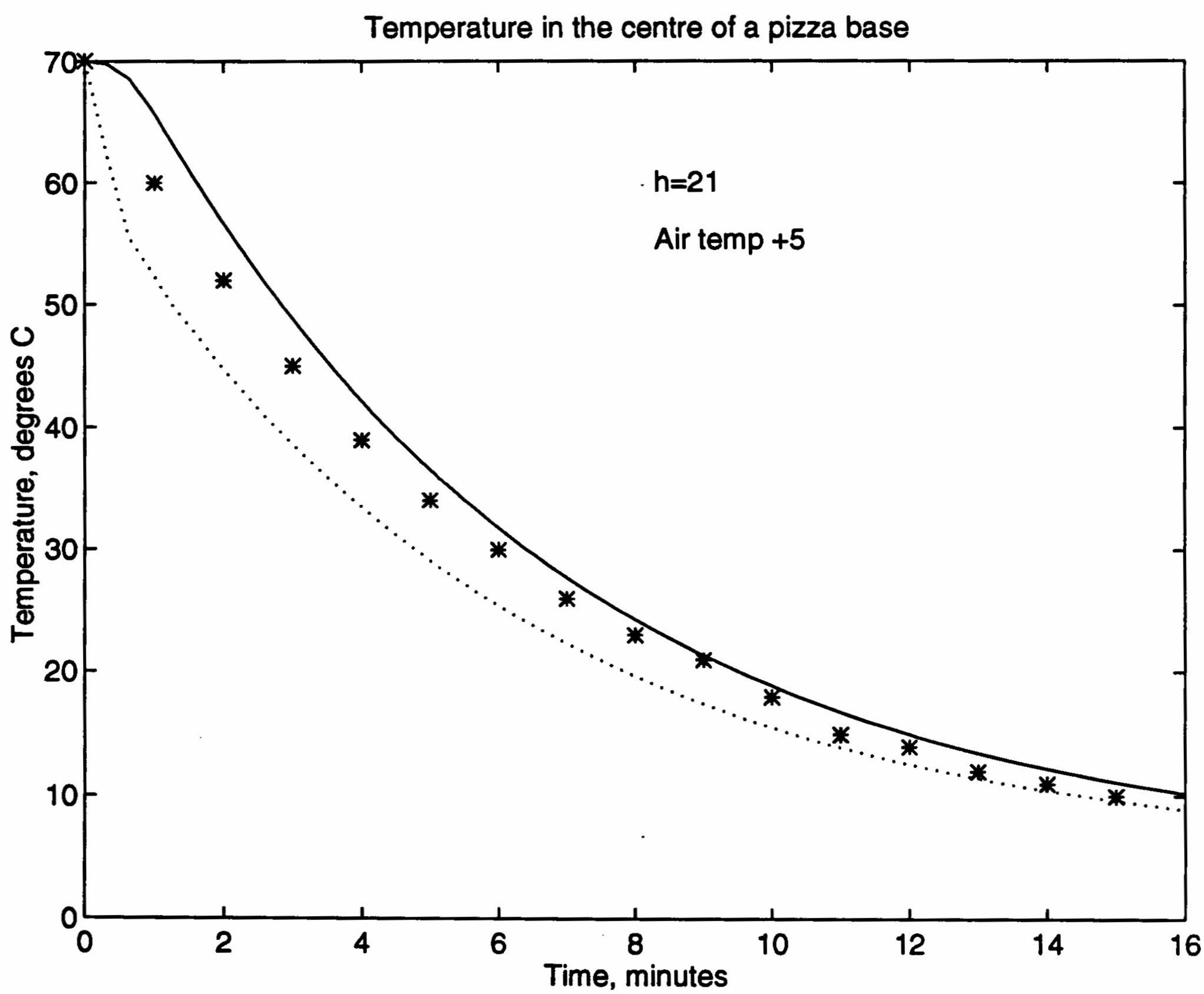

Temperature in the centre of a pizza base
h=21
Air temp +5
Temperature, degrees C
Time, minutes
0
10
20
30
40
50
60
70
0
2
4
6
8
10
12
14
16

