Determining the independence of various measures of financial risk

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1 Introduction

Trading Technology Australia (TTA) was founded in 1996 and has a background and client base in the financial markets. In 2002, TTA delivered its first Energy Markets project in the form of a risk management report to the CEO of a large electricity retailer. Since then, TTA has been working to further understand energy market issues such as the relevance of particular models used in trading and risk.

Common measures of financial risk include [1, 2, 5] Value-at-Risk (VaR) and Earnings-at-Risk (EaR). VaR measures the probable change of a portfolio's position due to market movements within a given confidence interval. For instance, suppose a portfolio has a 1-day VaR of \$40000 at the 99% confidence level. This means that, with a probability of 99%, the value of the portfolio will decrease by at most \$40000 during 1 day. Companies are required to report their VaR to the regulators of the market.

EaR measures the probable loss in earnings due to market or volumetric movements within a given confidence level. For instance, suppose a company calculates its 30-day EaR to be $-\$20000$ at the 99% confidence level. This means that, with a probability of 99%, the company's earnings will be at least $-\$20000$ on the 30^{th} day in the future. Presently, companies are not required to report their EaR externally, but it is a useful measure to guide financial strategy. EaR is an equivalent measure to "profit-at-risk" and "relative VaR", which are terms used in the literature.

The question posed by TTA to MISG2007 was: "How independent are VaR and EaR, specifically in the electricity market?" These quantities are computationally expensive to calculate, and if a map from one to the other could be found, significant savings would be generated for energy companies by making their risk reporting framework more streamlined, transparent and risk-compliant.

Attempting to answer this question, the MISG2007 team focused on simple example portfolios. These portfolios are particularly relevant for an electricity retailer.

In this paper we first give a brief introduction to the electricity market in Australia. Then we present the electricity-retailer portfolio that most succinctly captures the key properties and illustrates our key conclusions. Thirdly, formulae for VaR and EaR are derived for special cases where the forward price is equal to the spot price, in addition to spot price being related to demand. Finally, we state our conclusions.

The main conclusion is that for the electricity retailer, VaR is not strongly related to EaR in a way that can obviously reduce computational time. Of course, both measures will increase with market volatility, but accurately quantifying EaR given VaR does not seem to be possible. This is because of two factors. Firstly, VaR depends on forward prices, while EaR on spot prices, and these two prices may be unrelated. Secondly, VaR is sensitive to *drops* in prices, while EaR is sensitive to *increases* in prices and demands. Mathematically, VaR is looking at left tails of probability distribution functions, while EaR is looking at right tails. Since, in general, the distributions are non-symmetric there will be no obvious maps between VaR and EaR.

Throughout this paper we assume that the "risk free rate" (the rate of interest for money deposited in a bank) is zero. This simplifies formulae and is unlikely to change the main conclusions. We also calculate EaR and VaR at the 99% confidence level: other levels may be obtained by simply substituting another numerical value for the "0.01" that appears in all the formulae. A small amount of financial knowledge is assumed. A text that is immediately readable by applied mathematicians is [4], while a more advanced classic text is [3].

2 The electricity market in Australia

Generators, retailers and consumers form the electricity market in Australia. Generators generate electricity, consumers consume it, and retailers are the middlemen buying electricity from the generators and selling to the consumers. This paper concentrates on the electricity retailers. Retailers may also be generators, and generators may also be consumers.

The wholesale electricity market is administered and operated by the National Electricity Market Management Company Limited whose website, `www.nemmco.com.au`, contains a lot of detailed and current information concerning the market. For our purposes, we can think of the generators placing bids into a "pool": the bids describe the quantity and price of electricity that they are willing to supply. The demand is fixed in real time by the consumers, and electricity gets drawn from the pool to meet this demand, starting from the lowest priced electricity.

So, if the demand is low then electricity is only drawn from the generators who put in bids selling electricity at a low price, while if demand is high then electricity is also drawn (bought) from generators selling at a higher price. For instance, if generator A puts a bid of \$1/unit into the pool, and is prepared to generate 100 units at this price, while generator B's bid is \$2/unit, then if the demand, $D \leq 100$ units, the price will be \$1/unit. However, if the demand is greater than this the total price will be $\$100+2(D-100)$. Thus, total price (more strictly, "spot price") is a piecewise linear function of demand.

