SHELTER: HOMELESS POPULATIONS

1. OBJECTIVES

The aim was to derive and analyze a model for numbers of homeless and non-homeless people in a borough, in particular to see how these figures might be affected by different policies regarding housing various categories of people. Most attention was focused on steady populations although the stability of these and possible timescales of dynamic problems were also discussed.

2. THE MODEL

The population was not considered in detail but rather three main classes of household were considered: the homeless (registered as such and in temporary accommodation, e.g. temporary occupants of council housing not presently permanently occupied or in hostels), permanent occupants of council housing stock, and the remainder. The numbers of households of these categories in some boroughs are denoted by T, P and G respectively. The homeless were taken to be on the housing register for consideration towards permanent council housing. Numbers of families PR from those already in council houses and GR from the general population are also on the register (for changes of council houses - e.g. because of need for a larger home due to increased family size - and to move out of the private sector, respectively). The remaining numbers of council-housed and "general" households are PN and GN:

T = no. households in temporary accommodation (the homeless)

PR = no. households in council stock and seeking transfer

PN = no. households in council stock and not seeking transfer

GR = no. households in private sector and seeking council accommodation

GN = no. households in private sector and not seeking council accommodation

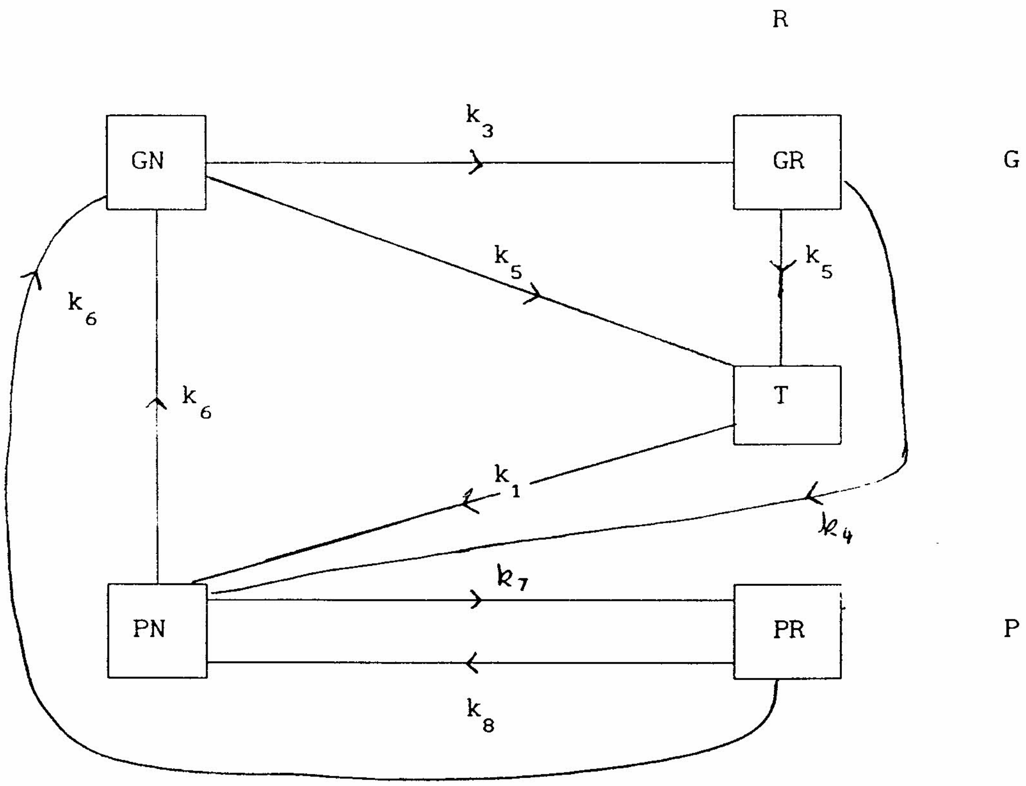
P = no. households in council stock = PR + PN (1)

G = no. households in private sector = GR + GN (2)

R = no. households on register = T + PR + GR (3)

The movement between the different categories is illustrated by figure 1.

Figure 1. Transfer between different types of household. The k's denote constants of proportionality governing the rates.



It is assumed that the homeless families only come from those in the private sector (councils will not evict tenants except for some misdemeanour, in which case the expelled parties are not not considered homeless) and that the rates are proportional to those populations, with the same rate constant k_5. The rates of moving from P to GN (due to improvement of circumstances), from GN to GP, and from PN to PR (perhaps some deterioration of conditions) are similarly taken to be proportional to the relevant populations (with constants of proportionality k_6, k_3, and k_7 respectively). The rates of rehousing were considered to be jointly proportional to the demand (GR, T, PR) and availability of council housing, $P_0 - P$, if P_0 = total housing stock (P = no. presently occupied), with respective constants k_4, k_1, k_8. (This law allows arbitrarily fast rates of rehousing. The inclusion of a factor inhibiting the speed could be considered.)

Movements from GR back to GN and from T directly into G were neglected.

