Chapter 2

Force-Control for the Automated Footwear Testing System

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2.1 Introduction

The Automated Footwear Testing System (AFTS) is a robotic system designed to replicated the movement and loading of a shoe as it contacts the ground during common human movements. By doing so, the AFTS can serve as a system for the functional testing of different footwear designs in a manner that is difficult to achieve by standard testing systems. The AFTS consists of four main components: a robotic Stewart platform, a rigid fixed frame, a load cell and a prosthetic foot. Motion of the foot relative to the ground is created by rigidly fixing the foot to the frame and moving the platform relative to the foot. See Figure 2.1. The Stewart platform has six degrees of kinematic freedom and can reproduce the required complex three-dimensional motion path within the limitations of its range of motion. While the platform is in contact with the footwear, the six-axis load cell measures the three-dimensional forces and moments acting on the prosthetic foot.

It has been shown that when a human subject performs the same movement with two different pairs of shoes, she will adjust her stride so that she feels similar forces on her legs, regardless of the footwear. That is, when testing footwear, it can be assumed that the force profiles will be the same for the different shoes, while the movement path will differ from shoe to shoe. Thus, a good shoe is a one that does not lead to an unstable or unnatural movement path, e.g., one that might lead to an overturned ankle, or one that might lead to the need for an overcompensation that could result in an 'over-use' injury. In order for the AFTS to be most effective at testing a wide variety of design features, it would be necessary to develop a means of determining, for any given shoe, a movement path that would

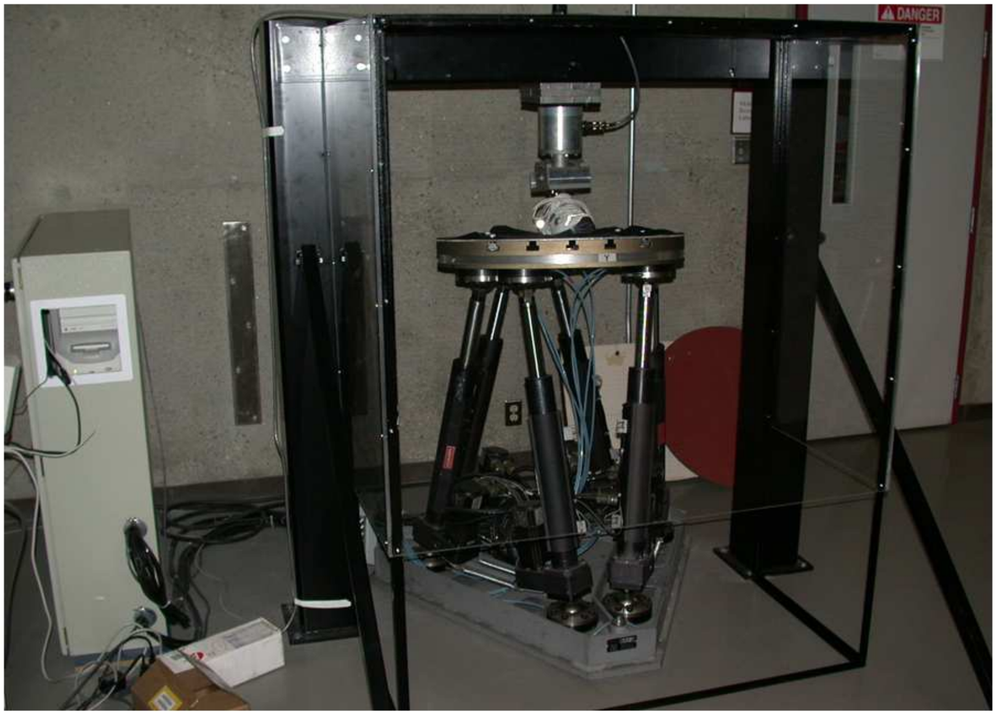

Figure 2.1: The Automated Foot Testing System consisting of the robotic Stewart platform, the prosthetic foot (shown here with shoe attached), the rigid frame, and the load cell (the cylinder just above the shoe).

generate some specified forces and moments that are representative of those that would be generated during the stride of some 'typical' human. This movement path could then be analyzed to determine if it is more or less likely to lead to injury.

The force profiles and movement paths for specific types of movements can be acquired experimentally. A time-series of forces can be acquired as a human subject's foot impacts a footplate during a stride, and markers on the shoe can be tracked in order to acquire a time-series of the position of the foot, i.e., a movement path. The position data includes the x, y, z positions, as well as the angles that the foot rotates about the x, y and z axes. These angles are generally referred to as roll, pitch and yaw, respectively, and we will denote them as α, β, and γ, respectively. The forces measured by the footplate are used to calculate forces in three directions F_x, F_y and F_z, and the moments M_x, M_y, and M_z, about the x, y and z axes, respectively, in the foot coordinate system. These forces and moments can be used as those felt by a typical human, i.e., the 'target forces'.

