# Equilibrium states of an electrostatically actuated MEMS switch

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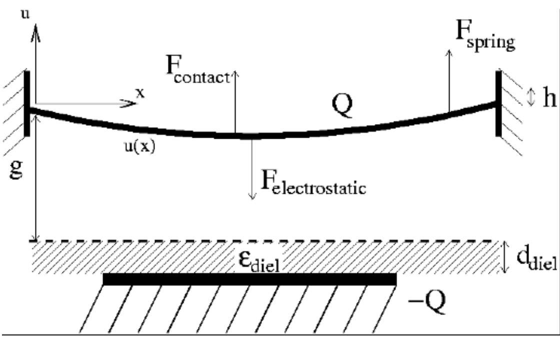
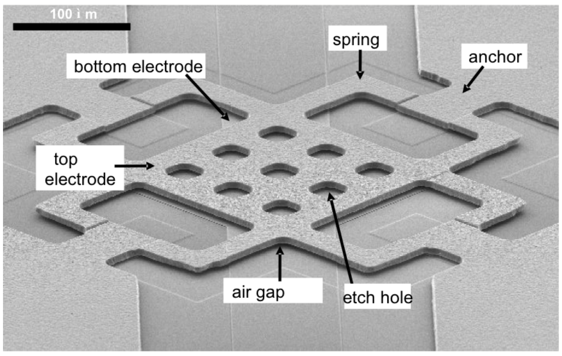
**Research on MEMS (MicroElectroMechanical Systems) has seen an amazing growth during the last 15 years, stimulated both by their interesting physical properties and their attractive application potential [1-7]. One of those appealing MEMS applications are radio frequency (RF) switches. Mobile phones need to address an increasing number of frequency bands and also need to become more energy efficient. With RF switches these needs can be met, because they enable reconfigurable and adaptive electronics, which can be placed just behind the antenna, thus increasing the versatility and energy efficiency of the phone. Advantages of RF MEMS switches over conventional semiconductor switches include lower power consumption, lower insertion loss, higher isolation and good linearity. However, because they are mechanical components in an electrical circuit they require a thorough understanding of electromechanical couplings. This problem deals with the static properties of these switches.**

Figure 1(a) shows a schematic cross-section of the RF MEMS switch. The thick black lines indicate the top electrode and bottom electrode. The dashed material is the dielectric with thickness $d_{\text{diel}}$. The presence of a charge $Q$ on the top electrode results in an electrostatic force $F_{\text{electrostatic}}$, which is balanced by the mechanical spring forces $F_{\text{spring}}$ on the anchors by which the top electrode membrane is suspended. If the top electrode touches the membrane, the contact force $F_{\text{contact}}$ prevents it from moving into the dielectric. The thickness of the top electrode is $h$, the anchors are separated by a distance $g$ from the bottom electrode. The shape of the top electrode is given by the function $u(x)$ (in two dimensions $u(x, y)$).

Figure 1(b) shows microscope picture of an actual capacitive MEMS switch. Electrodes, springs, gap $g$ and anchors are visible. Holes in the top electrode facilitate the manufacturing process.




(a)




(b)

Figure 1: (a) Schematic cross-section of a capacitive RF MEMS switch. (b) Scanning electron microscope picture of a capacitive RF MEMS switch.

**Problem formulation.** The main problem description is the following: find all the displacement states $i$ of the top electrode $u_{\text{eq},Q,i}(x, y)$ for which the forces on the top electrode are in equilibrium at a fixed charge $Q$ on the top electrode (or for a fixed voltage $V$ between the electrodes).

Sub-problems:

- Is there always a continuous path of equilibrium states $u_{\text{eq},Q,i}(x, y)$ between the open state $(u_{\text{eq},0,0}(x, y) = 0$ for all $x, y \in A_{\text{top}})$ and the closed state $(u_{\text{eq},\infty,N}(x, y) = -g$ for all $x, y \in A_{\text{bot}})$

(with continuous it is meant that no discontinuous changes in $u(x, y)$ occur along the path at any $x, y$).

- Is there a function $f(u_{\text{eq},Q,i}(x, y), Q)$ that is monotonically increasing along this path?
- Can it be shown that along this continuous path $dE_{\text{mech}}/(dC) > 0$ is always valid? Here $E_{\text{mech}}$ is the mechanical energy and $C$ is the capacitance.
- Is there a simple way to determine whether a state is stable or unstable at a fixed voltage or charge?
- For which geometries and boundary conditions is the problem analytically solvable? Most interesting is the situation in which the top electrode springs are clamped (zero displacement and zero slope) at some points of its boundary.
- The dynamics of the structure under the presence of gas damping is a related interesting problem (see reference [1]).