With these laws the differential equations describing the rates of change of the five populations are:

$$\frac{d}{dt} GN = - k_5 GN - k_3 GN + k_6 P; \qquad (4)$$

$$\frac{d}{dt} GR = - k_5 GR + k_3 GN - k_4 (P_0 - P)GR; \quad (5)$$

$$\frac{dT}{dt} = k_5 G - k_1 (P_0 - P)T; \quad (6)$$

$$\frac{d}{dt} PN = - k_6 PN - k_7 PN + (P_0 - P)(k_4 GR + k_1 T + k_8 PR); \quad (7)$$

$$\frac{d}{dt} PR = - k_6 PR + k_7 PN - k_8 (P_0 - P)PR. \quad (8)$$

In writing down these equations it has been implicitly assumed that:

(i) birth and death rates can be neglected (these should probably be allowed for if time scales of the model are similar to, or longer than one generation);

(ii) there is no migration between boroughs;

(iii) the numbers of households are large enough for them to be considered as continuum variables; the values of T are certainly not particularly big in which case some discrete variation (possibly random) ought to be used;

(iv) rates depend only upon present circumstances and any delays (due, say, to administrative procedures) are of little importance;

(v) movement from T to G is negligible.

We shall make some further remarks concerning (i) (iv) and (v) later. For this study seasonal variation has been ignored. This would cause cyclic variation of the populations but it might be expected to otherwise give rise to similar behaviour. (Periodic solutions, which depend upon the various rate constants, can be approached in the long term, against steady states being limits of the populations.)

3. THE "RICH-BOROUGH" MODEL

For cases where the vast majority of the population is happily resident in private sector accommodation

$$GN \simeq G_0 = \text{total no. of households in the borough.}$$

Now, P, T, and GR are much smaller than $GN(\simeq G \simeq G_0)$ and eqn. (4) becomes redundant while (5) and (6) can be replaced by

$$\frac{d}{dt} GR = - k_5 GR + k_3 G_0 - k_4 (P_0 - P)GR; \quad (9)$$

$$\frac{dT}{dt} = k_5 G_0 - k_1 (P_0 - P)T. \quad (10)$$

Steady solutions are then given by

$$k_5 GR + k_4(P_0 - P)GR = k_3 G_0; \qquad (11)$$

$$k_5 G_0 = k_1(P_0 - P)T; \qquad (12)$$

$$k_6 P = (P_0 - P)(k_4 GR + k_1 T); \qquad (13)$$

the last being given by (7) + (8) with $\frac{dP}{dt} = 0$. Eqn. (12) represents balance between families becoming homeless and getting rehoused: this housing rate appears in (13), is just $k_5 G_0$, and is independent of k_1!. Then (13) is

$$k_6 P = k_4(P_0 - P)GR + k_5 G_0. \qquad (14)$$

Adding (11) and (14),

$$k_5 GR + k_6 P = (k_3 + k_5)G_0 \qquad (15)$$

so $k_5 k_6 P = k_4(P_0 - P)[(k_3 + k_5)G_0 - k_6 P] + k_5^2 G_0$.

This quadratic equation,

$$k_4 k_6 P^2 - [k_5 k_6 + k_4 k_6 P_0 + k_4(k_3 + k_5)G_0]P + [k_4(k_3 + k_5)P_0 + k_5^2]G_0 = 0, \qquad (16)$$

has one positive root (all the constants are positive) less than P_0 (no. of families housed should be less than no. of houses) provided that

$$k_5 G_0 < k_6 P_0,$$

i.e. the rate at which households become homeless is no greater than the maximum possible rate at which council houses are vacated.

(A stronger condition is needed to ensure a sensible solution of (15). It can be assumed that in a "rich borough" families can be rehoused and move back into satisfactory private property rapidly compared with their transfer into temporary housing and onto the register:

$$k_1 P_0,\ k_4 P_0,\ k_6 \gg k_3,$$

k_5.

Also, as previously noted, $GN \simeq G_0$ is much greater than GR etc. This suggests a rescaling, $G_0 = N/\varepsilon$, $k_3 = \varepsilon K_3$, $k_5 = \varepsilon K_5$, for some small number ε. Eqn. (16) be becomes

$$k_4\{k_6P^2 - [k_6P_0 + k_4(K_3 + K_5)N]P + k_4(K_3 + K_5)P_0N\} - \varepsilon(K_5k_6P - K_5^2N) = 0.$$

Neglecting the ε term the equation becomes

$$(k_6P - (K_3 + K_5)N)(P - P_0) = 0$$

with a root $P_0 = (K_3 + K_5)N/k_6 < P_0$ provided that

$$P_0 > (K_3 + K_5)N/k_6 = (k_3 + k_5)G_0/k_6;$$

the total rate at which households come on to the housing register is less than the maximum rate at which houses are vacated. Including the neglected $-\varepsilon(K_5k_6P - K_5^2N)$ term gives a negative correction to P, thus ensuring that GR is positive (and small compared to G_0).)

As well as having

$$GR = \frac{(k_3 + k_5)G_0 - k_6P}{k_5}$$

other numbers of interest are:

$$PR = k_7P/(k_6 + k_7 + k_8P_0 - k_8P),$$

$$PN = (k_6 + k_8P_0 - k_8P)P/(k_6 + k_7 + k_8P_0 - k_8P),$$

$$\text{and}\quad T = k_5G_0/k_1(P_0 - P).$$

It is observed that k_1 only appears in the expression for T. This is a consequence of the fact that in the steady equations k_1 and T only appear multiplied together (with $(P_0 - P)$) - it is the rate $k_1(P_0 - P)T = k_5G_0$ which really plays a role. It follows that a reduction in k_1 (giving less priority to housing the homeless) simply has the effect of increasing the number in temporary accommodation.