The demand changes continually, and the generators' bids into the pool may change frequently. The generators' bids dictate the *spot price* of electricity. Some generators may choose to place low bids, guaranteeing their electricity will be used almost continually, while some generators may prefer to place exceedingly highly priced bids hoping that the demand will spike one day and their electricity will be bought.

The retailer sits in the middle. The important point is that, they sell electricity to the consumers at a *fixed* price, while, in the simplest situation, they buy electricity from the pool at the spot price. This means that the energy retailers face substantial financial risk. The spot price is set by the generators, and because the demand fluctuates, the spot price may be significantly greater than (over ten times) the fixed price paid by the consumers. Therefore energy retailers enter into forward contracts with the generators.

3 The example portfolio for an electricity retailer

Recall that the risk-free rate is zero by assumption. Our canonical energy retailer portfolio is set up as follows. Define the following variables.

- t: the time variable. The current time is set to be $t = 0$.
- P_c: the price the customers pay the retailer per unit of electricity. Assume this is fixed.
- $P(t)$: the spot price per unit of electricity at time t. This is a stochastic variable with expected value at time t of $\overline{P(t)}$. Without any forward contracts $P(t)$ is the price the retailer would have to pay per unit of electricity at time t.
- $Q(t)$: the demand by customers for electricity at time t. This is a stochastic variable with expected value at time t of $\overline{Q(t)}$.

For the purposes of calculating VaR and EaR, the retailer uses models, historical data, or some other technique to find the probability distribution functions for $P(t)$ and $Q(t)$, and hence their expected values for future times $t > 0$.

Now suppose the retailer enters into the following long forward contract with the generator.

- The retailer will buy from the generator $\overline{Q(T)}$ units of electricity at price P_f at time $t = T$.

P_f is known as the forward price.

Evidently, this is a very simple portfolio, consisting of just $\overline{Q(T)}$ forward contracts with the same price expiring at the same time $t = T$. A real-life situation would consist of many forwards and other derivatives expiring at different times. It is difficult to see how the extra sophistication would change our conclusions, however.

Finally, define the forward price in the future:

- $F(t,T)$ is the price specified in a forward contract set up at time t which will expire at time T. It is a stochastic variable, but again assume the retailer can calculate its probability distribution.

Clearly, $P_f = F(0,T)$.

3.1 VaR

The VaR can be calculated for any time, t, in the future. Suppose at time $t \leq T$ the retailer would like to sell the $\overline{Q(T)}$ forward contracts to another party. It would sell them at price $F(t,T)$, so the payoff would be

$$\text{payoff}(t) = \overline{Q(T)}(F(t,T) - P_f). \tag{1}$$

The t-day VaR is calculated from this payoff by constructing the probability distribution function for the payoff, Pr(payoff) (at time t) as illustrated in Figure 1. Mathematically,

$$\Pr(\text{payoff}(t) < \text{VaR}) = 0.01 \ , \tag{2}$$

for the 99% confidence level. In this formula, VaR will typically be negative, but its absolute value is reported.