For the AFTS, a movement path is specified, translated into platform coordinates and executed on the machine. During the execution, the load cell measures the forces and moments that act on the prosthetic foot. We wish to find the particular movement path of the Stewart platform that will generate the target force profile. Thus, we are interested in solving an inverse problem. The main goal of the workshop was to investigate potential solution methods for this 'force-control' problem, including looking into its feasibility.

When the same shoe used by the human subject is mounted on the prosthetic foot of the AFTS, and the experimentally measured movement path is replicated on the Stewart platform, the forces and moments measured to be acting on the prosthetic foot do not match the experimental data. The forces acting normal to the ground/platform are similar in magnitude for both cases. However, the forces acting parallel to the ground/platform are not similar. Thus, before using the AFTS to test different footwear, it is necessary to determine the platform movement path that leads to the target force profiles for the 'control' shoe, i.e., the shoe used during the acquisition of the target force profile. This may also be viewed as a force-control problem. It may be reasonable to use such an approach if the discrepancies between the movement paths for the human subject and platform are relatively small. However, if they are sufficiently large, it would lead to difficulty in the interpretation of any testing results. That is, the causes of these discrepancies may reveal information regarding the feasibility of using force-control as a means of testing footwear. Thus, we seek possible origins of the discrepancies.

We first investigate the possibility of performing a closed-loop control of the forces. That is, we investigate the possibility of adjusting the position of the platform at discrete points along the movement path until the forces measured by the load cell of the AFTS match the target forces at that point. The results, discussed in Section 2.2, indicate that there are some fundamental issues that must be considered before the AFTS can reliably be used as a testing system. We study two such issues. The first study looks at the effects due to the choice of the origin of the platform coordinate system. See Section 2.3. This choice might effect how the Stewart platform executes the specified motion, and thus might effect the measured forces. In the second study, presented in Section 2.4, the system is modelled as a simple elastic body in order to gain some information regarding the feasibility of solving the inverse problem. The results suggest that it may be more appropriate to take a global rather than local approach to controlling the forces. In Section 2.5, we discuss the possibility of parameterizing the movement path using cubic splines, and then minimizing, with respect to the parameters of the curves, a functional that is small when the measured forces are near the target forces. Conclusions follow.

2.2 Closed-loop Force Control

We wish to find the series of platform positions (i.e. the movement path) that will lead to the force and moment profiles that are measured in the human subject (i.e. the target forces). One possible means of achieving this would be to perform a 'closed-loop' force control. That is, at discrete intervals along the path, the platform position is adjusted until the forces and moments that are measured at the load cell match the target forces. Ideally, it would be possible to perform a Newton-type iteration, where the initial guess could be either the experimentally measured position or the position found at the previous step, and an approximate Jacobian could be computed by measuring the changes that occur in the three forces and three moments as the six position variables are incremented successively by a small amount, while the other position variables are held constant at their initial values. That is, the approximate Jacobian could be given by

$$J = \begin{bmatrix} \frac{\Delta F_x}{\Delta x} & \frac{\Delta F_x}{\Delta y} & \frac{\Delta F_x}{\Delta z} & \frac{\Delta F_x}{\Delta \alpha} & \frac{\Delta F_x}{\Delta \beta} & \frac{\Delta F_x}{\Delta \gamma} \\ \frac{\Delta F_y}{\Delta x} & \frac{\Delta F_y}{\Delta y} & \frac{\Delta F_y}{\Delta z} & \frac{\Delta F_y}{\Delta \alpha} & \frac{\Delta F_y}{\Delta \beta} & \frac{\Delta F_y}{\Delta \gamma} \\ \frac{\Delta F_z}{\Delta x} & \frac{\Delta F_z}{\Delta y} & \frac{\Delta F_z}{\Delta z} & \frac{\Delta F_z}{\Delta \alpha} & \frac{\Delta F_z}{\Delta \beta} & \frac{\Delta F_z}{\Delta \gamma} \\ \frac{\Delta M_x}{\Delta x} & \frac{\Delta M_x}{\Delta y} & \frac{\Delta M_x}{\Delta z} & \frac{\Delta M_x}{\Delta \alpha} & \frac{\Delta M_x}{\Delta \beta} & \frac{\Delta M_x}{\Delta \gamma} \\ \frac{\Delta M_y}{\Delta x} & \frac{\Delta M_y}{\Delta y} & \frac{\Delta M_y}{\Delta z} & \frac{\Delta M_y}{\Delta \alpha} & \frac{\Delta M_y}{\Delta \beta} & \frac{\Delta M_y}{\Delta \gamma} \\ \frac{\Delta M_z}{\Delta x} & \frac{\Delta M_z}{\Delta y} & \frac{\Delta M_z}{\Delta z} & \frac{\Delta M_z}{\Delta \alpha} & \frac{\Delta M_z}{\Delta \beta} & \frac{\Delta M_z}{\Delta \gamma} \end{bmatrix}. \tag{2.1}$$