Regarding waiting times on the register, those for GR and PR are $1/k_4(P_0 - P)$ and $1/[k_8(P_0 - P) + k_6]$, respectively, and are also unaffected by any change of k_1, whereas for T the corresponding time is $1/k_1(P_0 - P)$, again with the reciprocal dependence upon the constant k_1.

Linearizing the dynamic problem about this steady state it was found that it was stable: different starting populations give rise to the same long term static values.

4. THE "FULL EQUATIONS"

Returning to the system (4) - (8) it can be observed that the right-hand sides sum to zero (flow into one category matches flow out of another). Hence

$$GN + GR + T + PN + PR = G + T + P = \text{constant}$$

$$= G_0 = \text{total no. of households in the borough.}$$

It is then possible to reduce the system of five differential equations to one of four by, for example, writing

$$GN = G_0 - P - T - GR \tag{17}$$

in (5) to give

$$\frac{d}{dt} GR = - k_5 GR + k_3 (G_0 - P - T - GR) - k_4 (P_0 - P)GR \tag{18}$$

as well as

$$G = G_0 - P - T \tag{19}$$

in (6).

Also suming (7) and (8) leads to

$$\frac{dP}{dt} = - k_6 P + (P_0 - P)(k_4 GR + k_1 T). \tag{20}$$

The system is now one of three ordinary differential (6), (18), (20). The sub-categories PN and PR can be determined from

$$\frac{d}{dt} PR = - k_6 PR + k_7 PN - k_8 (P_0 - P)PR$$

$$= k_7 P - [k_6 + k_8 (P_0 - P) + k_7] PR. \tag{21}$$

Equilibrium solutions are found by setting the left-hand sides of (6), (18), (20) (and (21)) to zero. This now results in a cubic equation for P, on eliminating GR and T:

$$T = \frac{k_5 (G_0 - P)}{k_5 + (P_0 - P)k_1};$$

$$GR = \frac{k_6 P[k_5 + (P_0 - P)k_1] - k_1 k_5 (G_0 - P)(P_0 - P)}{k_4 (P_0 - P)[k_5 + k_1 (P_0 - P)]}$$

$$\frac{k_4 k_6}{k_3 + k_5} P(P_0 - P)[k_5 + k_1(P_0 - P)] - k_4(G_0 - P)(P_0 - P)[k_5 + k_1(P_0 - P)]$$

$$+ k_6[k_5 + k_1(P_0 - P)]P + k_5(k_4 - k_1)(G_0 - P)(P_0 - P) = 0. \qquad (22)$$

Since the left-hand side $= - k_1 G_0 P_0 (k_4 P_0 + k_5) < 0$ for $P = 0$

and $= k_5 k_6 P_0 > 0$ for $P = P_0$

there is always a feasible solution P (one with the number of occupied council houses being positive but no greater than that actually available).

It appeared unproductive to try to proceed further with the model in its full generality during the Study Group so attention focused on a case with the constants suggested by the figures and estimates supplied.

5. NUMERICAL SOLUTION

More quantitative consideration was based upon the following information for a steady(ish) situation in a "typical" English metropolitan borough ($\frac{d}{dt}GR = \frac{dT}{dt} = \ldots. = 0.$)

Population = 210,000 people.

Taking an average of 3 people/household, this gives

$$G_0 = 7 \times 10^4 \text{ house} \qquad (23)$$

$$\text{Council houses} = P_0 \simeq 1.8 \times 10^4 \text{ house.} \qquad (24)$$

Annual relet rate $= (P_0 - P)(k_4 GR + k_1 T + k_8 PR)$

$$= k_6 P + k_8(P_0 - P)PR \simeq 1.3 \times 10^3 \text{ house yr}^{-1}. \qquad (25)$$

Annual transfer rate = annual rate of arrival on register

- rate of going instead into private accommodation

$= k_8(P_0 - P)PR$

$$= k_7 PN - k_6 PR \simeq 300 \text{ house yr}^{-1}\ . \qquad (26)$$

Annual lets to homeless and others

$$= (P_0 - P)(k_4 GR + k_1 T) \simeq 10^3 \text{ house yr}^{-1}. \qquad (27)$$

Annual lets to homeless = annual rate of families becoming homeless

$$= k_1(P_0 - P)T = k_5 G \simeq 330 \text{ house yr}^{-1}. \qquad (28)$$

So annual lets to others = annual rate of registering - rate of becoming homeless

$$= k_4(P_0 - P)GR = k_3 GN - k_5 GR \simeq 670 \text{ house year}^{-1}. \quad (29)$$

Families in temporary accommodation = T = 35 house. (30)

New non-homeless applicants on housing list = GR = 4.5 × 10^3 house. (31)

Transfer applicants = PR ≃ 3 × 10^3 house. (32)

No. unoccupied council houses (1.9% - to be expected small!)

$$= P_0 - P \simeq 330 \quad (33)$$

From (28), (30), (33) $k_1 \simeq \dfrac{330}{330 \times 35} \simeq 3 \times 10^{-2} \text{ house}^{-1} \text{ yr}^{-1}$.

From (29), (31), (33) $k_4 \simeq \dfrac{670}{330 \times 4.5 \times 10^3} \simeq 4 \times 10^{-4} \text{ house}^{-1} \text{ yr}^{-1}$.