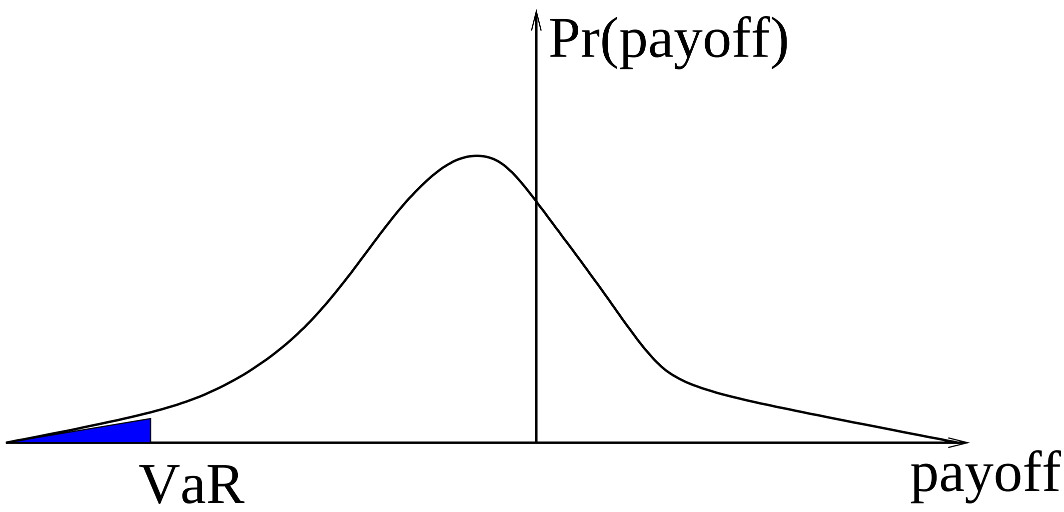

Figure 1: The probability distribution, Pr(payoff) for the payoff at time t. The shaded region has area 1%.

3.2 EaR

The EaR can also be calculated for any time, t, in the future. For times $t < T$, the retailer must sell $Q(t)$ units of electricity to the customers at price P_c, and buy $Q(t)$ units at the spot price, $P(t)$. Thus, its cashflow is

$$\text{cash flow}(t) = Q(t)(P_c - P(t)) \ . \tag{3}$$

The t-day EaR is calculated from this cash flow by constructing the probability distribution function for the cash flow, Pr(cash flow), (at time t) as illustrated in Figure 2. Mathematically,

$$\Pr(\text{cash flow}(t) < \text{EaR}) = 0.01 \ , \tag{4}$$

for the 99% confidence level calculation.

For time $t = T$, the retailer must sell $Q(T)$ units to the customers at price P_c, so its revenue will be $Q(T)P_c$, as before. However, now the retailer must buy the $\overline{Q(T)}$ units of electricity as specified in the forward contract, which costs $\overline{Q(T)}P_f$. In addition, if $Q(T) > \overline{Q(T)}$ it must also buy the excess units at the spot price, $P(T)$. Its total cash flow is therefore

$$\text{cash flow}(T) = Q(T)P_c - \left[\overline{Q(T)}P_f + (Q(T) - \overline{Q(T)})P(T)\Theta(Q(T) - \overline{Q(T)})\right] \ , \tag{5}$$

where

$$\Theta(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{otherwise} \ . \end{cases}$$

In the following section, for simplicity we shall use equation (3) to calculate EaR. Use of the more complicated equation (5) is not expected to alter our conclusions. EaR is sensitive to those situations where cash flow is drastically negative. In both situations (equation (3) and (5)) this is when Q is large, implying a large P, and the cash flow is dominated by the large $-QP$ term.