In practice, this could be computed by incrementing a single position variable, measuring the forces, incrementing that position variable back to its original value, and repeating this for each of the position variables. Once the approximate Jacobian is computed, it could be used to choose the next iterate. The platform would then be moved into the corresponding position, and the forces would be measured. If the forces are still not sufficiently close to the targets, another iteration could be performed, perhaps via a quasi-Newton iteration, or perhaps a new Jacobian could be computed. This procedure would continue until the desired forces are obtained to within a given tolerance. We could then proceed to the next point along the movement path, and find the platform position corresponding to the target forces that are required at this new point.

For this method to be feasible, the computed Jacobians must be non-singular. In order to test this, we computed the approximate Jacobian on the AFTS at two points along the movement path. The most striking result was that when certain position variables were incremented and returned to their starting values, the measured forces did not return to their original values. Even when the position variables were incremented by as little as 0.1mm and returned to their original values, the forces and moments could be as much as 5% different from their starting values.

Upon inspection of the AFTS in use, it was found that certain movements caused the shoe to slip along the platform. Such irreversible behaviour will greatly hinder any force-control procedure. Indeed, the discrepancies between the forces measured for the human subject and those measured for the platform for the same movement path could be caused to a large extent by the slipping. This is consistent with the observation that the forces normal to the ground/platform are sufficiently similar, while the tangent forces are not.

It is not surprising that when the approximate Jacobian was formed, we found that it was singular. It was seen, however, that only certain directions were irreversible, and it was speculated that this was caused by slipping when increments were made in these directions. In order for force-control to be possible, steps must be taken to reduce the slipping as much as possible. The platform being used for the data acquisition was quite worn, which likely exacerbated the problem. Thus, it is possible that the installation of a new platform surface designed to limit slipping would greatly improve the prospects. Either way, a method that minimizes slipping, in particular in the specific directions, will greatly increase the chances of success.

2.3 Stewart Platform Dynamics

It is expected that even when steps are taken to reduce slipping, a path-dependence of the forces measured at the load cell will likely linger. Thus, it is not only important to improve reversibility, but also to maximize reproducibility.

A run of the AFTS begins by raising the platform until it comes in contact with the shoe. The origin of the platform coordinate system is chosen as this initial point of contact. Currently, care is not taken to ensure that this point of contact is the same for each run. However, due to the method that is used to transform the experimentally measured movement path into platform positions, the choice of the origin of the platform coordinate system will affect the resulting platform movement path (see below). We therefore investigate the magnitude of this effect so that we may determine whether this is a potential cause of error, and whether care must be taken to choose the origin to be the same for each run of the AFTS.

Thus, we need to look into the dynamics of the Stewart platform. The platform has six degrees of freedom determined by the length of the actuators (legs), where each set of leg lengths corresponds to a unique position and orientation of the platform.

The actual platform path is not directly specified by the user. The user supplies a series of 'way-points', which are a series of positions that the platform must pass through, but the user does not have control over the path that is taken to go from one way-point to the next. The path between each pair of way-points is determined by an algorithm that requires all actuators to start and stop at the same time. Thus, these intermediate paths may be quite different depending on where the shoe initially contacts the platform (i.e., the choice of origin for the platform coordinate system). Because we were not able to test this on the AFTS itself, we performed a theoretical investigation of the path differences that might occur for three different origin locations, as shown in Figure 2.2. A central location was chosen, then the two other locations were chosen 5cm away from this central location along the x and y axes, respectively. We computed the platform paths by first computing the leg lengths corresponding to a series of way-points using the software package designed for this purpose. By assuming that during the transition between the way-points all the actuators would start and stop at the same moment, we determined a series of leg lengths that would occur between each of the way-points. We then used numerical methods to invert the nonlinear relation between the leg lengths and platform position, and obtained the intermediate platform positions that corresponded to the intermediate leg lengths.

A sample of results is plotted in Figure 2.3. An interesting observation is that the movement path has kinks at the way-points. It can also be seen that indeed the paths are different depending on the location of the coordinate system, although they are not more than 0.002mm for any given position variable. However, we did see that increments as little as 0.1mm could cause significant changes in the forces. Furthermore, it might be expected that there would be a cumulative effect. Thus, it is not clear that these small path differences would not have an effect on the measured forces. Therefore, to be sure that errors are not introduced, we suggest that care be taken to ensure that the origin is chosen as much as possible in the same location for each run. This may increase the reproducibility, and thus the reliability of the testing system.