From (26), (32), (33) $k_8 \simeq \dfrac{300}{330 \times 3 \times 10^{-2}} \simeq 3 \times 10^{-4} \text{ house}^{-1} \text{ yr}^{-1}$.

Also $P \simeq P_0 = 1.8 \times 10^4$ house, (34)

$PN = P - PR \simeq 1.5 \times 10^4$ house, (35)

$G \simeq G_0 - P \simeq 5.2 \times 10^4$ house, (36)

and $GN = G - GR \simeq 4.7 \times 10^4$ house. (37)

Using (28), (36), $k_5 \simeq \dfrac{330}{5.2 \times 10^4} \simeq 6 \times 10^{-3} \text{ yr}^{-1}$,

and then, from (29), (37), $k_3 \simeq \dfrac{(670 + 30)}{4.7 \times 10^4} \simeq 1.5 \times 10^{-2} \text{ yr}^{-1}$.

Returning to (25) and employing (34), $k_6 \simeq \dfrac{10^3}{1.8 \times 10^4} \simeq 5 \times 10^{-2} \text{ yr}^{-1}$.

Finally, (26), with (32) and (35) gives $k_7 \simeq \dfrac{300 + 150}{1.5 \times 10^4} = 3 \times 10^{-2} \text{ yr}^{-1}$.

These figures are very approximate. During the Study Group calculations were carried out with numbers of people rather than families (three people/household) and the following values were found and used:

$$
\left.
\begin{aligned}
G_0 &= 2 \times 10^5 \text{ people} \\
P_0 &= 6 \times 10^4 \text{ people} \\
k_1 &= 10^{-2} \text{ people}^{-1} \text{ yr}^{-1} \\
k_3 &= 1.5 \times 10^{-2} \text{ yr}^{-1} \\
k_4 &= 10^{-4} \text{ people}^{-1} \text{ yr}^{-1} \\
k_5 &= 5 \times 10^{-3} \text{ yr}^{-1} \\
k_6 &= 5 \times 10^{-2} \text{ yr}^{-1} \\
k_7 &= 3 \times 10^{-2} \text{ yr}^{-1} \\
k_8 &= 10^{-4} \text{ people}^{-1} \text{ yr}^{-1}.
\end{aligned}
\right\} \qquad (38)
$$

Taking these values gave, by solving (22), steady values of

$$
\left.
\begin{aligned}
&P \simeq 5.6 \times 10^4 \text{ people, } GR \simeq 5.2 \times 10^3 \text{ people, } T \simeq 18 \text{ people,} \\
&PR \simeq 3.5 \times 10^3 \text{ people, } GN \simeq 1.4 \times 10^5 \text{ people, } R \simeq 8.7 \times 10^3 \text{ people,} \\
&(P_0 - P)/P \simeq 0.07 \ (7\%).
\end{aligned}
\right\} \qquad (39)
$$

These figures can be seen to differ substantially from the original data. The difficulty appears to lie with the cubic equation (22). Its three roots lie quite close together (one just less than P_0 and the others just above). The consequent sensitivity on the data means that the determination of the constants should be carried out with more care and accuracy.

A local analysis indicated that the steady state is stable. Numerical simulation of the differential equations (4) - (8) or, equivalently, (16), (18), (20) and (21) using the values (38) was carried out; see figure 2.

It should be observed that there is a very rapid initial change but the populations only settle down on their steady values over a time scale of 30 years. This suggests that the dynamics are quite important (policies may change a few times in such a period) and that births, deaths, new families, ... should also be taken into account.

Computations were also done, starting at the above steady state, but with constant k_1 reduced by a factor of ten ($k_1 = 10k_4$) so much less priority is given to the homeless; see figure 3.

The graphs indicate very little change, except that the number of homeless rapidly rise to ten times the previous value (consistent with the "rich-borough" model).

The final numerical solution, figure 4, was the "fair" case of $k_1 = k_4 = k_8 = 10^{-4}$ people^{-1} yr^{-1}.

Again the populations changed little, except that there was another eventual ten-fold increase in the number temporarily housed, T. The amount of "vacant" council property, $P_0 - P$, which can be used to accommodate the homeless temporarily, was seen to increase a little, but not sufficiently to cope with the change in homeless families. (The model allows for only households in permanent accommodation to move into satisfactory private property.)

6. ASYMPTOTIC SOLUTION

The constants used (and the numerical solution) indicate some substantially different sizes of populations and time scales. To try and exploit these variations to achieve approximate solutions the equations were scaled to identify useful large or small parameters:

$$P = (1 - p)P_0, \quad G = (k_4P_0/k_1)g, \quad GR = (k_4P_0/k_1)h, \quad t = \tau/k_6;$$

$$\frac{dg}{d\tau} = -\frac{k_5}{k_6}g + \frac{k_1}{k_4}(1 - p) - \frac{k_4P_0}{k_6}pg\ ;$$

$$\frac{dh}{d\tau} = -\frac{k_5}{k_6}h + \frac{k_3}{k_6}(g - h) - \frac{k_4P_0}{k_6}pg\ ;$$

and $$\frac{dp}{d\tau} = 1 - p - \frac{k_4P_0}{k_6}\left[\frac{G_0-P_0}{P_0} + P + \frac{k_4^2}{k_1^2}h - \frac{k_4}{k_1}g\right]p.$$