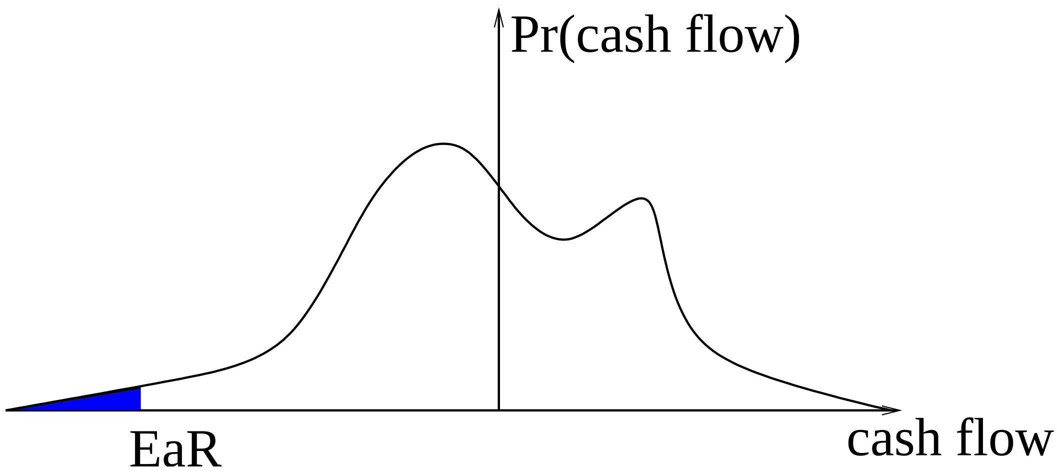

Figure 2: The probability distribution, Pr(cash flow) for the payoff at time t. The shaded region has area 1%.

3.3 Summary

Equations (1), (3) and (5) demonstrate that in this situation:

- the t-day VaR depends on the probability distribution of the forward prices of the contracts in the portfolio $F(t,T)$;
- the t-day EaR depends on the probability distributions of the demand, $Q(t)$, and the spot price $P(t)$.

The problem thus reduces to finding relations between integrals of the three probability distributions. It is immediately obvious that if these three are completely uncorrelated then VaR and EaR will be unrelated. This conclusion may be generalised to more complicated portfolios containing more stochastic variables. In the following sections, simple correlations are assumed which lead to potentially useful relationships between VaR and EaR.

4 Detailed analysis of special cases

In this section we present formulae for VaR and EaR for special situations which are mathematically nongeneric, but which are hopefully relevant to financial situations.

To make some headway, first assume that the spot price and the forward price are equal:

$$P(t) = F(t,T) \ . \tag{6}$$

This is widely used in other markets, such as gold, but may be less relevant to the electricity markets.

Furthermore, in many markets the price is strongly correlated with the demand. This is certainly true for electricity markets where generators bid into the pool at certain prices and the demand dictates whose electricity gets drawn from the pool. This means that at any one time — say $t = 0$ — the spot price is a *deterministic* increasing function of demand; in fact, as described in Sec 2, total price is a piecewise linear function of demand:

$$\text{Total price} = Q(0)P(0) = \text{piecewise linear}(Q(0)) \ .$$

Unfortunately, for forecasting purposes, the generators' bids are unknown in the future. They can be modelled by a stochastic variable. This implies that at time t in the future the relationship between spot price and demand is unknown, but can be modelled by a positive linear correlation. We present the *linearly* correlated case: more general correlations can be similarly studied.

4.1 Correlations in general

4.1.1 Perfect linear correlation

Two variables, P and Q, are perfectly (linearly) correlated if and only if their joint probability distribution function, $\mathrm{Pr}_{P,Q}$, is of the form

$$\mathrm{Pr}_{P,Q}(P,Q) = \delta(P - \alpha Q - \beta)\mathrm{Pr}_Q(Q) \ ,$$

where Pr_Q is normalised to unity: $1 = \int \mathrm{Pr}_Q$, and δ is the Dirac delta function. The variables are positively correlated if and only if $\alpha > 0$. As the name suggests, Pr_Q is the marginal probability distribution for Q after integrating over all P. That is,

$$\mathrm{Pr}_Q(Q) = \int \mathrm{d}P \ \mathrm{Pr}_{P,Q}(P,Q) \ .$$

Similarly, the distribution function for P reads

$$\mathrm{Pr}_P(P) = \frac{1}{\alpha}\mathrm{Pr}_Q\left(\frac{P-\beta}{\alpha}\right) \ .$$

Evidently the delta function is enforcing the linear relation $P = \alpha Q + \beta$.

4.1.2 Completely uncorrelated variables

Conversely, two variables, P and Q, are completely uncorrelated if and only if their joint probability distribution function, $\mathrm{Pr}_{P,Q}$, is a product:

$$\mathrm{Pr}_{P,Q}(P,Q) = \mathrm{Pr}_P(P)\mathrm{Pr}_Q(Q) \ .$$

We shall study these two cases — correlated and uncorrelated — below.