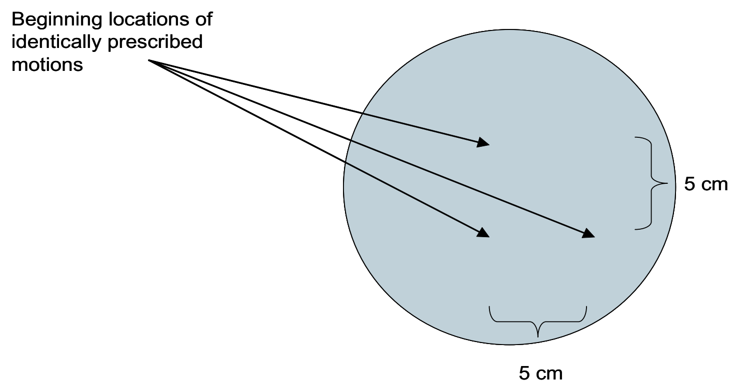

Figure 2.2: Three different origins of the platform coordinate system.

2.4 Feasibility Study

In the problem described above, we are trying to determine the displacements that must be imposed at the lower boundary (i.e., the bottom of the shoe) in order to generate some specified forces at the upper boundary (i.e., the load cell). To demonstrate the difficulty involved in solving these types of inverse problems for elastic bodies, we study a simple forward problem. We determine the displacements that occur in a planar elastic block for three different sets of lower boundary conditions (i.e., displacements that are imposed at the lower boundary). See Figure 2.4. We then calculate the forces that are generated at the upper boundary due to the resulting displacements. If we assume that the lower boundary is linear (i.e. that the displacements at the lower boundary vary linearly), we can choose the three boundary conditions such that they form a 'basis' for all possible boundary conditions. The situation when the lower boundary is linear corresponds to the case when the platform is in contact with the whole shoe. Although this is not a good assumption for many of the motions of interest, it is sufficient for the purposes of this feasibility study.

The displacements in the elastic block are described by the Navier equations

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} = 0, \tag{2.2}$$

$$\mu\frac{\partial^2 v}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} = 0, \tag{2.3}$$

where $u(x,y)$ is the displacement from the 'no force' position in the x direction, $v(x,y)$ is the displacement in the y direction, and the constants λ and μ are the Lamé coefficients. The lower and upper boundary are taken to be at $y=0$ and $y=1$ respectively, while the side boundaries are taken to be at $x=0$ and $x=1$. At the upper boundary, we assume no displacements, i.e. we have $u(x,1)=0$ and $v(x,1)=0$, while we take the boundary conditions on both sides to be stress free, i.e. we take

$$(\lambda+\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y}=0 \quad \text{and} \quad \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0 \tag{2.4}$$

at $x=0$ and $x=1$.

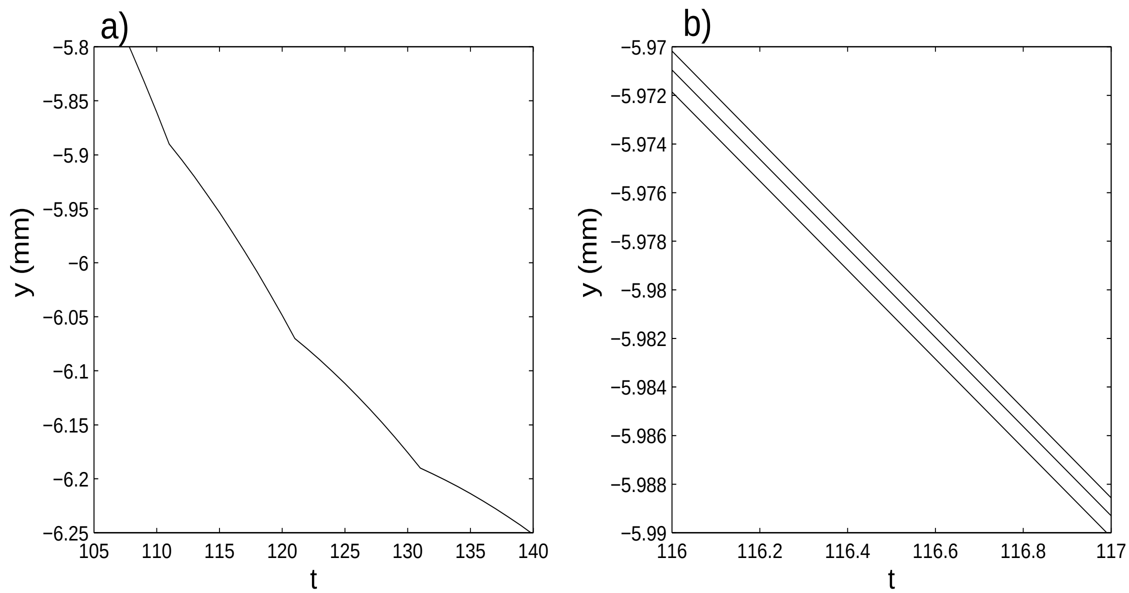

Figure 2.3: Platform dynamics. (a) A portion of a typical path taken by platform; kinks occur at the way-points, (b) three different paths corresponding to the three different 'platform origins' depicted in Figure 2.2.