FIGURE 5b

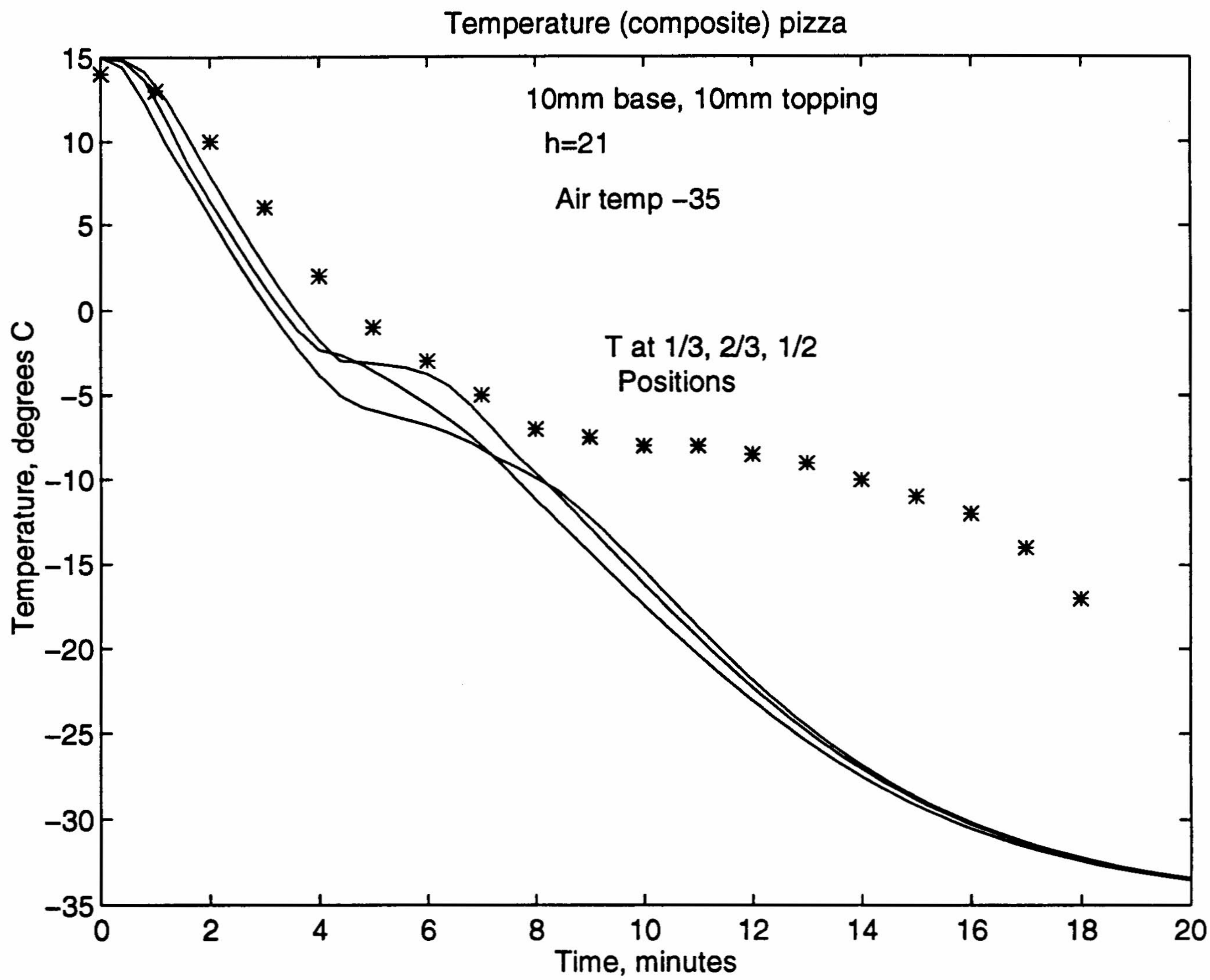

Temperature (composite) pizza
10mm base, 10mm topping
h=21
Air temp −35
T at 1/3, 2/3, 1/2
Positions
Temperature, degrees C
Time, minutes
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
0
2
4
6
8
10
12
14
16
18
20

FIGURE 6