4.1.3 Expectation values and the correlation coefficient

Denote the expectation value of a variable by $\mathbb{E}$. So

$$\mathbb{E}(P) = \int \mathrm{d}P \ P\mathrm{Pr}_P(P) = \int \mathrm{d}P\mathrm{d}Q \ P\mathrm{Pr}_{P,Q}(P,Q) \ ,$$

for instance. Statisticians sometimes use the correlation coefficient of two variables, P and Q, which is defined as

$$\mathrm{Corr}(P,Q) = \frac{\mathbb{E}(PQ) - \mathbb{E}(P)\mathbb{E}(Q)}{\sqrt{\mathbb{E}(P^2) - \mathbb{E}^2(P)}\sqrt{\mathbb{E}(Q^2) - \mathbb{E}^2(Q)}} \ .$$

Substituting the two cases above into this formula yields

$$\begin{aligned} \text{completely correlated} \quad &\Rightarrow \quad \text{Corr} = \text{sign}(\alpha) \ , \\ \text{completely uncorrelated} \quad &\Rightarrow \quad \text{Corr} = 0 \ . \end{aligned}$$

It is not true that if Corr $= 0$ then the variables are uncorrelated, for Corr only picks up *linear* correlations. In our case, Corr is less useful than considering the underlying probability distribution functions.

4.1.4 Partially correlated variables

Now introduce the joint probability distribution function for partially correlated variables:

$$\text{Pr}_{P,Q}(P,Q) = \lambda f(P) g(Q) + (1-\lambda)\delta(P - \alpha Q - \beta) h(Q) \ . \tag{7}$$

If $\lambda = 0$ then the variables are completely (linearly) correlated, while if $\lambda = 1$ the variables are completely uncorrelated. The functions f, g and h must satisfy the normalised condition:

$$1 = \lambda \left(\int f \right) \left(\int g \right) + (1-\lambda) \int h \ .$$

We show below how the correlation coefficient may be expanded in powers of λ or $(1-\lambda)$.

4.1.5 VaR and EaR

We shall perform an expansion of VaR and EaR around $\lambda = 0$ and $\lambda = 1$. In general, VaR (at the 99% level), is defined by equation (2), which in this case reads

$$\begin{aligned} 0.01 &= \int_{\bar{Q}(P-P_f)<\text{VaR}} \text{d}P\text{d}Q \ \text{Pr}_{P,Q}(P,Q) \\ &= \lambda \int_{-\infty}^{P_f+\text{VaR}/\bar{Q}} \text{d}P \ f(P) \left(\int g \right) \\ &\quad +(1-\lambda) \int_{-\infty}^{(P_f-\beta+\text{VaR}/\bar{Q})/\alpha} \text{d}Q \ h(Q) \ . \end{aligned} \tag{8}$$

The last expression holds only if $\alpha > 0$ (positive correlation). Otherwise the formula reads

$$0.01 = \lambda \int_{-\infty}^{P_f+\text{VaR}/\bar{Q}} \text{d}P \ f(P) \left(\int g \right) + (1-\lambda) \int_{(P_f-\beta+\text{VaR}/\bar{Q})/\alpha}^{\infty} \text{d}Q \ h(Q) \ .$$

Similarly, EaR, defined by equations (3) and (4), reads

$$\begin{aligned} 0.01 &= \int_{Q(P_c-P)<\text{EaR}} \text{d}P\text{d}Q \ \text{Pr}_{P,Q}(P,Q) \\ &= \lambda \int_{Q(P_c-P)<\text{EaR}} \text{d}P\text{d}Q \ f(P)g(Q) \\ &\quad +(1-\lambda) \int_{Q(P_c-\alpha Q-\beta)<\text{EaR}} \text{d}Q \ h(Q) \ . \end{aligned} \tag{9}$$

Now let us examine particular cases more thoroughly.