For the three different cases, we chose three different lower boundary conditions. See Figure 2.4. In the first case, we choose $u(x, 0) = c$ and $v(x, 0) = 0$, where c is some constant. This case corresponds to pure shear in the x direction. For the second case, we consider pure compression in the positive y direction, i.e., we have $u(x, 0) = 0$ and $v(x, 0) = a$, where a is some constant. In the third case, we take $u(x, 0) = 0$ and $v(x, 0) = bx$, where b is some constant, which corresponds to a lower boundary that ramps linearly from zero compression at $x = 0$ to a maximum compression at $x = 1$. Any linear condition on the displacements at the lower boundary can be represented as linear combinations of these three input displacements.

We solve this system of partial differential equations (2.2–2.3) numerically using finite differences on a 50×50 grid. We choose the constants $\lambda = 1$, $\mu = 1/2$, $a = 0.5$, $b = 0.5$, and $c = 0.5$.

Once the displacements have been found, the forces and moment can be calculated using

$$F_y = \int \left((\lambda + 2\mu)\frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} \right) dx \qquad \text{(normal)} \tag{2.5}$$

$$F_x = \int \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) dx \qquad \text{(shear)} \tag{2.6}$$

$$M = \int x \left((\lambda + 2\mu)\frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} \right) dx \qquad \text{(moment).} \tag{2.7}$$

We are interested in the values for these forces at the upper boundary ($y = 1$). We denote the normal force at the upper boundary as F_1, the stress along the upper boundary as F_2, and the moment as M, and calculate each of these for each of the three boundary conditions. We obtain a 3×3 matrix that

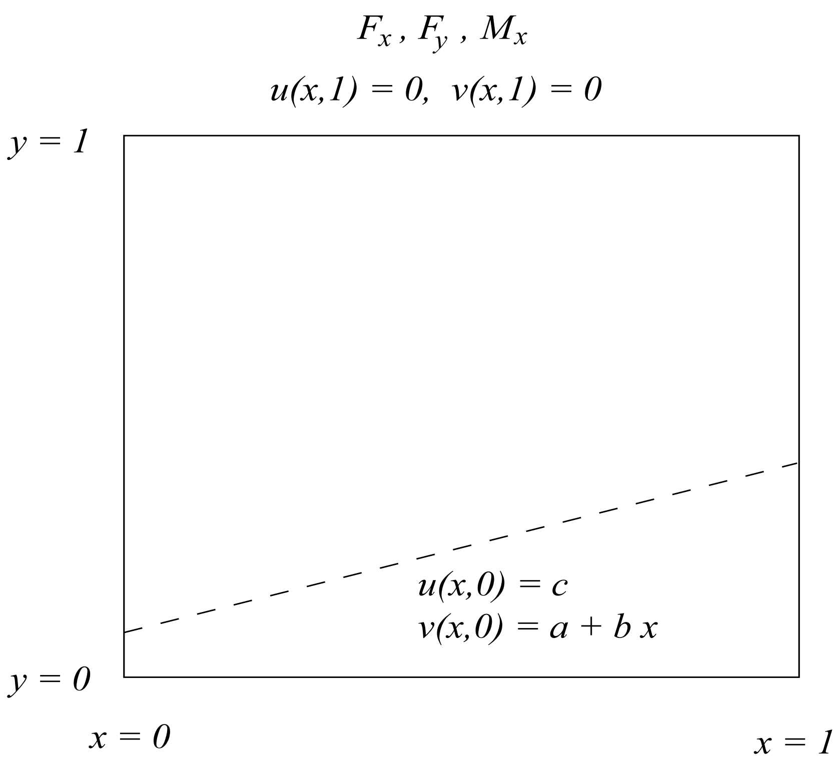

Figure 2.4: An elastic block of unit length and height. Lower boundary conditions for the displacements $u(x, y)$ and $v(x, y)$ are different for the three different cases that are studied (corresponding to different values of the constants a, b, and c), while the side and upper boundary conditions are the same.

defines the relationship between our input displacements and our output forces:

$$A = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ M_1 & M_2 & M_3 \end{bmatrix}$$

where F_{ij} is the force i in the jth case, and M_j is the moment in the jth case. With the displacement fields computed above, this matrix becomes

$$A = \begin{bmatrix} -0.0053 & 0.5768 & 0.2845 \\ -0.0147 & 0.0041 & -0.0099 \\ 0.0112 & 0.2845 & 0.1426 \end{bmatrix}.$$

The condition number of this matrix, which is the ratio of singular values, indicates the sensitivity of the forces to the changes in the boundary conditions. If the condition number is small, then we would expect that changes in the lower boundary would cause changes of a similar magnitude in the force, which would indicate that the inverse problem was well-conditioned. However, if the condition number is large, then we might expect that the matrix is close to singular, which would imply that the columns are close to being linearly dependent, which in turn would imply that different combinations of inputs would produce very similar outputs. That is, the forces are not very sensitive to changes in the lower boundary, and thus, the inverse problem is not very well conditioned.