Setting $\alpha = k_4/k_1$, $\beta = k_5/k_6$, $\gamma = k_3/k_6$, $\delta = (G_0 - P_0)/P_0$, $\mu = k_1P_0/k_6$, these dimensionless equations become:

$$\frac{dp}{d\tau} = 1 - p - \mu(\delta + p + \alpha^2 h - \alpha g)p; \qquad (39)$$

$$\frac{dg}{d\tau} = \frac{1-p}{\alpha} - \beta g - \alpha\mu ph; \qquad (40)$$

$$\frac{dh}{d\tau} = \gamma g - (\beta + \gamma)h - \alpha\mu ph. \qquad (41)$$

The parameters are, according to (38),

$$\alpha \simeq 10^{-2}, \ \beta \simeq 10^{-1}, \ \gamma \simeq 3 \times 10^{-1}, \quad \delta \simeq 2, \quad \mu \simeq 10^4.$$

The last is the most extreme and can be exploited to simplify the problem. The appearance of μ terms (not multiplied by α) in (39) indicates a rapid change (over a time scale of a day or so) to

$$p \sim \alpha(g - \alpha h) - \delta.$$

This value can be substituted into (40) and (41) to give

$$\frac{dg}{d\tau} \sim \frac{1-p}{\alpha} - \beta g - \alpha\mu[\alpha(g - \alpha h) - \delta]h$$

$$\text{and} \quad \frac{dh}{d\tau} \sim \gamma g - (\beta + \gamma)h - \alpha\mu[\alpha(g - \alpha h) - \delta]h.$$

The large size of $\alpha\mu$ in the latter can then be exploited to give a fairly rapid (months) change of h to

$$h \sim \gamma g/\alpha\mu[\alpha(g - \alpha h) - \delta] \sim \gamma(\alpha g)/(\alpha^2\mu)((\alpha g) - \delta)$$

$$\text{and then} \quad \frac{d}{d\tau}(\alpha g) \sim 1 - (\alpha g) + \delta - \beta(\alpha g) - \gamma(\alpha g)$$

$$= 1 + \delta - (1 + \beta + \gamma)(\alpha g).$$

So $\alpha g \to (1 + \delta)/(1 + \beta + \gamma)$ as $\tau \to \infty$, the approach being as $\exp[-(1 + \beta + \gamma)\tau]$ (a decay time of about 15 years).

This procedure is better carried out by taking $\alpha \ll 1$, $\mu = \lambda/\alpha^2$ and $g = n/\alpha$.

The steady g can be used to determine the other static values.

$$h \sim \gamma(1 + \delta)/\alpha^2\mu(1 - \delta(\beta + \gamma)), \quad p \sim (1 - \delta(\beta + \gamma))/(1 + \beta + \gamma).$$

These can also be found from the nondimensional version of (22):

$$(\beta + \gamma)(\beta + \alpha\mu p)[(1 - p)(1 + p\mu) - \mu(1 + \delta)p]$$

$$- \alpha\gamma\mu p(1 - p) + \mu p(1 - p)(\beta + \gamma + \alpha\mu p) = 0.$$

For $p = 0$ the left-hand side $= \beta(\beta + \gamma) > 0$, for $p = 1$ the left-hand side $= - \mu(1 + \delta)(\beta + \gamma)(\beta + \alpha\mu) < 0$, for $p = 0$ $\frac{d}{dp}$ (left-hand side) $= (\beta + \gamma)[- \beta(1 + \mu\delta) + \alpha\mu] + \mu(\beta + \gamma - \alpha\gamma) \simeq \mu(\beta + \gamma)(1 - \beta\delta) > 0$ for the present values, and there is precisely one root between 0 and 1 (and no solution greater than 1).

The positive root is $p \sim (1 - \delta(\beta + \gamma))/(1 + \beta + \gamma)$, as long as $\delta < 1/(\beta + \gamma)$.

Note that k_1 only appears through α and μ so that:

p is, to leading order, independent of k_1;

g and h are approximately proportional to k_1 so G and GR are, to leading order, independent of k_1.

Turning our attention to the homeless,

$$T = \frac{k_5(G_0 - P)}{k_5 + (P_0 - P)k_1} = \frac{P_o(\delta + p)}{1 + \mu p/\beta} \sim \frac{\beta P_0(\delta + p)}{\mu p}$$

$$\sim \frac{\beta P_0(1 + \delta)}{\mu(1 - \delta(\beta + \gamma))} = \frac{k_5 k_6}{k_1[k_6 P_0 - (k_3 + k_5)(G_0 - P_0)]},$$

and is again inversely proportional to k_1 (approximately).

The insensitivity of all the steady values, other than of T, to the crucial homeless-housing-rate constant k_1 indicates the relative smallness of numbers of homeless. Should k_1 be reduced to such an extent that T becomes a significant part of the total population then it might have a more noticeable effect on the other category sizes.

7. OTHER MODELS

(i) Councils may wish to put extra effort into housing homeless families if T gets above a certain size. Ideally this would mean constraining T but the procedure could be represented by replacing k_1T by $k_1T + \hat{k}_1T^2$.

(ii) Administrative delays can be important. These might be represented by replacing terms in flow rates such as $P(t)$ by $P(t - t_0)$ with t_D = delay time. One early model did try to model this effect but extra care needs to be taken to ensure that no quantity becomes negative.

This variation would change the dynamics but leave the steady states unaltered.

(iii) One possible reading of the new legislation is that after providing one year of temporary housing a council will have discharged its duty to a homeless family. If the council gives automatic reassessment as homeless to such families the model considered here can still apply. However, should one apply the regulations very stringently, and deny further temporary accommodation, some of the equations, particularly that concerning the homeless population, need substantial modification.