4.2 Perfect positive correlation between price and demand

In this case, the parameter λ in equation (7) is zero, $\alpha > 0$, and h becomes the probability distribution function for Q. Equations (8) and (9) read

$$0.01 = \int_{-\infty}^{(P_f-\beta+\mathrm{VaR}/\bar{Q})/\alpha} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \tag{10}$$

$$\begin{aligned} 0.01 &= \int_{Q(P_c-\alpha Q-\beta)<EaR} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \\ &= \int_{Q_+}^{\infty} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) + \int_{-\infty}^{Q_-} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \ . \end{aligned} \tag{11}$$

Here

$$Q_\pm = \frac{P_c - \beta \pm \sqrt{(P_c-\beta)^2 - 4\alpha\mathrm{EaR}}}{2\alpha} \ . \tag{12}$$

In most financial situations the probability distributions will be such that EaR is negative (it is greater than 1% likely the firm will have negative cash flow), so $Q_- < 0$ and the final term vanishes, since physically we should have $\mathrm{Pr}_Q(Q) = 0$ for $Q < 0$.

4.3 Completely uncorrelated price and demand

In this case, the parameter λ in equation (7) is unity, f is the probability distribution function for P, and g for Q. Equations (8) and (9) read

$$0.01 = \int_{-\infty}^{P_f+\mathrm{VaR}/\bar{Q}} \mathrm{d}P \ \mathrm{Pr}_P(P) \tag{13}$$

$$\begin{aligned} 0.01 &= \int_{Q(P_c-P)<\mathrm{EaR}} \mathrm{d}P\mathrm{d}Q \ \mathrm{Pr}_P(P)\mathrm{Pr}_Q(Q) \\ &= \int_0^{\infty} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \int_{P_c-\mathrm{EaR}/Q}^{\infty} \mathrm{d}P \ \mathrm{Pr}_P(P) \\ &\quad + \int_{-\infty}^{0} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \int_{-\infty}^{P_c-\mathrm{EaR}/Q} \mathrm{d}P \ \mathrm{Pr}_P(P) \ . \end{aligned}$$

Once again in real financial situations the probability distributions will be zero for $Q < 0$ (negative demand), the final term vanishes, so the EaR expression takes the form

$$0.01 = \int_0^{\infty} \mathrm{d}Q \ \mathrm{Pr}_Q(Q) \int_{P_c-\mathrm{EaR}/Q}^{\infty} \mathrm{d}P \ \mathrm{Pr}_P(P) \ . \tag{14}$$

Finally note that solving the first expression for VaR would probably yield VaR < 0, but that we would report, by convention, $|\mathrm{VaR}|$ to the regulatory bodies.

4.4 Strongly positively correlated price and demand

In this case, the parameter λ in equation (7) is close to zero and $\alpha > 0$. Further simplification of equations (8) and (9) is not possible without assuming some form for the functions f, g and h.

Assume

$$\mathrm{Pr}_{P,Q}(P,Q) = \frac{1}{\sqrt{\pi\epsilon}} \exp\left(-\frac{(P-\alpha Q-\beta)^2}{\epsilon}\right) h\left(\frac{P+\alpha Q-\beta}{2\alpha}\right) \ . \tag{15}$$

The probability distribution must be normalised which implies (after changing variables to $u = P - \alpha Q - \beta$ and $v = (P + \alpha Q - \beta)/2\alpha$),

$$1 = \int h \ .$$

This probability distribution implies

$$\begin{aligned} \mathbb{E}(P-\alpha Q-\beta) &= 0 \ , \\ \mathbb{E}(P-\alpha Q-\beta)^2 &= \epsilon/2 \ . \end{aligned}$$

Hence, this probability distribution is describing the situation when, on average, P and Q are linearly related ($P = \alpha Q + \beta$), but their perfect correlation is spoiled by Gaussian white noise, with zero mean and variance $\epsilon/2$.