The condition number for the matrix A is given by

$$\operatorname{cond}(A) = 147.7,$$

indicating that the forces are not very sensitive to changes in the lower boundary. This can be seen more clearly in Figure 2.5, in which the forces (represented by the three plots on the right side of the figure) generated by variation of the lower boundaries conditions (shown on the left of the figure) are presented on the same plot. It can be seen that even large differences in the lower boundary conditions can result in only small changes in the forces.

This example provides evidence regarding the difficulty that may be involved in attempting to determine the lower boundary conditions given the forces at the upper boundary. That is, the inverse problem may not be well-conditioned. In such cases, finding solutions becomes difficult. Iterative methods tend to converge slowly, and it is possible that they may not converge at all.

However, these results depend on the specific choices of the parameters of the problem λ and μ. Because we did not know the actual values of these parameters for the shoe, reasonable approximations were chosen. Errors in this choice may affect the conclusions of this example.

Factors that effect the conditioning that we have not considered include the movement of the shoe on the prosthetic foot, which is expected to lead to poorer conditioning. Such movement would decrease the sensitivity of the forces due to changes in the lower boundary, and thus increase the condition number.

The zero displacement condition assumed at the top the prosthetic foot is almost certainly not satisfied by the human foot and hence, no matter what continuum model is used for the foot-shoe combination, any attempt to examine the problem analytically will lead to different results for the two problems even if the displacement conditions at the shoe plate interface can be accurately reproduced.

2.5 Movement Path Parameterization

The evidence presented above indicates the difficulty involved in using closed-loop force control to solve this problem. We, therefore, explore the possibility of non-locally controlling the forces along a parametrized movement path. It is expected that the conditioning of the inverse problem will still be an issue for this approach. However, variation of the parameters of the movement path would not lead to unnatural movements, which would reduce (perhaps eliminate) the need to make platform adjustments in directions that would cause unavoidable slipping. Because we did not have sufficient time during the workshop for a full investigation, we describe only briefly how one might go about using path parametrization in this problem.

We begin by parameterizing the position data obtained from the human subject. We proceed by choosing several points on the curves of each of the position variables. Examples for the spatial coordinates are shown in Figure 2.6. The number and position of the points are chosen in such a way as to maximize the reproduction of the qualitative features of the curves while minimizing the number of parameters needed. For example, for the z position data, it was judged that four points were necessary to obtain a parametrized curve that could approximate both the sharp increase and decrease that is observed at the beginning and end, respectively, of the time-series. After the points have been chosen, cubic spline interpolation can be used to obtain the parametrized approximation to each of the curves.

For data acquired during the contact phase of a heel-toe run, the time-series of the six position variables could be reproduced reasonably well by fitting cubic splines to a total of 34 points. The position data for the 3 spatial coordinates is shown in Figure 2.6; data for the 3 angles is not shown. Thus, the path can be written as a function of the 34 parameters

$$\mathrm{Path} = P(p_1, p_2, \ldots, p_{34}),$$

where p_i are the path parameters that can be adjusted to vary the movement path. An example of how the path might change when one of the parameters is varied is shown in Figure 2.7.

The forces can now be measured as the AFTS executes the initial parameterized path taken from the human subject data. That is, we have

$$\mathrm{Force} = F(P),$$

and we would like to find the 34 parameters p_i that will reproduce the target force profile F_{target}. In practice, we will try to minimize some functional (e.g. with respect to the L^2 norm) of the force and target force profiles over all possible parameter values. That is,

$$\min_P \|F_{\text{target}} - F(P)\|_2.$$

A standard method, perhaps a non-Jacobian method such as a polytope algorithm, might be used for the minimization.

For the closed-loop force-control problem, we look for zeros of a function of six variables for each interval along the path, whereas, here we are minimizing a single functional over 34 parameters. Although only one minimization problem needs to be solved, a large number of parameters are involved. The question arises whether such a method is feasible. Indeed, even if a Jacobian need not be calculated, a single ‘function evaluation’ consists of a full run of the AFTS, which took much longer than one minute. It is not known how many such function evaluations would need to be executed to determine the path parameters. However, it could possibly be in the hundreds. It is possible that a sufficient solution could be obtained by variation over only a smaller subset of the path parameters. These and other considerations require extensive further study before such a method could be implemented effectively.