Writing $H(t,\tau)$ = "density" of homeless at time t who became homeless at time τ, $t - t_D < \tau < t$, (t_D = one year for the "one-year rule"),

$$T(t) = \int_{t-t_D}^{t_D} H(t,\tau)d\tau = \text{total no. homeless households},$$

$\frac{\partial H}{\partial t} = -k_1(P_0 - P)H = -$ rate of rehousing for the families who became homeless at time τ, $\tau < t < \tau + t_D$,

$$H(\tau,\tau) = k_5 G(\tau) = \text{rate of becoming homeless at } t = \tau,$$

and now $\frac{d}{dt} GN = - k_5 GN - k_3 GN + k_6 P + H(t, t - t_D)$, where the additional and final term represents the rate at which still-homeless families leave their temporary housing compulsorily. (This term could alternatively be included in the GR equation.)

(iv) Councils may aim to relet houses a set time t_D after they become vacant (typically a few weeks). They may be let to households in different categories according to some weighting (assigned priority).

If $V(t)$ = rate of houses becoming vacant, and hence ready for reletting at time $t + t_D$,

$$V = k_6 P + w_8 PR\, V(t - t_D)/S,$$

$$\frac{d}{dt} GR = - k_5 GR + k_3 GN - w_4 GR\, V(t - t_D)/S$$

$$\frac{dT}{dt} = k_5 GR - w_1 TV(t - t_D)/S$$

$$\frac{d}{dt} PN = - k_5 PN - k_7 PN + V(t - t_D),$$

and $$\frac{d}{dt} PR = - k_6 PN + k_7 PN - w_8 PR\, V(t - t_D)/S$$

where $S = w_4 GR + w_1 T + w_8 PR$. The w's are (constant) weights and their relative sizes indicate the priority given to housing the three categories on the register.

(It is not immediately obvious that P cannot exceed P_0, the total available council stock, especially as P_0 no longer appears in the differential equations. This difficulty can be resolved by recasting the system in a similar form to the model (iii). Letting $E(t,\tau)$ be the "density" of houses at time t which became empty at time τ, $t - t_D < \tau < t$,

$E(\tau,\tau)$ = rate of vacancy at time τ

$$= V(\tau) = k_6 P(\tau) + w_8 PR(\tau)E(\tau,\tau - \tau_D)/S(\tau) > 0,$$

$\frac{\partial E}{\partial t} = 0$, so $E(t,\tau) = V(\tau)$ = rate at which houses became empty for $\tau \le t \le \tau + \tau_D$, and in particular,

$$V(t - t_D) = E(t, t - t_D).$$

Then $P(t) + \int_{t-t_D}^{t} E(t,\tau)d\tau = P_0$. This condition should be necessary to fix steady states.)

(v) Discrete, rather than continuous, time may be used. This could again change the dynamics but leave steady states unaltered. Delays might be more easily incorporated using this approach (taking step length equal to the delay).

(vi) Allowance for difference types (sizes, ages ....) of households might be made. Although this would make models more accurate there would be considerable complication.

8. CONCLUSIONS

The main outcome of this brief study is the identification of the key role played by the constant k_1 - the constant which fixes the speed at which the homeless are rehoused in permanent council property. Reducing this constant, i.e. making the system "fairer" with less priority to accommodating homeless families, appears to have little effect on the sizes of other categories on the waiting list but there is a marked increase in the number of households in temporary accommodation.

The model, indicated by the size of its longest time-scale, should be modified to allow for births etc..

It could be varied by including flows for GR to GN (people removing themselves from the register) or by allowing the rates at which registered and unregistered people become homeless to differ, but these modifications are unlikely to substantially change the main result.

The inclusion of movement from the homeless to the general population (from T to G) would have the effect of limiting the numbers in temporary accommodation. However, it is thought this effect is very small so a great reduction in k_1 (and vast swelling of T) would be needed for this flow to become significant.

John Byatt-Smith, Craig Glasgow, Michael Grinfeld, Shirley Harris, Andrew Lacey, Stefan Llewellyn Smith, Tim Lowe, Christos Nikolopoulos, Richard Norgate, David Parker, Kevin Parrott, Colin Please, Warren Smith, Angeli Thomis, Jonathan Wattis