A useful result is that in the limit of small ϵ,

$$\begin{aligned} \int_{-\infty}^{\infty} \mathrm{d}x \ \frac{1}{\sqrt{\pi\epsilon}} \exp\left(-(x-x_0)^2/\epsilon\right) f(x) &= \int_{-\infty}^{\infty} \mathrm{d}y \ \frac{1}{\sqrt{\pi}} \exp\left(-y^2\right) f(x_0+\epsilon y) \\ &= f(x_0) + \tfrac{1}{4}\epsilon f''(x_0) + O(\epsilon^2) \ . \end{aligned}$$

for (integrable, etc) functions f. This may be used to expand $\mathrm{Pr}_{P,Q}$ in small ϵ so that it looks more like the previous definition for partially correlated variables, Eq (7):

$$\mathrm{Pr}_{P,Q}(P,Q) = \delta(P-\alpha Q-\beta)h(Q) + \frac{1}{4}\epsilon\delta''(P-\alpha Q-\beta)h\left(\frac{P+\alpha Q+\beta}{2\alpha}\right) + O(\epsilon^2) \ .$$

This motivates the use of the notation 'h' for the probability distribution function in equation (15) when comparing with the general expression of equation (7). It also demonstrates that ϵ is $O(\lambda)$ in the notation of the previous sections. We shall now perform a power series expansion in ϵ of equations (8) and (9).

The formula for VaR reads

$$0.01 = \int_{-\infty}^{(P_f-\beta+\mathrm{VaR}/\bar{Q})/\alpha} \mathrm{d}Q \left(h(Q) + \frac{\epsilon}{16\alpha^2}h''(Q) + O(\epsilon^2)\right) \ . \tag{16}$$

The formula for EaR reads

$$0.01 = \left(\int_{Q^+}^{\infty} \mathrm{d}Q + \int_{-\infty}^{Q_-} \mathrm{d}Q\right)\left(h(Q) + \frac{\epsilon}{16\alpha^2}h''(Q) + O(\epsilon^2)\right) \ , \tag{17}$$

where $Q_\pm$ is given in equation (12). Boundary terms encountered during the derivation of these expressions are zero through assuming a well behaved h. As mentioned before, it is likely that $Q_- < 0$, so for financially realistic cases with $h(Q) = 0$ for $Q < 0$, the second integral is zero (more generally, much smaller than the first integral).

4.5 Weakly positively correlated price and demand

In this case, the parameter λ in equation (7) is close to unity and $\alpha > 0$. Further simplification of equations (8) and (9) is not possible without assuming some form for the functions f, g and h. It is not possible to make the large ϵ expansion of equation (15), since this does not yield $\mathrm{Pr}_{P,Q}(P,Q) = f(P)g(Q)$ as the first term.

Instead, assume

$$\mathrm{Pr}_{P,Q}(P,Q) = f\left(P - \epsilon\alpha Q\right) g\left(Q - \epsilon P/\alpha\right) \ ,$$

with $\alpha > 0$. For $\epsilon = 0$ this reduces to the completely uncorrelated situation, $\mathrm{Pr}_{P,Q} = fg$. The probability distribution must be normalised. Upon changing variables, $u = P - \epsilon\alpha Q$ and $v = Q - \epsilon P/\alpha$, and expanding to first order in ϵ, the normalisation condition reads $1 = (\int f)(\int g)$. The functions f and g may be arbitrarily scaled without changing the form for $\mathrm{Pr}_{P,Q}$, so let us choose

$$\int f = 1 = \int g \ .$$

The normalisability also implies that both f and g vanish at $\pm\infty$, which shall be employed below when we use integration by parts.

Expanding to first order in ϵ gives

$$\mathrm{Pr}_{P,Q}(P,Q) = f(P)g(Q) - \epsilon\alpha Q f'(P)g(Q) - \epsilon\frac{P}{\alpha}f(P)g'(Q) + O(\epsilon^2) \ .$$

Hence

$$\begin{aligned}
\mathrm{Pr}_P(P) &= f(P) - \epsilon f'(P)\int \mathrm{d}Q\ \alpha Q g(Q) + O(\epsilon^2) \ , \\
\mathrm{Pr}_Q(Q) &= g(Q) - \epsilon g'(Q)\int \mathrm{d}P\ P f(P)/\alpha + O(\epsilon^2) \ ,
\end{aligned}$$

which yield

$$\begin{aligned}
\mathbb{E}(P) &= \int \mathrm{d}P\ P f(P) + \epsilon\alpha\mathbb{E}(Q) \ , \\
\mathbb{E}(Q) &= \int \mathrm{d}Q\ Q g(Q) + \epsilon\mathbb{E}(P)/\alpha \ ,
\end{aligned}$$

to first order. The expectation value of P depends on the expectation value of Q, and vice versa, so there is a weak positive correlation between the two variables.