**Finite element method.** The equilibrium problem can be solved using finite element packages. Two examples are shown in figures 2 and 3. However it is not straightforward to find all solutions, especially since there can be multiple solutions for each value of $V$, $Q$ and $C$. Some of the solutions are stable and other are unstable.

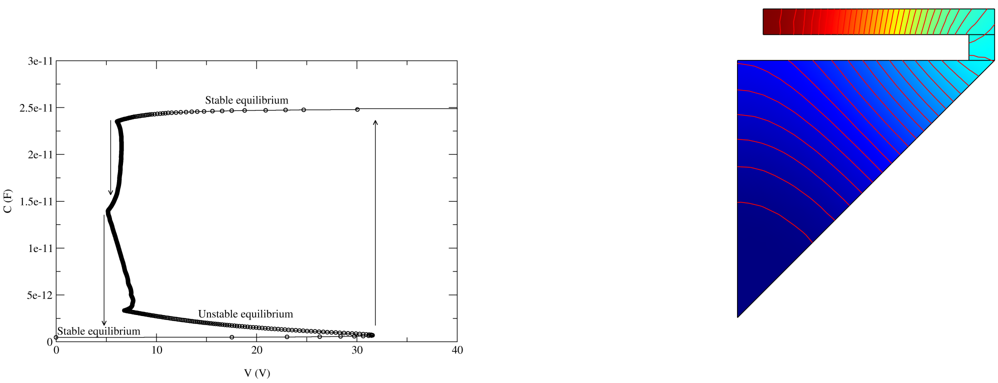



Figure 2: Calculated capacitance-voltage ($C - V$) characteristic (left) of the RF MEMS switch shown on the right (which is similar to that in figure 1(b)). Only 1/8 of the total switch is shown. The color indicates the height, anchor is red, dielectric height is blue. Arrows indicate discontinuities that occur under voltage controlled actuation.

**Mathematics.** The equilibrium states are given by the local minima and local maxima of the total energy function $E_{\text{tot}}$. The total energy is given by the sum of the electrical energy $E_{\text{el}}$ and the mechanical energy $E_{\text{mech}}$.

$$E_{\text{tot}} = E_{\text{mech}} + E_{\text{el}}. \tag{1}$$

If we assume that the parallel plate approximation is valid, the capacitance $C$ is given by the integral over the area of the bottom electrode $A_{\text{bot}}$:

$$C(u(x, y)) = \int_{A_{\text{bot}}} \frac{\epsilon_0 dA}{g + u(x, y) + d_{\text{diel}}/\epsilon_{\text{diel}}}, \tag{2}$$

$$E_{\text{el}} = \frac{Q^2}{2C}. \tag{3}$$

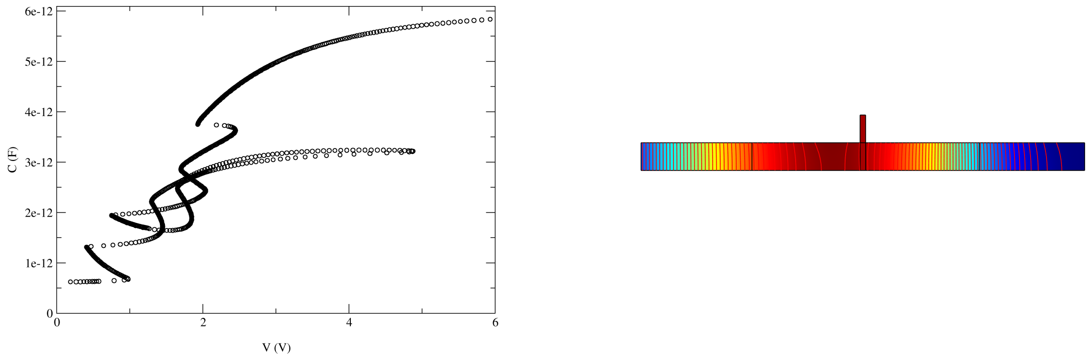



Figure 3: Calculated capacitance-voltage $(C - V)$ characteristic (left) of the 'see-saw' RF MEMS switch shown on the right. Only $1/2$ of the total switch is shown.

If only the bending forces are taken into account, and if it is assumed that the thickness is very small and initial stress is zero, the mechanical energy is given by:

$$E_{\text{mech}} = \int_{A_{\text{top}}} \frac{D}{2} |\Delta u|^2 dA, \tag{4}$$

$$D = \frac{2h^3 Y}{3(1 - \nu^2)} \tag{5}$$

where $Y$ and $\nu$ are the Young's modulus and Poisson ratio of the material.

**References.**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