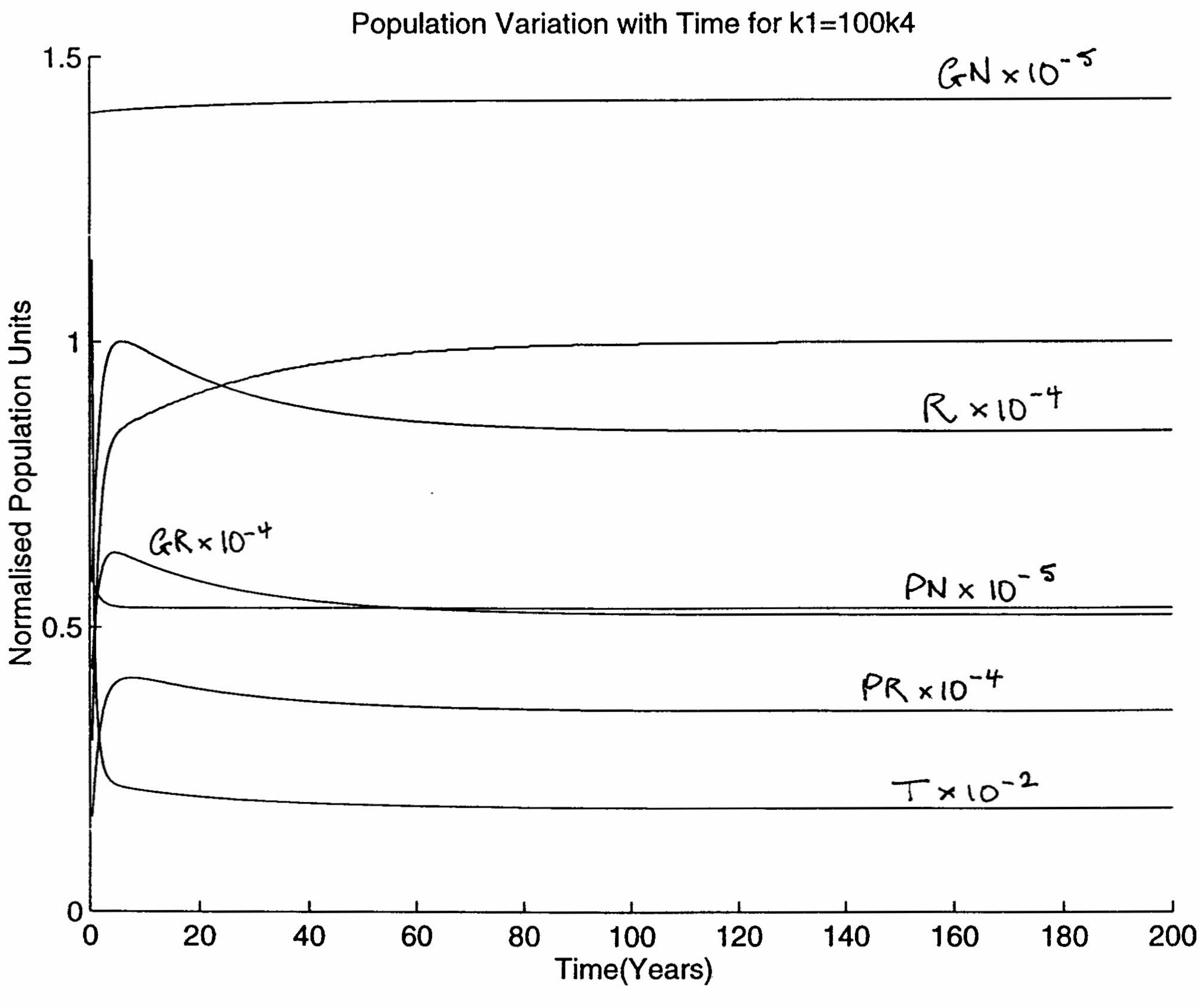


Fig 2 : Steady-state solution timescale.