VaR, equation (8), becomes

$$0.01 = \left(\int_{-\infty}^{P_f + \mathrm{VaR}/\bar{Q}} \mathrm{d}P\ f(P)\right) - \epsilon\alpha f(P_f + \mathrm{VaR}/\bar{Q})\mathbb{E}(Q) \tag{18}$$

EaR, equation (9), becomes

$$\begin{aligned}
0.01 \ =\ & \int_0^\infty \mathrm{d}Q\ g(Q)\int_{P_c - \mathrm{EaR}/Q}^\infty \mathrm{d}P\ f(P) + \int_{-\infty}^0 \mathrm{d}Q\ g(Q)\int_{-\infty}^{P_c - \mathrm{EaR}/Q} \mathrm{d}P\ f(P) \\
& -\epsilon\alpha\left(\int_{-\infty}^0 - \int_0^\infty\right)\mathrm{d}Q\ Q g(Q) f(P_c - \mathrm{EaR}/Q) \\
& -\frac{\epsilon}{\alpha}\left(\int_{-\infty}^{P_c} - \int_{P_c}^\infty\right)\mathrm{d}P\ P f(P) g(\mathrm{EaR}/(P_c - P))
\end{aligned} \tag{19}$$

Once again, the integrals over the positive domains dominate since it is likely that the functions f and g are small or zero for negative argument.

4.6 Discussion

In this main section of the paper, we have studied a number of situations in which VaR and EaR might be able to be related. Examining equations (10), (13), (16) and (18) demonstrates that VaR depends upon the *left* tail of a distribution. Conversely, equations (11), (14), (17) and (19) demonstrate that EaR depends upon the *right* tail of a distribution. Physically this corresponds to VaR measuring the likelyhood of a large *decrease* in derivative price (which is equal to spot price in this section), while EaR measures the likelyhood of a large *increase* in demand or spot price.

5 Conclusions

VaR and EaR are calculated from integral equations involving (joint) probability distribution functions for spot prices, forward prices and demands, as well as potentially other more exotic option prices. The MISG team was asked by Technology Trading Australia whether VaR and EaR were related in some way which might reduce time spent computing these risk measures while additionally making risk reporting more streamlined, transparent and risk-compliant.

In some cases the probability distribution functions involved are related and this implies some simplification of the computations required to extract the two measures of risk, since in real-life situations a large amount of computational time is spent using historical data to build the probability distribution functions. However, in all but the most trivial situations, such as symmetric probability distribution functions, this does not appear to imply simple algebraic expressions relating VaR to EaR.

Our main conclusion is that for electricity retailers, VaR and EaR are not related in any useful way. In rough financial terms, the VaR for an electricity retailer is sensitive to *drops* in (forward) prices, since then the value of a portfolio is reduced. However, the EaR is usually more sensitive to *increases* in electricity prices and demands since these will cause losses in earnings. This strongly suggests there is no simple map from VaR to EaR. Other markets may *not* share these features. However, in these cases any theoretical, simple relationships between VaR and EaR may be inaccurate in practice due to the large variety of rather indpendent stochastic variables that they depend upon.

Our recommendations for reducing time spent on computing VaR and EaR are:

- Identify the probability distribution functions required for VaR and Ear, and compute any that are common between VaR and EaR only once.
- It may be easier to use the stochastic set (demand, bid structure) than the set (demand, spot price) since in some cases the bid structure fluctuates only a little, or may even be considered fixed.
- In some cases it may be appropriate to assume the forward prices are equal to the spot prices, which will reduce the number of variables in the problem.
- The probability distributions would be simple to calculate if the underlying stochastic processes could be found and calibrated from historical data. In this method, it is the calibration (and continual recalibration required by regulators) that is difficult.

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