2.6 Conclusions

We would like to determine the particular movement path that would generate a specified target force profile. We examined the feasibility of performing a closed-loop control of the forces, and found that the nature of the problem does not lend itself well to this method. We do not conclude that it is impossible to use closed-loop force control to solve the problem. However, the evidence indicates that it would be very difficult to do so.

Parametrization of the movement path is one possible alternative. The promising feature of this method is that it would not lead to unnatural movements that could cause the shoe to slip along the platform. Thus, it is expected that the reproducibility of force measurements would be significantly improved. The conditioning of this method is not known; further study is required before conclusions regarding the method's feasibility can be made. Such investigations would require extensive data acquisition using the AFTS itself.

Regardless of the method used, we discovered that it is necessary to reduce slipping of the shoe along the platform as much as possible. Simply resurfacing the platform may lead to significant improvements in this respect. We also found that the platform will follow a different trajectory depending on the origin of the platform coordinate system. Although the path differences are small, significant cumulative errors may arise. Thus, it would be prudent to ensure that the platform origin does not vary from run to run.

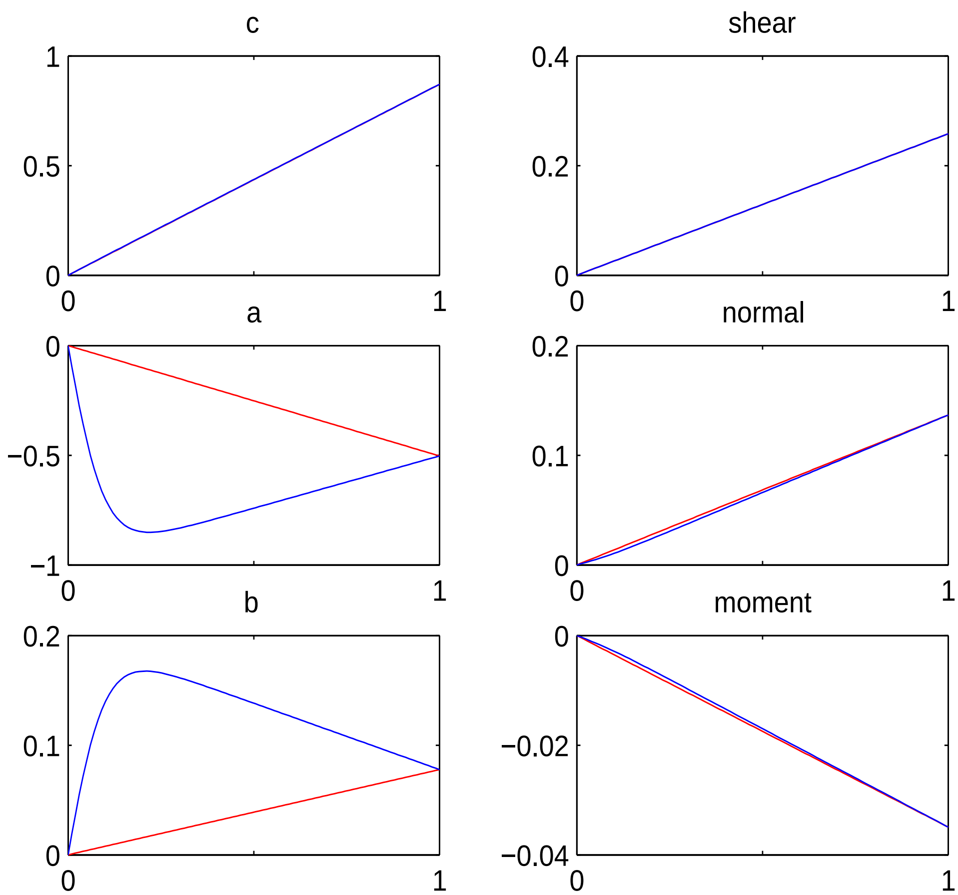

Figure 2.5: The two curves in the left panels represent two different sets of boundary conditions, i.e. of the constants a and b representing the amount of compression at the bottom boundary and slope of the bottom boundary, respectively, where the y-axis gives the values of the constants and the x-axis represents a parametrization for the changes in the constants. The curves in the right panels show the resulting differences in the forces, i.e., large changes in the boundary conditions only result in small changes in the forces, where the y-axis gives the values of the forces and the x-axis represents the same parametrization as in the left panels.