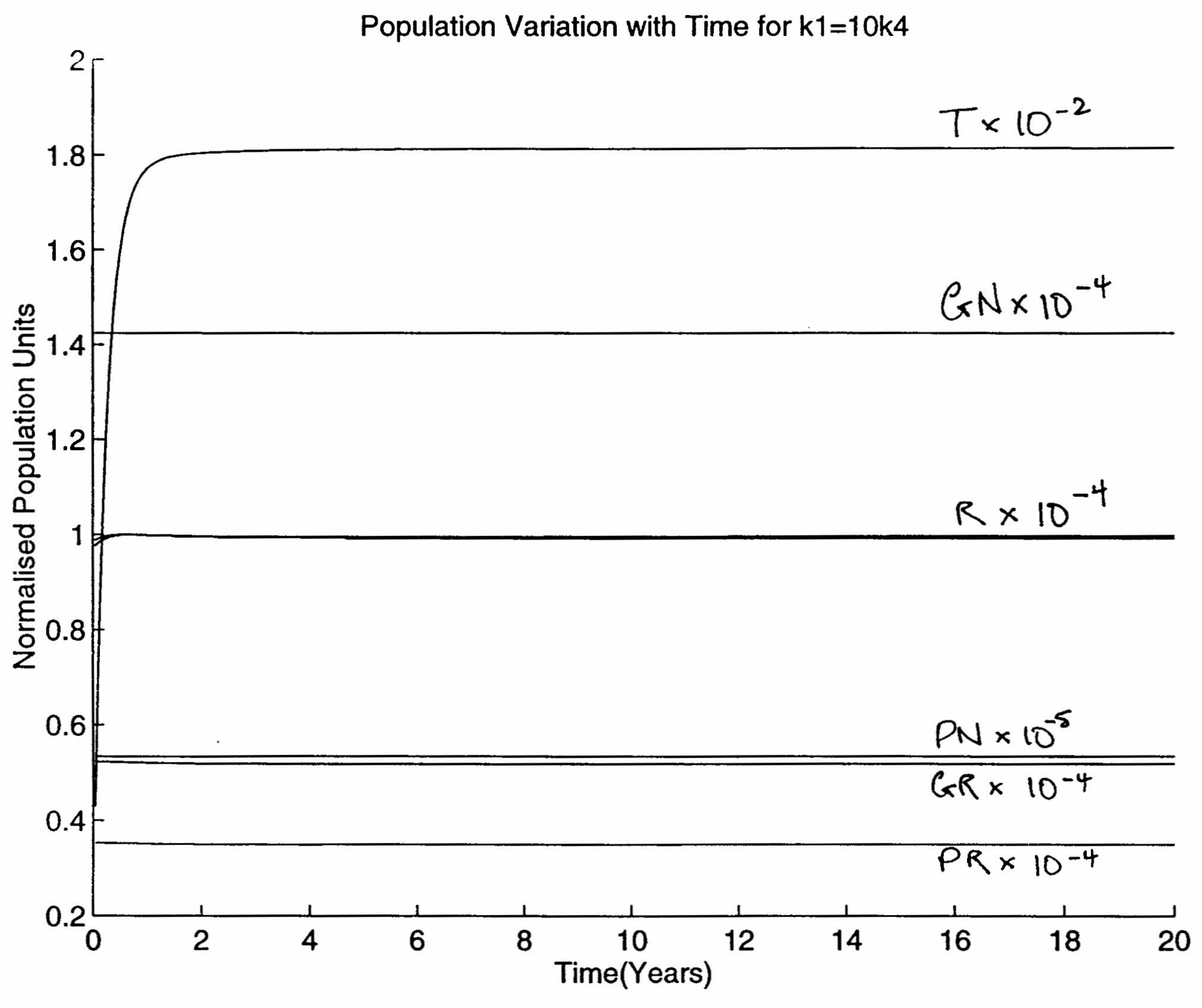


Fig 3 : Population trends with $k_1 = 10\,k_4$

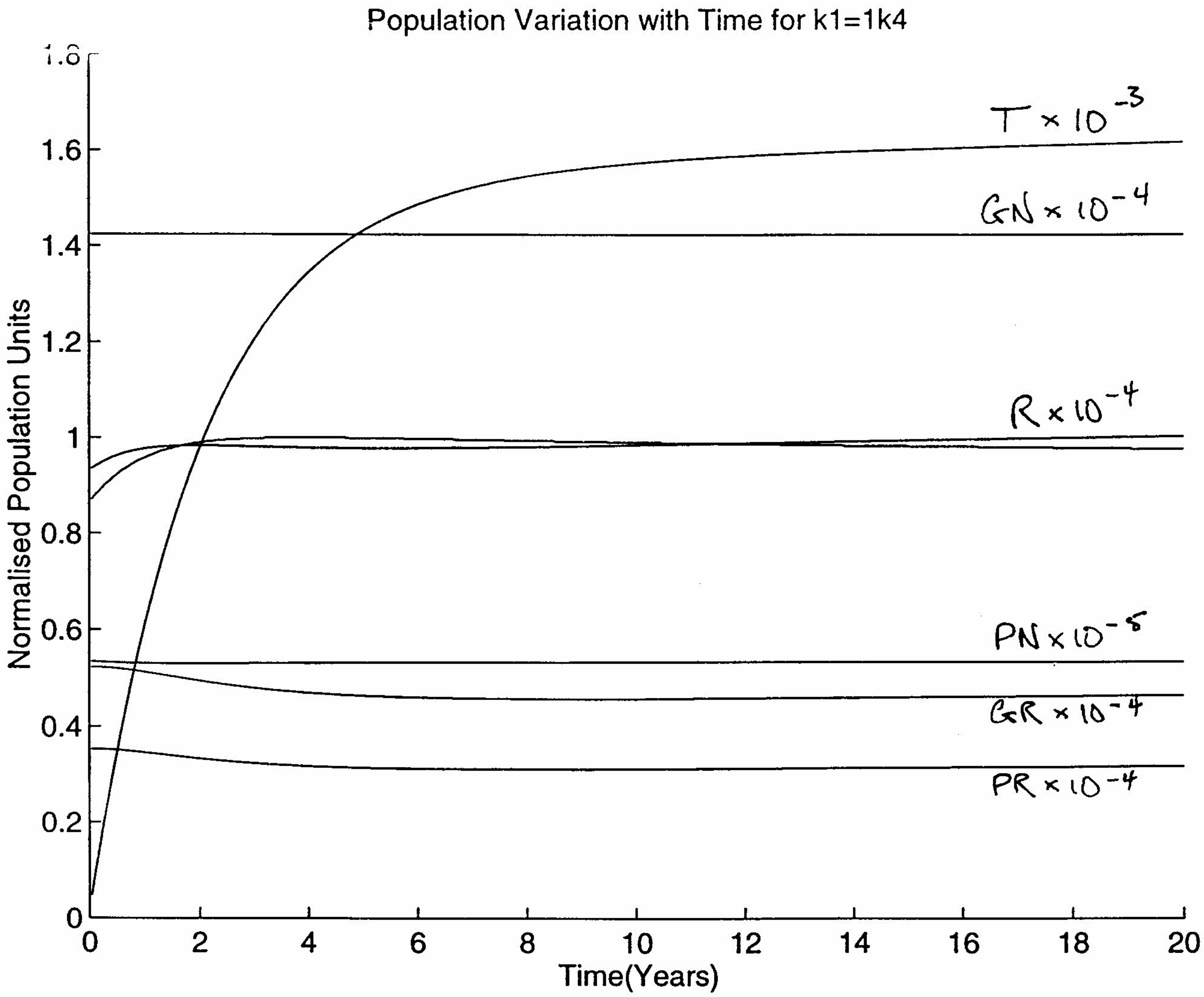


Fig 4: Population trends with $k_1 = k_4$