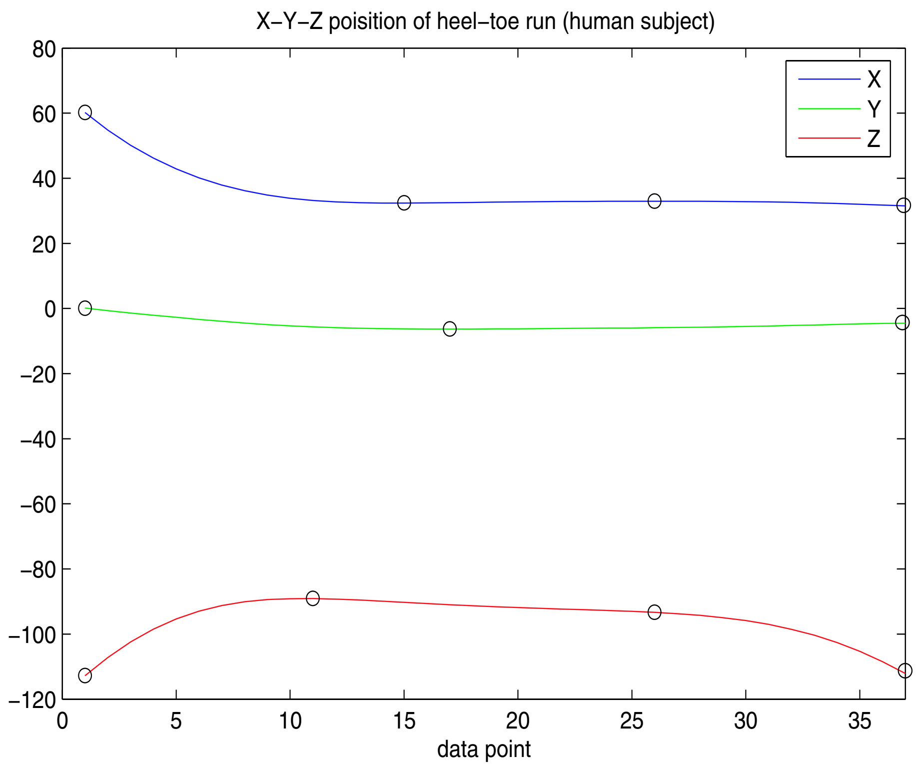

Figure 2.6: Position data in the x, y, and z directions for a heel-toe run of a human subject. Points along the curves (the circles) have been chosen such that spline interpolants through these points will reasonably reproduce the curves.

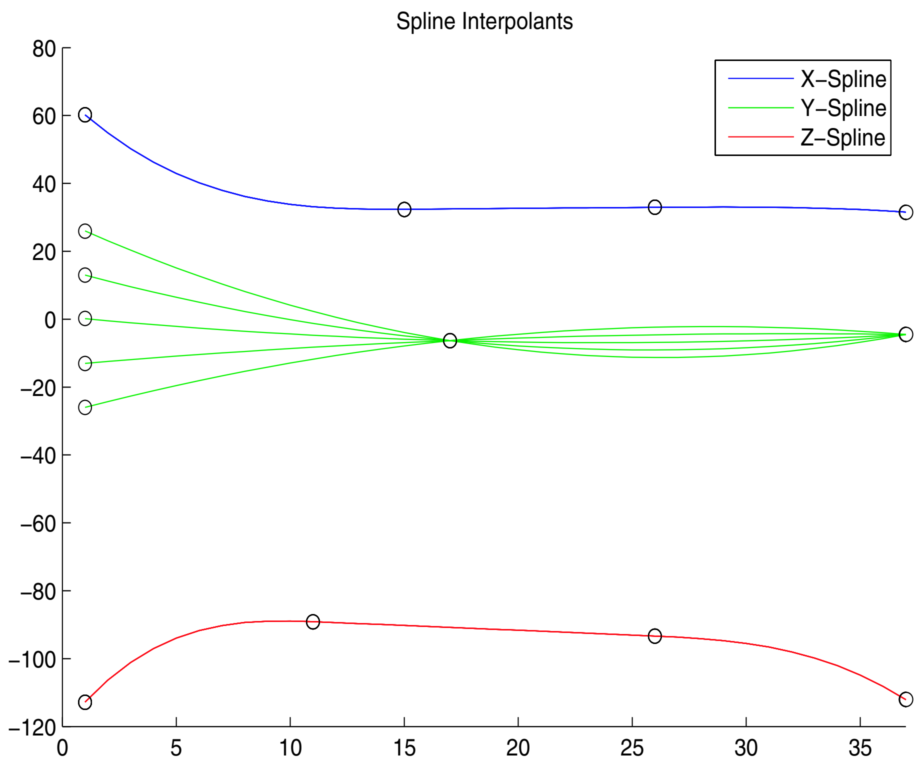

Figure 2.7: Spline interpolants of the points taken from the x, y, and z position data of a heel-toe run of a human subject, as shown in Figure 2.6. Variation of one of the points represents how the path can change as the path parameters are varied.